A Mixed Stream of Viscoelastic Liquid Through a Porous Medium is Situated in a Vertical Channel with Permeable Walls

L. Hari Krishna, A. Hemantha Kumar

Abstract: We examined the completely developed mixed convection flow of a visco-elastic fluid via a porous medium in a vertical channel with a permeable wall. The non-linear control equations have been resolved using the conventional disturbing method for the speed and temperature domain. Graphs will be used to detail the effects on speed and temperature of the viscoelastic Reynold number, the cross flow parameter, the number of Grashof , and Prandtl temperature.

Keywords: Viscoelastic fluid, porous medium, flow, vertical channel, mixed convection.

I. INTRODUCTION

Many recent papers have been published on the issue of convective fluid flow in saturated porous media. The interest in understanding pores material transport processes is growing thanks to the growth of geothermal technologies, high-quality insulating buildings and cold stores, increased interest in energy efficient drying methods. The nuclear industry also has an interest in the assessment of heat dissipation in hypothetical accidents and in the effective insulation of a nuclear reactor. None of us have examined the convective flux of the mixed viscoelastic fluid, fully developed in a permeable vertical flux through a porous fluid. In the vicinity of the porous medium, the flow of non-Newtonian liquids finds essential applications in improved oil extraction, filtration, insulation systems and development of composites, etc. Some of the studies [1] can be mentioned here. The combined effects of viscosity changes and convective cooling in an unstable nano-fluid circulation via a permeable tube were studied by Kamiset et al. [2] later, according to a Buongiorno method. In a vertical porous tube, Singh [3] investigated thermal radiation with a viscous-elastic sliding mixed MHD mixture. Idowu et al [4] studied the dynamic stream of MHD in an oblique magnetic field between the two infinite parallel flat surfaces. In a porously saturated porous channel, Falade et al. [5] analyzed the MHD oscillating present. Recently studied heat and mass transfers in the non-Newtonian MHD fluid on the infinitely vertical porous plate were made by Raghunat and Siva Prasad [6]. Saleh et al. [7], which focused on observations of reversal of convective flows.

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II. MATHEMATICAL FORMULATION

As shown in figure 1, we use the laminar convection stream of viscoelastic fluids in a permeable vertical flow through the porous matrix. The rate of injection on one wall shall equal the level of suction on the other wall. The x-axis has to be paralleled by a rectangular (x and y) coordination unit, but it crosses the walls of the channel opposite the x-axis. At a constant temperature of T_1 the left side (i.e. Y = 0), the right side of the wall (i.e. at y = h) is retained every time $T_1 > T_2$ is possible.

The stream is theoretically stable and fully developed, i.e. zero cruising speed. The continuity formula then comes down to $\partial u / \partial x = 0$.

Rivlin-Ericksen constitutive equation can be modeled on viscoelastic fluids

$$S = -pI + \mu A_1 + \alpha_1 A_2 + \alpha_2 A_1^2 \qquad 1$$

Scalar pressure p, μ, α_1 and α_2 surface constants, commonly referred to as viscosity, elasticity and cross-viscosity coefficients, are the places where the Cauchy stress sensor is found. The product constants of a particular liquid can be calculated by viscometric fluxes.

A_1 and A_2 are tensors from Rivlin-Ericksen, showing the degree of distortion and acceleration respectively. A_1 and A_2 are set by

$$A_1 = \nabla V + (\nabla V)^T \qquad 2$$

$$A_2 = \frac{dA_1}{dt} + A_1(\nabla V) + (\nabla V)^T A_1 \qquad 3$$

$$\mu \geq 0, \quad \alpha_1 > 0, \quad \alpha_1 + \alpha_2 = 0 \qquad 4$$

Visco-elastic liquids are called second-degree liquids when they are modeled by Rivlin-Ericksen. Dunn and Rajagopal are known for their detailed description of the properties of second-degree fluids. In the context of dissipative inequality (Clausius-Duhem), Rajagopal and Gupta [8] study thermodynamics and generally agree that Helmholtz's special free energy must be at least balanced. From the thermodynamic consideration that they assumed

According to the approach of Boussinesq, the basic equations of momentum and energy control such a stream

$$\rho v_0 \frac{du}{dy} = -\frac{dp}{dx} + \mu \frac{d^2u}{dy^2} + \alpha_1 v_0 \frac{d^3u}{dy^3} - \frac{\mu}{k_0} u + \rho g \beta (T - T_0) \tag{5}$$

$$v_0 \frac{dT}{dy} = \alpha \frac{d^2T}{dy^2} \tag{6}$$

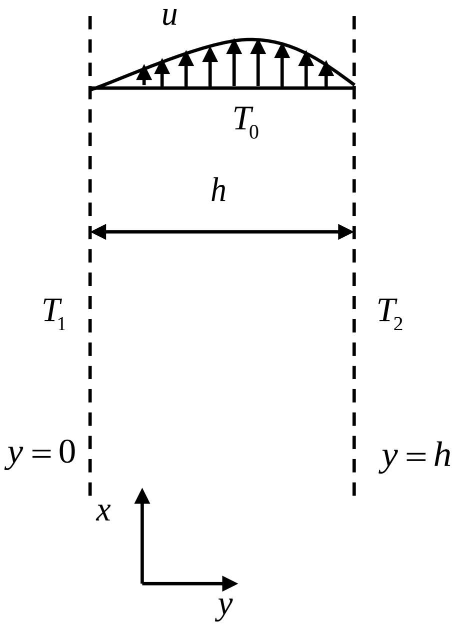

Fig.1 The physical model

The limits shall be determined by

$$u(0) = u(h) = 0, \quad T(0) = T_1, \quad T(h) = T_2 \tag{7}$$

Presentation of the following parameters

$$\bar{y} = \frac{y}{h}, \quad \bar{u} = \frac{u}{U}, \quad \theta = \frac{T - T_0}{T_2 - T_0}$$

We obtain

$$kR\frac{d^3u}{dy^3} + \frac{d^2u}{dy^2} - R\frac{du}{dy} - \frac{1}{Da}u + \frac{Gr}{\mathrm{Re}}\theta + A = 0 \tag{8}$$

$$\frac{d^2\theta}{dy^2} - R\,\mathrm{Pr}\frac{d\theta}{dy} = 0 \tag{9}$$

The appropriate dimensional limit conditions are defined by

$$u(0) = u(1) = 0, \quad \theta(0) = r_T, \quad \theta(1) = 1 \tag{10}$$

III. SOLUTION

The BVP solution of first order (4) - (6) is considered for the small ones. Because the formula implies that (1) is only extracted from the first order of magnitude, the disturbance solution must therefore be reasonably logical and reasonable if the terms remain the same magnitude. We write

$$u = u_0 + ku_1 \tag{11}$$

$$\theta = \theta_0 + k\theta_1 \tag{12}$$

By replacing and limiting equations (11) and (12) in (8) and (9) and assimilating them with comparable powers of, we obtain the

3.1 Zeroth-order system (k^0)

$$\frac{d^2u_0}{dy^2} - R\frac{du_0}{dy} - \frac{1}{Da}u_0 = -\frac{Gr}{\mathrm{Re}}\theta_0 - A \tag{13}$$

$$\frac{d^2\theta_0}{dy^2} - R\,\mathrm{Pr}\frac{d\theta_0}{dy} = 0 \tag{14}$$

Together with boundary conditions

$$u_0(0) = u_0(1) = 0, \quad \theta_0(0) = r_T, \quad \theta_0(1) = 1 \tag{15}$$

3.2 First-order system (k^1)

$$\frac{d^2u_1}{dy^2} - R\frac{du_1}{dy} - \frac{1}{Da}u_0 = -R\frac{d^3u_0}{dy^3} - \frac{Gr}{\mathrm{Re}}\theta_1 \tag{16}$$

$$\frac{d^2\theta_1}{dy^2} - R\,\mathrm{Pr}\frac{d\theta_1}{dy} = 0 \tag{17}$$

Together with boundary conditions

$$u_1(0) = u_1(1) = 0, \quad \theta_1(0) = 0, \quad \theta_1(1) = 0 \tag{18}$$

3.3 Zero th Order (or Newtonian Fluid Solution)

We get the solution of the equations (13) and (14) with the limit conditions (15),

$$\theta_0 = \frac{\left(1 - r_T e^{R\mathrm{Pr}}\right) + (r_T - 1)e^{R\mathrm{Pr}\,y}}{\left(1 - e^{R\mathrm{Pr}}\right)} \tag{19}$$

$$u_0 = c_1 e^{ay} + c_2 e^{by} + \frac{Gr}{\mathrm{Re}}\left(f_1 - f_2 e^{R\mathrm{Pr}\,y}\right) + ADa \tag{20}$$

3.4 First-order (or Second-grade solution-fluid)

Eq. Resolution. (3.7) we obtain with the appropriate limit conditions

$$\theta_1 = 0 \tag{21}$$

The formula (20) and (21) are replaced. (16) The resulting correlation with the appropriate conditions is then resolved

$$u_1 = c_3 e^{ay} + c_4 e^{by} - f_6 y e^{ay} - f_7 y e^{by} + f_5 e^{R\mathrm{Pr}\,y} \tag{22}$$

Finally, up to first order and are the disturbance solutions

$$\theta = \frac{\left(1 - r_T e^{R\mathrm{Pr}}\right) + (r_T - 1)e^{R\mathrm{Pr}\,y}}{\left(1 - e^{R\mathrm{Pr}}\right)} \tag{23}$$

$$u = (c_1 + kc_3 - kf_6 y)e^{ay} + (c_2 + kc_4 - kf_7 y)e^{by} + \frac{Gr}{\mathrm{Re}}\left(f_1 - f_2 e^{R\mathrm{Pr}\,y}\right) + kf_5 e^{R\mathrm{Pr}\,y} + ADa \tag{24}$$

Please note that our findings, and those from Aung and Worku were reduced.

IV. DISCUSSION OF THE RESULTS

In order to see the effects of $Da, k, R, \mathrm{Pr}, Gr, \mathrm{Re}$ and r_T on the velocity u, we have plotted Figs.2-10. Darcy number's Da effect on u for $A = 1$, $k = 0.02, r_T = 0.5, R = 5,$ Grashof number $Gr = 1, \mathrm{Pr} = 2$ and Reynolds number $\mathrm{Re} = 1$ is shown in Fig.2. The velocity u increases with the increase Da are noticed. Fig.3 shows the viscoelastic parameter k effect on for cross flow parameter $R=5$, $r_T=0.5$, $A=1$, $Da=0.1$, Grashof number $Gr=1, \Pr=2$ and Reynolds number $\mathrm{Re}=1$. With an increase in k the velocity u decrease is observed. The peak velocity is shifted from the edge by the increased value of the viscoelastic parameter. For's influence cross flow parameter R on u for $\Pr=2$, Grashof number $Gr=1$, $Da=0.1$, $r_T=0.5$, $k=0.02$, $A=1$, and Reynolds number $\mathrm{Re}=1$ is presented in Fig.4. The velocity is reduced when an increase cross flow parameter R is observed. Fig. 5 shows the effect of the number of Prandtl $\Pr$ on u for cross flow parameter $R=5, A=1$, $Da=0.1$, Grashof number $Gr=1, k=0.02$, $r_T=0.5$, and Reynolds number $\mathrm{Re}=1$. It is found that as the number of Prandtl increases, the acceleration increases. The effect of the number of Grashof on u for $Da=0.1, r_T=0.5$, $k=0.02, \Pr=2$, cross flow parameter $R=5$, $A=1$,and Reynolds number $\mathrm{Re}=1$, is presented in Fig.6. It is observed that as the number of Grashof increases the frequency increases.

Fig.7 shows the effect of the number of Reynolds on u for Grashof number $Gr=1, \Pr=2, k=0.02$, $Da=0.1, r_T=0.5$, cross flow parameter $R=5, A=1$ and Reynolds number $\mathrm{Re}=1$. It is noted that as the number of Reynolds increases, the velocity decreases. The effect of the parameter of the wall temperature r_T on u for Grashof number $Gr=1, \Pr=2$ $k=0.1$, $Da=0.1$, cross flow parameter $R=5$, $A=1$, and Reynolds number $\mathrm{Re}=1$ is shown in Fig.8. It is observed that with rising velocity u, r_T Increase. Fig.9 displays the temperature θ effect R for $r_T=0.5$ and $\Pr=2$. It is observed that, on the rise cross flow parameter R, the temperature θ decreases. The effect of the number of Prandtl on the temperature for $r_T=0.5$ and cross flow parameter $R=5$ is shown in Fig. 10. That's right. The temperature θ decreases as the amount of Prandtl number $\Pr$ increases. Fig.11 shows the effect of r_T on temperature effect for cross flow parameter $R=5$ and Prandtl number $\Pr=2$. The temperature θ is observed to rise with an increase in r_T.

V. CONCLUSIONS

In a vertical channel with permeable walls we investigated the fully developed mixed convective flux of viscoelastic through a porous medium. For the velocity and temperature fields, the non-linear equations are determined using traditional disturbance techniques. As it increases or decreases as it rises or decreases, the area of velocity increases. As it decreases with increase or decrease, the temperature range increases as well.

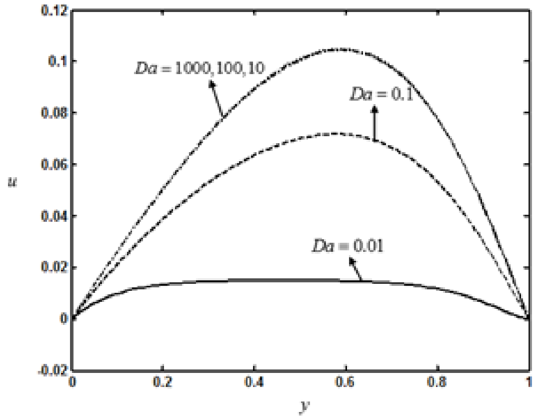

Fig. 2. Effect of Darcy number Da on u for $k=0.02, r_T=0.5, R=5$, $A=1$, $Gr=1, \Pr=2$ and $\mathrm{Re}=1$.

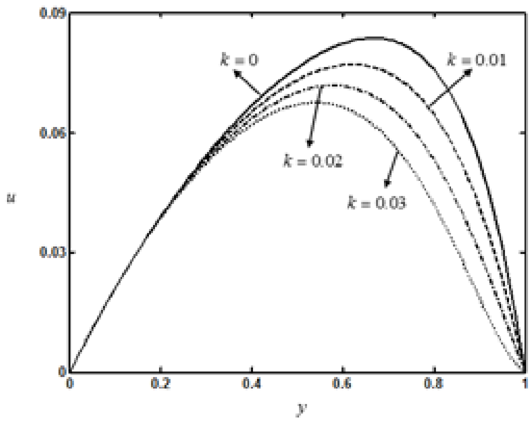

Fig. 3. Effect of viscoelastic parameter k on u for $Da=0.1, r_T=0.5$, $R=5$, $A=1$, $Gr=1, \Pr=2$ and $\mathrm{Re}=1$.

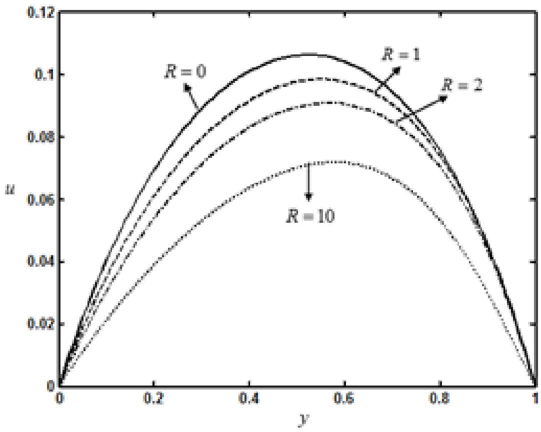

Fig. 4. Effect of R on u for $Da=0.1, r_T=0.5, k=0.02, A=1$ $Gr=1, \Pr=2$ and $\mathrm{Re}=1$.

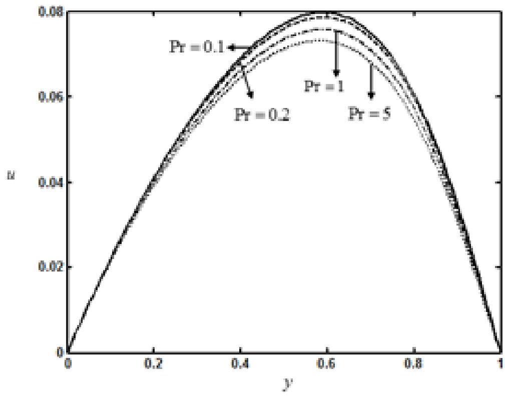

Fig. 5. Effect of Prandtl number Pr on u for $Da = 0.1, r_T = 0.5, R = 5, A = 1, Gr = 1, k = 0.02$ and $\mathrm{Re} = 1$.

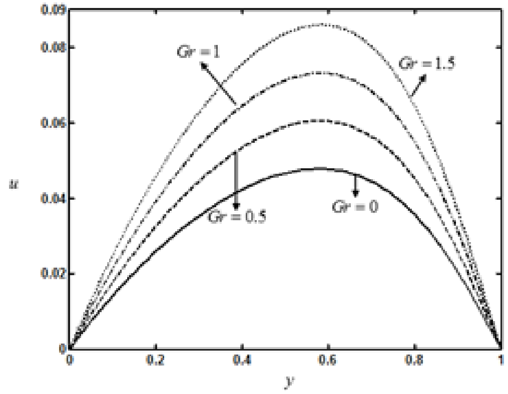

Fig. 6. Effect of Grashof number Gr on u for $Da = 0.1, r_T = 0.5, R = 5, k = 0.1, A = 1, \mathrm{Pr} = 2$ and $\mathrm{Re} = 1$.

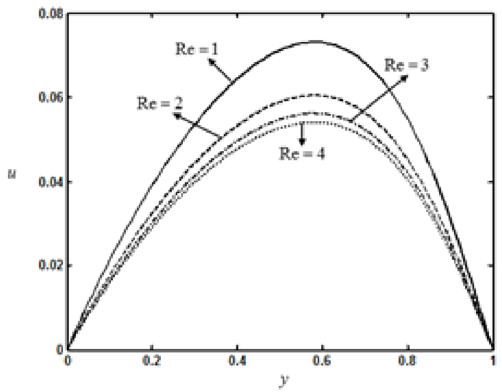

Fig. 7. Effect of Reynolds number Re on u for $Da = 0.1, r_T = 0.5, R = 5, Gr = 1, \mathrm{Pr} = 2, A = 1$ and $\mathrm{Re} = 1$.

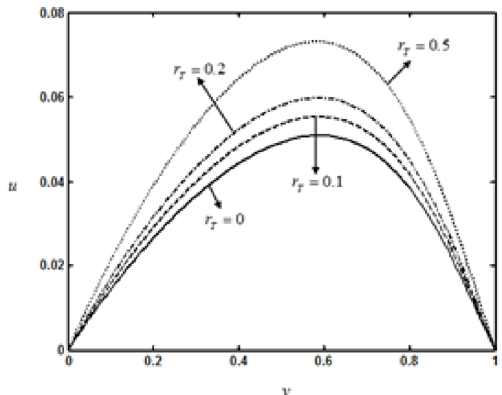

Fig. 8. Effect of wall temperature parameter k on u for $Da = 0.1, k = 0.1, R = 5, A = 1, Gr = 1, \mathrm{Pr} = 2$ and $\mathrm{Re} = 1$.

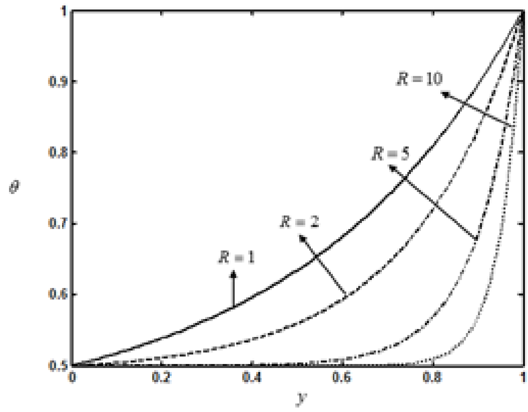

Fig. 9. Effect of R on θ for $r_T = 0.5$ and $\mathrm{Pr} = 2$.

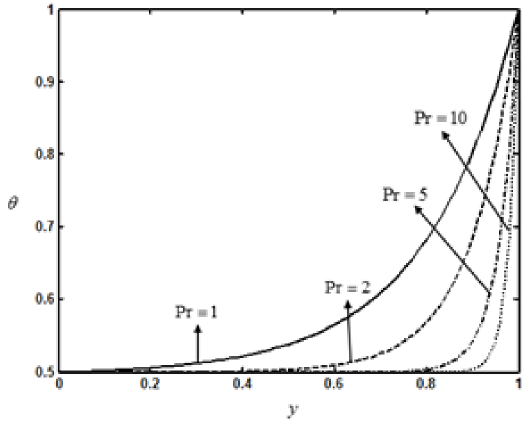

Fig. 10. Effect of Pr on θ for $r_T = 0.5$ and $R = 5$.

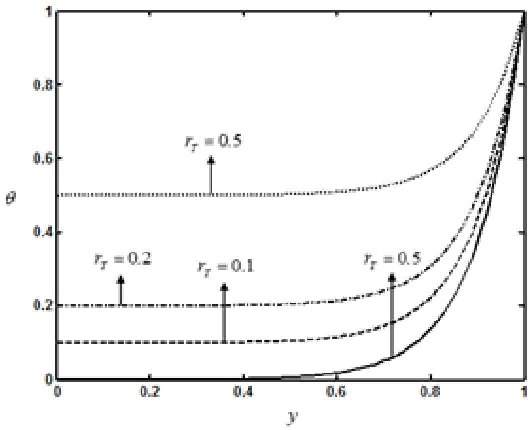

Fig. 11. Effect of r_T on θ for $R = 5$ and $\mathrm{Pr} = 2$.

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