

Locational Marginal Price Calculation Implying Different System Uncertainties

Mostafa Atef, R. A. Swief, T.S. Abdelsalam

Abstract: This paper presents locational marginal price (LMP) calculation technique facing different operation conditions. Uncertainties in power systems such as losing transmission line of load change could be consider a common case. In calculating LMP, the effect of line flow limits will affect the prices at each bus. Also, the effect of shifted the load from bus to another is investigated. LMP is a technique used in market to determine the cost of supplying the next incremental electrical energy at a specific bus. Four different case of LMP are discussed in this paper. Three and four simple bus networks are used to demonstrate different cases of LMP.

Keywords: Locational marginal pricing (LMP), offer price, Dispatch, transmission flow limit, congestion, Losses

I. INTRODUCTION

Locational marginal Pricing (LMP) is a technique used in electrical market to calculate the cost of supplying one incremental MW to load. It is also a method of determining prices in which market clearing prices are calculated for a number of locations on transmission grid called nodes or buses. [1] LMP methodology has been a dominant approach in energy market operation and planning to identify the nodal price and manage the transmission congestion. [2] Each bus or nodal is a location on the transmission at which energy is injected by generator or consumed by load. The LMP difference from bus to bus due to losses, transmission congestion. There are multi constrained that impact on LMP such as removal of transmission line, change in power flow through transmission line, change in transformer tap ratio and load MW increase. LMP affected with load characteristics, the cost of fuel, and transmission congestion and generation facility. [1]

LMPs is used also for co-optimization such ancillary service as reserves and regulation as well. Another definition of LMP is a change of cost generation to supply an incremental of MW at the location while satisfying all the constraints. [3] LMP is derived from AC OPF or DC OPF. It is also called a spot drive or a nodal price. [4] LMP calculated from the sum three component. The three components are marginal energy price, marginal congestion, and marginal loss price. [8] Electrical Power flows into all available paths to reach to consumption point from supply point.

The following components determine the Power flow distribution on transmission system: location of generation

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units and its magnitude, Paths impedance between load and generation unit. Power flows are managed by dispatching the system. LMP difference between any two buses or nodes, this due to the transmission cost between generation unit and load.[5] The day ahead market determine the LMP based on the dual variable of the of the optimal power flow. [6]. LMP has a significant role in maintaining the economic efficiency. Another definition for LMP is defined as the marginal cost that is required to supply electric power for incremental load at a certain bus that requires electrical power with achieving the balance of electric power system. [7]

This paper illustrates the calculation of LMP at each bus (node) for two system (3-bus system and 4-bus system). It also discusses the LMP for different cases. The cases are as following:

- No congestion (ignoring line flow limit) or losses
- With congestion (LMP considering line limit flow) and with no losses.
- LMP considering the losses of transmission line

The process and methodology of calculation as following

- Calculation of the cost of supplying electric power to the load according to the dispatch of each generator unit according to the following:
 - a- The offer price of each generator
 - b- The limit of each generator
- Check that power flow that must achieve the constrains of power system to avoid the exceeding of line capacity limit as following:
 - a- Calculate the power flow at each line according to the impedance of each line and the ratio between the impedance of lines
 - b- check the limit of each line
 - c- If the power flow exceeds the line capacity, redispatch of the generator units
- Determine LMP by calculating the cost of supplying additional 1MW at each bus .[1]
- Repeat the calculation of power flow after increasing of 1MW as discussed before.

II. THREE BUS SYSTEM LMP CALCULATIONS

The data of system is in Fig. 1 Shows that the cost of electrical power from station G1 is cheaper than Station G2. As illustrate in Fig.1 that the load of 150MW at bus3 and connected to bus1 and bus2 through line2 and line3 respectively. The impedance of all transmission lines is equal and losses are ignored. So, if G1 that has the cheaper cost supply the load at bus 3, the impedance path of line 1 and 3 has two times the impedance for path of line 2.



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The flow divides inversely to impedance. The power flow of line 2 is two times line 1 and 2. The plan of any market is to obtain the minimum cost with achieving the security of system.

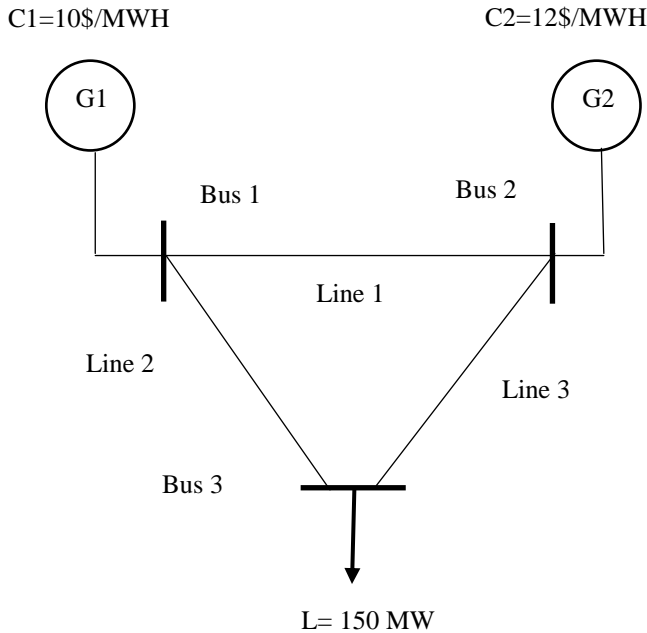


Fig. 1 Three Bus System Topology [1]

Case 1: the calculation of LMP ignoring line flow limit.

Based on the generation offer of stations G1 and G2, the G1 station will supply all MW of the load at bus 3 because G1 is the cheaper. The total cost of supplying 150 MW load at bus 3 is $150 \text{ MWh} * 10\$/\text{MWh} = 1500 \$$

As all lines have equal impedance, line1 and line3 has two times the impedance of line2. Since, the power flow divides inversely to impedance. And hence the power flow in path of line1 and line3 is half power flow in line2. So, 100 MW flows line 2 and 50 MW flows line 1 and 2.

The calculation of LMP at each bus is the cost supplying an additional load of 1MW at each bus. [8] So, the LMP at each bus is 10 \$ since the system is unconstrained, i.e. $LMP_1 = LMP_2 = LMP_3 = 10\$/\text{MWh}$. Table.1 shows the results of case1.

Table.1 the results of case 1 when ignoring line flow limits

The total cost of supplying load	LMP ₁	LMP ₂	LMP ₃
1500 \$	10\$/MWh	10\$/MWh	10\$/MWh

As shown in table1, LMP are the same for all buses when transmission limit is not exceeded and when losses are zero. [9]

Case 2: the capacity of Line 1 is 30 MW and line 2 & 3 is 150 MW. Dispatch can be solved by supplying 120 MW of load from G1 because the capacity of line 1 is limited and supply the remaining load 30 MW from the more expensive station G2. The cost of supplying all 150MW of load is 1560\$ as illustrates in the next equation.

$$120 \text{ MW} * 10\$/\text{MWh} + 30 \text{ MWh} * 12\$/\text{MWh} = 1560\$$$

The constrained flows are different from the case1 and hence the LMPs at each bus different from the first case.

The calculation of LMP at bus 3 is calculated by increase 1MW at bus 3. The extra 1MW would not supply by G1 due to the constrained flow of Line1. And an increase of 1MW by either G1 or G2 alone will increase the flow of line1 over limit. So, the system must be redispatch using both generators. This redispatch will give a different LMP at each bus. In order to calculate LMP₃, ΔP_1 will supply by G1 and ΔP_2 will supply by G2 these to give additional load 1MW at bus3. As the pervious assumption in case1 that all lines have the same reactance, so $2/3 \Delta P_1$ that supply by G1 would reach the load through line 2 and $1/3 \Delta P_1$ through line 1 and line 3. Also, ΔP_2 will be divided by $2/3 \Delta P_2$ would flow on line 3 and $1/3 \Delta P_2$ on line 1 and then line2. As the line1 reach to maximum limit of capacity, so $1/3 \Delta P_1 - 1/3 \Delta P_2 = 0$. In addition, in order to supply additional load of 1MW at bus3, $\Delta P_1 + \Delta P_2 = 1\text{MW}$. And hence to keep the flow on line1 not exceed the permissible capacity and for economic dispatch. So, by solving the next two equations:

$$1/3 \Delta P_1 - 1/3 \Delta P_2 = 0 \text{ ----- (1)}$$

$$\Delta P_1 + \Delta P_2 = 1\text{MW} \text{ ----- (2)}$$

0.5MW of incremental load at bus 3 is served by unit G1 and 0.5MW by unit G2.so, $\Delta P_1 = \Delta P_2 = 0.5 \text{ MW}$ as shown in Fig.2 and hence, LMP₃ is calculated as following:

$$LMP_3 = 0.5 * 10\$/\text{MWh} + 0.5 * 12\$/\text{MWh} = 11\$/\text{MWh}.$$

Similarly, in order to calculate LMP1. The extra 1MW will supply by G1 without affecting on line 1(net flow change is zero) and since G1 is the cheaper offer price. So, $LMP_1 = 10 \text{ }/\text{MWh}$. And also, LMP2 calculated with the technique. $LMP_2 = 12\$/\text{MWh}$.

Table.2 the results of case 2 when the capacity limit of Line1 is 30 MW

The total cost of supplying load	LMP ₁	LMP ₂	LMP ₃
1560 \$	10\$/MWh	12\$/MWh	11\$/MWh

As illustrated in the table.2 and Fig2. Illustrate that LMP₃ is higher than the marginal cost of cheaper station G1 due to the congestion of line1

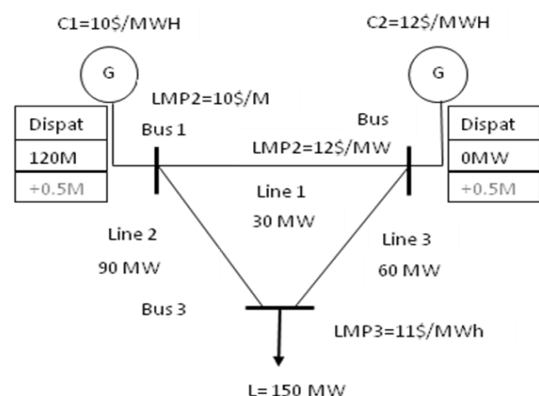


Fig.2 Case 2 constrained condition

Case 3: by using the same system shown at Fig1. And set the capacity of line 2 equal to 80 MW and those of line1 and line 3 to 100 MW. So, according to limit capacity of line2,

$$P_1+P_2=150 \quad \text{----- (3)}$$

$$2/3P_1+1/3P_2=80 \quad \text{----- (4)}$$

By solving eqn.3&4, $P_2 = 60\text{MW}$ & $P_1=90\text{MW}$. Since P_1 is supplied by G_1 and P_2 by G_2 . The total cost of supplying load at bus3 = $90\text{MW} \times 10\$/\text{MWh} + 12\text{MW} \times 60\$/\text{MWh} = 1620\text{\$}$.

The cost of supplying all 150MW of load at bus3 is increased from case1 and case2 due to the limit capacity of line2 as illustrated in the above equation.

LMP_3 is calculated with the same way that has been illustrated in case2. Eqn.5 and eqn.6 should be satisfied to supply the increase of 1MW at bus3.

$$2/3\Delta P_1+1/3\Delta P_2=0 \quad \text{----- (5)}$$

$$\Delta P_1+\Delta P_2= 1\text{MW} \quad \text{----- (6)}$$

By solving eqn.5&6, $\Delta P_1= -1\text{ MW}$ and $\Delta P_2=2\text{ MW}$. So, G_1 should reduce its output by 1 MW and G_2 should increase its output by 2 MW.

$$LMP_3=(-1\text{MW} \times 10\$/\text{MWh})+(2\text{MW} \times 12\$/\text{MWh})=14\$/\text{MWh}$$

It is obvious from this case that LMP is greater than the offer price of the most expensive unit G_2 .

Table.3 the results of case 3 when the capacity limit of Line1 &3 is 100 MW and Line 2 is 80 MW

The total cost of supplying load	LMP_1	LMP_2	LMP_3
1620 \$	10\$/MWh	12\$/MWh	14\$/MWh

As illustrated from table.3 that an incremental load at bus3 will obstruct the usage of the cheaper station G_1 due to its more positive contribution to flow from bus 1 to bus 3 on the constrained line 2 than that from the more expensive station G_2 . In order to supply the additional load of 1 MW at bus 3 and maintain the system security, the generation cost has to be up by 14\$.

Case 4: In this case, the load of 150 MW is shifted from bus3 to bus2. Also set the capacity of line3 to 30MW and those of line1 and line2 to 100MW. Since, the cheaper station G_1 cannot supply all load at bus2 due to the limit capacity of line2. Hence, G_1 and G_2 will share the supplying of load at bus2. And also, there is no flow from G_2 in line 2. So,

$$1/3P_1 = 30 \quad \text{----- (7)}$$

$$P_1+P_2= 150 \quad \text{----- (8)}$$

By solving eqn.7& 8:

$$P_1 = 90\text{ MW} \quad P_2 = 60\text{ MW}$$

The total cost of supplying load of 150 MW at bus2= $90\text{MW} \times 10\$/\text{MWh} + 60\text{MW} \times 12\$/\text{MWh} = 1620\text{\$}$

As obvious that the total cost increased when the load is shifted at bus3 and lin3 is limited .To calculate LMP_3 (incremental increase of 1MW at bus3), the following condition should be satisfied for economic dispatch:

$$\Delta P_1+\Delta P_2= 1\text{MW} \quad \text{----- (9)}$$

$$1/3\Delta P_1+2/3\Delta P_2=0 \quad \text{----- (10)}$$

By solving eqn.9&10, $\Delta P_2= -1\text{ MW}$ and $\Delta P_1=2\text{ MW}$. So, G_2 should reduce its output by 1 MW and G_1 should increase its output by 2 MW.

$$LMP_3=(2\text{MW} \times 10\$/\text{MWh})+ (-1\text{MW} \times 12\$/\text{MWh}) =8\$/\text{MWh}$$

It is clear from this case that LMP_3 is less than the cheapest station G_1 . This case illustrates that an incremental load bus3 will improve the usage of the cheaper station G_1 due to less negative contribution to flow between bus3 and bus 2 on the constrained line3 than that from the more expensive unit G_2 . In order to supply the additional load of 1 MW at bus3 and maintain the system constraints, the generation cost is increased by 8\$.

Table.4 the results of case 4 when the load shifted from bus3 to bus 2

The total cost of supplying load	LMP_1	LMP_2	LMP_3
1620 \$	10\$/MWh	12\$/MWh	8 \$/MWh

II. COMPARISON OF THE RESULT OF FOUR CASES

The results of different cases illustrate that LMP impact by the constraints of transmission line. Table.5 compare the different above cases and effect of constraints on transmission line.

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Table.5 Comparison of the result of four cases

<i>Case / LMP = \$/MWh</i>	<i>LMP₃</i>	<i>LMP₂</i>	<i>LMP₁</i>	<i>conclusion</i>
- Ignoring line flow limit. Unconstrained case. - Load at bus3=150MW	10	10	10	- LMP is same for all buses - Load is supplied only by the cheaper unit G1
- Considering line flow limit The capacity of Line1=30MW Line2=150MW Line3=150MW -Load at bus3=150MW	11	12	10	-LMP is different - Load is supplied by G1 and G2
- Capacity of Line1=100MW Line2=80MW Line3=100MW -Load at bus3=150MW	14	12	10	-LMP ₃ is greater than the offer price of most expensive station G2 -An incremental of load at bus 3 will obstruct the usage of the cheaper station G1
The capacity of Line1=100MW Line2=100MWLine3=30MW -Load is shifted from bus3 to bus 2=150MW	8	12	10	-LMP ₃ is less than the offer price of the cheapest station G1 - This case illustrates that an incremental load bus 3 will improve the usage of the cheaper station G1

III. FOUR BUS SYSTEM LMP CALCULATIONS

In the next example, four bus system has two generation unit. G1 and G2 have 125MW generation limit. All lines have the same reactance. Path of line4, line3 and line2 has three times the reactance of path line1. Flow divides inversely to impedance. G1 has the cheaper offer of generation as shown in Fig.3. If G1 supply load at bus2, flow of path line4, line3 and line4 is one third flow of path line1. If G1 supply load of 100 MW at bus2, 75 MW flow on path of line1 and 25 MW flow the path of line4, line3 and line2. [10]

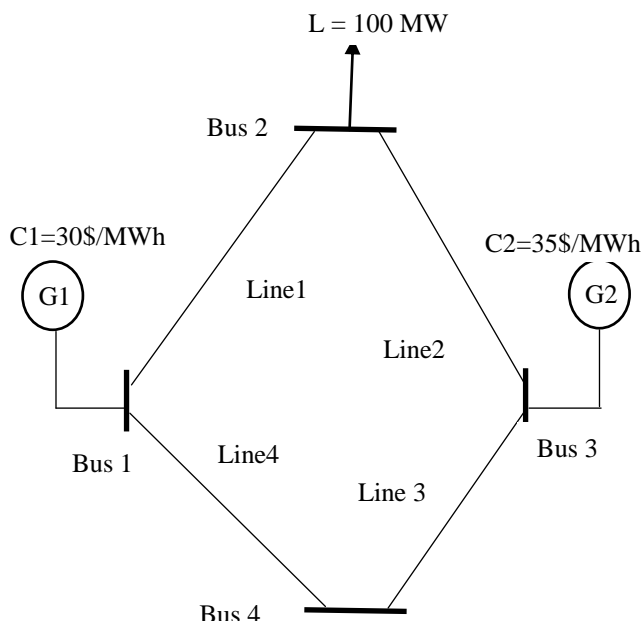


Fig.3 Four Bus System Topology. [10]

Case 1: No congestion and no losses. Transmission limit of line1 = 85MW

Based on G1 offer price, G1 supply all 100 MW at bus2. And also, this is under the generation limit of G1 and within the transmission limit of line1. According to the assumption that all lines have the same reactance as discussed above, so the power flow in line is 75MW and 25MW flow in path of line4, 3 and 2. As discussed previously, that LMP is the cost of serving the next MW. LMP2 is the cost of supplying next 1MW to bus2. And also based on the generation offer price, G1 supply the next MW to bus2 since it is within limit of generation G1 and within limit of line1. LMP2 = 30\$. Price at bus1 is the cost of supplying next 1MW to bus1 and based on offer price G1 supply the incremental MW on bus1 and net flow change in line 1 is zero. So, LMP1 = 30\$. Price at bus3 is the cost of supplying next 1MW to bus3. By the same way, G1 supply the next MW to bus3 based on the offer price. This is resulting incremental flow in Line1, line 4 and line3 by 0.5MW and reduction flow in line2 by 0.5MW. Price at bus4 is the cost of supplying next 1MW to bus4. G1 supply the next MW to bus4 based on the offer price. LMP4 = 30\$.

Table.6 the results for 4 bus system in case 1 when no congestion and no losses

The total cost of supplying load	LMP ₁	LMP ₂	LMP ₃	LMP ₄
\$/MWh	\$/MWh	\$/MWh	\$/MWh	\$/MWh
3000 \$	30	30	30	30

It is illustrates from table.6 that in the absence of losses and congestion, all buses have the same LMP.

Case 2: congestion and no losses. Transmission limit of line1 = 75.2MW

Assume the transmission limit of line is reduced. Dispatch can be solved as in the no congestion case and with the same concept that G1 supply all load at bus2 with the same power flow in lines since it is the power flow in line1 is less than the transmission limit. LMP₂ is the cost of supplying an incremental of 1MW at bus2. An increase of 1MW by either G1 or G2 alone will increase the line1 flow over the limit, so the system must be redispatch using both generators according to the following equations.

$$\Delta P_1 + \Delta P_2 = 1\text{MW} \quad \text{----- (11)}$$

$$3/4\Delta P_1 + 1/4\Delta P_2 = 0.2\text{MW} \quad \text{----- (12)}$$

By solving eqn.9 &8, $\Delta P_1 = -0.1$, $\Delta P_2 = 1.1$

So, G1 must be reduced by 0.1 and hence 75% of 0.1MW will appear on line1. G2 supply by 1.1MW and hence 25% of 1.1 will appear on line1. So, the net effect on line 1 is a flow increase by 0.2 from G1 and G2.

$$LMP_2 = 1.1\text{MW} * 35\$/\text{MWh} - 0.1\text{MW} * 30\$/\text{MWh} = 35.5\$\$$

LMP₃ can be calculated by the following eqn.

$$\Delta P_1 + \Delta P_2 = 1\text{MW} \quad \text{----- (13)}$$

$$1/2\Delta P_1 + 0 = 0.2 \quad \text{----- (14)}$$

$$\Delta P_1 = 0.4\text{MW}, \Delta P_2 = 0.6\text{MW}$$

$$LMP_3 =$$

$$0.6\text{MW} * 35\$/\text{MWh} + 0.4\text{MW} * 30\$/\text{MWh} = 33\$\$$

LMP₄ can be calculated by the following eqn.

$$\Delta P_1 + \Delta P_2 = 1\text{MW} \quad \text{----- (15)}$$

$$1/4\Delta P_1 - 1/4\Delta P_2 = 0.2 \quad \text{----- (16)}$$

$$\Delta P_1 = 0.9\text{MW}, \Delta P_2 = 0.1\text{MW}$$

$$LMP_4 = 0.1\text{MW} * 35\$/\text{MWh} + 0.9\text{MW} * 30\$/\text{MWh} = 30.5\$\$$

Calculation of LMP₁ is different because G1 can supply the next MW to bus1 based on the offer price G1 and without affecting on the transmission limit of line 1 because the net flow change in line1 is zero.

Table.7 the results for 4 bus system in case 2 when considering congestion and no losses

The total cost of supplying load	LMP ₁ \$/MWh	LMP ₂ \$/MWh	LMP ₃ \$/MWh	LMP ₄ \$/MWh
3000 \$	30	35.5	33	30.5

As obvious from table.7 that bus2, 4 and 4 affected by the congestion of line 1 and bus1 not affected by the limit capacity of line1.

Case 3: Losses and no congestion

Assume that the losses of above system in Fig.3 is equal to 4MW. And hence G1 supply load at bus2 due to its lower offer price but due to losses G1 must generate 104MW. The price at bus2 is the cost of supplying next 1MW. G1 must generate an additional 1.04MW to bus2 to deliver 1MW to load at bus2. LMP₂ = 1.04MW * 30\$/MWh = 31.2\$. Price at bus3 and bus4 would be similarly calculated. LMP₃ = LMP₄ = 31.2\$. LMP₁ = 30\$ as an increment of load can be supplied from G1 with no impact to transmission flows.

Table.8 the results for 4 bus system in case 3 when considering losses and no congestion

The total cost of supplying load	LMP ₁ \$/MWh	LMP ₂ \$/MWh	LMP ₃ \$/MWh	LMP ₄ \$/MWh
3120 \$	30	31.2	31.2	31.2

It is illustrated from the results in table.8 that the losses affected on the total cost of supplying load because G1 must generate extra 4MW to cover the losses.

IV. CONCLUSION

The concept of LMP is used to determine the cost of next incremental 1MW. The constraints and losses of the system occurs on the calculation of LMP. LMP can be calculated by using the nodal power balance equations. When neglecting the congestion and losses, the LMP at each bus is the generation cost of the cheaper generation unit. When taken the congestion into account and neglecting the losses of transmission line, LMP components depend on the location of the reference bus and the price may be higher than the cheaper generation unit. Also, LMPs differ from buses to another depend on the location of bus. In the studied cases, constrained flows of transmission line could obstruct the cheaper unit to supply all load. When more than one generator is on the margin, prices may be higher than any offer or lower than any offer. Also, in case of considering losses, the generation unit must generate excess power to cover the losses. And hence, the total cost of generation is increased due to the cost of supplying the addition power to cover the losses. Finally, LMP acts as a price indicator for both losses and congestion, it would provide an efficient mechanism for managing the competitive wholesale energy market.

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