Modeling Worm Proliferation in Wireless Sensor Networks with Discrete Fractional Order System

A. George Maria Selvam, S. Godfrey Winster, R. Janagaraj, G. Maria Jones

Abstract*: Wireless sensor networks (WSNs) are at risk to cyber attacks and thus security is of vital concern. WSN is a soft target for worm attacks due to fragile defence mechanism in the network . A single unsecured node can essentially propogate the worm in the complete network via communication. Mathematical epidemic models are useful in the study of propagation of worms in WSNs. This work considers a fractional order discrete model of attacking and spreading dynamics of worms in WSNs of the form The proposed epidemic model is probed with the assistance of stability theory. Basic reproduction number (R_0)is determined for the analysis of the dynamics of worm propagation in WSNs. The equilibrium states are computed and analyzed the stability. Basic reproduction number R_0 enables to discover the threshold values for communication radius and node density distribution. If reproduction number is less than one, the worm free equilibrium state (WFE) is locally asymptotically stable (LAS) and if reproduction number is more than one then the endemic equilibrium state (EE) is asymptotically stable. Numerical illustrations affirm the consistency of the theoretical analysis and stimulating dynamical behavior of the system is observed.

***Keywords:* Epidemic model, fractional order, locally asymptotically stability, reproduction number, wireless sensor network, worm propagation.**

$$S(\kappa+1) = S(\kappa)+\frac{h^{\alpha}}{\Gamma(1+\alpha)}\left[\mu-\beta S(\kappa)I(\kappa)-\mu S(\kappa)-\omega S(\kappa)+\varepsilon R(\kappa)\right]$$

$$I(\kappa+1) = I(\kappa)+\frac{h^{\alpha}}{\Gamma(1+\alpha)}\left[\beta S(\kappa)I(\kappa)-\mu I(\kappa)-\gamma I(\kappa)\right]$$

$$R(\kappa+1) = R(\kappa)+\frac{h^{\alpha}}{\Gamma(1+\alpha)}\left[\omega S(\kappa)+\gamma I(\kappa)-\mu R(\kappa)-\varepsilon R(\kappa)\right]$$

I. INTRODUCTION

Wireless Sensor Networks (WSN) have potential applications in defense, commercial, industrial and domestic areas. A WSN comprises of nodes usually called devices or sensors. Each node has the ability to their own establish and the goal is to collect, monitor and control the data. The nodes progression the collected data, store and assign it to other devices for further action. WSN operates vastly in hostile inaccessible areas and the data gathered by the sensor nodes might be extremely sensitive and valuable. Weak defense capabilities in a WSN due to limited power resources makes it vulnerable to worm attacks targeting at hijacking the data and yet times to destroy the network.

Epidemiological models have been widely considered and used by researchers to analyse the propagation of malwares in wireless networks as it holds similarity with the spread of disease in the population [2, 4]. Compartmental models which are quite effective in investigating the spread of epidemics assume that the model can be split into following three compartments: (i) susceptible population (S), who can be infected, (ii) infected population (I) and (iii) recovered or removed population (R) who can no longer transmit or contract the infection.

Worm attacks hold maximum threat to the safty and functionality of a WSN. Mathematical Modeling plays a very important role in epidemiological aspect of computer virus/worm propagation in networks. Mathematical models of propagation are advantageous in comprehending the patterns of transmission of worms from one node to another and controlling the transmission.

Epidemiology holds a central field of research in population dynamics. Numerous reserchers have modelled and investiagted the dynamics of infectious diseases. Formulation of epidemic models using differential equations by Kermack and Mckendrick [9] laid the basis for the development of epidemiological mathematical models. Compartmental models are based on the assumption that every individual is in one of several predefined compartments with regards to its infection status [10].

The paper is organised as follows. The subsequent segment presents the correlated work on computer worm propagation. Section III describes worm propagation model and by means of a discretization process, discrete fractional order system is obtained. Existence and local stability of the equilibrium states is discussed in section IV. Sections V and VI provide numerical examples to demonstrate the theoretical analysis with suitable parameter values and a brief conclusion is given.

II. RELATED WORK

Ossama et al [11] examined vulnerable - exposed - infectious - secured - vulnerable model for worm attack and it was effective to measure the impact of security processes on worm propagation. With the use of reproduction rate, the authors analyzed global stability of a worm-free state and local stability of a unique worm-epidemic state. In [12], authors investigated a SLBS propagation model with delay and incomplete antivirus ability and discussed the existence of local as well as global stability of the virus-free and virus equilibria, respectively.

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Paper [13] proposed the interaction dynamics between a malicious worm and an benign worm, considering a- SEIAR (susceptible - exposed - infectious - benign worm - recovered) mathematical model.

Paper [8] developed a refined SEIRS (Susceptible - Exposed - Infectious - Recovered - Susceptible) computer worm propagation model. They found worm-free equilibrium state and worm-endemic equilibrium state and analyzed its stability conditions. The paper also analysed control strategies based on the basic reproduction number R_0. A SIQRS model [3] describes the dynamics of worm propagation in a wireless network. The authors obtained reproduction number and equilibrium states, stability at varied equilibrium states was analyzed with numerical simulations.

System of Fractional order differential equations are used in article [5]. This paper develops propagation behavior of computer virus with classical SIR epidemic model under human intervention and discussed the spread of computer virus across the Internet. [7] considered a SIQR(susceptible-infected - quarantined - recovered) model of fractional order to study worm proliferation in WSN with quarantine. Also basic reproduction number R_0 is calculated and is used in the analysis of the spread of worms in the network.

III. WORM PROPAGATION MODEL

A worm is a stand alone malware computer program that can use scanning techniques to penetrate a computer network. A computer worm replicates or copies itself and does not require a host or human intervention to propagate to other machines in a network. An infected network can collapse within a matter of hours resulting in the loss of data, assets and money.

Globally 4.021 billion people use Internet and malicious programs propagate through Internet. Recent decades have witnessed several types of viruses and worms causing serious damage to IT industries world wide. Today WSN is available for domestic purposes like weather monitoring, target tracking, environmental/earth sensing, pollution monitoring, forest fire detection, landslide detection and so on. Due to its application in a wide variety of branches and its inexpensive infrastructure, WSN is gaining popularity. WSN is weak in security aspect because of their limited power consumption and this makes it a soft target for cyber attack.

Validated by the present epidemic model [6], the nodes in WSNs are classified into three states. Let us consider the following worm propagation model

$$\begin{aligned}\frac{dS}{d\kappa} &= \mu - \beta S(\kappa)I(\kappa) - \mu S(\kappa) - \omega S(\kappa) + \varepsilon R(\kappa)\\ \frac{dI}{d\kappa} &= \beta S(\kappa)I(\kappa) - \mu I(\kappa) - \gamma I(\kappa) \\ \frac{dR}{d\kappa} &= \omega S(\kappa) + \gamma I(\kappa) - \mu R(\kappa) - \varepsilon R(\kappa)\end{aligned} \tag{1}$$

In this model , it is assumed that N(*k*)=1 such that $S(\kappa)+I(\kappa)+R(\kappa)=1$.

- Susceptible state (S) : nodes which are vulnerable to worms attack but uninfected in WSNs.
- Infected state (I) : nodes infected by worms and capable of infecting other nodes in WSNs.
- Recovered state (R) : disinfected nodes or nodes damaged by worms and isolated from the network.

Transfer diagram for the transitions states of the nodes of model (1) is shown in Figure 1. The notations used in this model are explained in Table I.

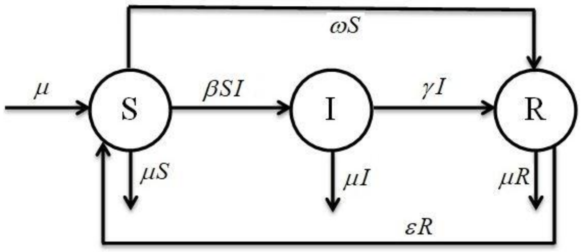

Fig.1 Example of a figure caption

Table I. Notation of the model

S. No	Notation	Meaning
1	β	Effective infection rate of S and I
2	μ	Probability of death nodes when the energy of nodes is exhausted
3	ω	Probability of immunity rate of S
4	γ	Probability of immunity rate of I
5	ε	Probability of some recovered nodes

A. Model Formulation

The fractional order systems have risen to prominence due to its distinct features namely they are realistic and practical in their approach, it has non-local property, as it takes into consideration the past and distributed effects of the model which the integer order differential equations lack, and these systems are convenient to model biological systems with memory effect. The non-local property implies that the subsequent status of the model depends not only upon its present stage but also upon all its previous stages.

In [6], the authors investigated the worm propagation in a WSN by formulating continuous time model and analyzed its stability by determining the equilibrium states. This paper examines a discrete fractional order model of attacking and spreading behavior of possible worms in WSNs. In order to harness the advantage of the properties of fractional order systems, fractional order is introduced in (1) to yield the following system

$$\begin{aligned}D^{\alpha} S(\kappa) &= \mu - \beta S(\kappa)I(\kappa) - \mu S(\kappa) - \omega S(\kappa) + \varepsilon R(\kappa)\\ D^{\alpha} I(\kappa) &= \beta S(\kappa)I(\kappa) - \mu I(\kappa) - \gamma I(\kappa) \\ D^{\alpha} R(\kappa) &= \omega S(\kappa) + \gamma I(\kappa) - \mu R(\kappa) - \varepsilon R(\kappa)\end{aligned} \tag{2}$$

here α is the fractional order $\alpha \in (0,1]$ and ${}_aD_t^{\alpha} f(t) = \frac{1}{\Gamma(1-\alpha)}\int_a^t \frac{f^{(n)}(\tau)}{(t-\tau)^{\alpha-n+1}}d\tau$, for $n-1 < \alpha < n$ (Caputo, 1967) is the standard Caputo definition of fractional derivatives. There are several methods to construct the discrete time model from continuous time model such as Runge - Kutta method, Euler's method, predictor-corrector method, and other finite difference methods. In [1] a discretization process was introduced for the discretization of FDEs. This is an estimate for the right hand side of the FDE $D^{\alpha}x(t) = f(x(t)), t>0, \alpha \in (0,1)$. This method is now applied to the model (2) of fractional-order worm propagation in WSNs. Consider that $S(0)=S_0, I(0)=I_0$ and $R(0)=R_0$ are the initial conditions of system (2).

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In applied mathematics, the process of transforming continuous models described by differential equations into its discrete counterpart is termed as discretization. The discrete version of the model is more suitable for implementing numerical methods and computer simulations. The system (2) is discretized with piecewise constant arguments and initial conditions. Repeating the discretization process *k*-times, system (2) is reduced to the resulting form:

$$\begin{aligned} S(\kappa+1) &= S(\kappa)+\frac{h^{\alpha}}{\Gamma(1+\alpha)}[\mu-\beta S(\kappa)I(\kappa)-\mu S(\kappa)-\omega S(\kappa)+\varepsilon R(\kappa)] \\ I(\kappa+1) &= I(\kappa)+\frac{h^{\alpha}}{\Gamma(1+\alpha)}[\beta S(\kappa)I(\kappa)-\mu I(\kappa)-\gamma I(\kappa)] \\ R(\kappa+1) &= R(\kappa)+\frac{h^{\alpha}}{\Gamma(1+\alpha)}[\omega S(\kappa)+\gamma I(\kappa)-\mu R(\kappa)-\varepsilon R(\kappa)] \end{aligned} \quad (3)$$

where $\alpha \in (0,1]$ and $h > 0$ is defined as time interval of production. Further, Euler discretization of SIR worm propagation model is obtained if $\alpha \to 1$ in (3).

B. Existence and Uniqueness

Sufficiency condition for existence & uniqueness of the solution of system (2) are provided as follows:

THEOREM 1.

In the specified region $\Omega \times (0,T]$, sufficient condition for existence and uniqueness of the solution of system (2) is

$$\Delta = \max\{\mu+2[\beta\lambda+\omega], \mu+2[\beta\lambda+\gamma], (\mu+2\varepsilon)\}$$

PROOF: We prove a sufficient condition for existence and uniqueness of the solutions of fractional order system (2) in the region $\Omega \times (0,T]$ where

$$\Omega = \{(S_t, I_t, R_t) \in \mathbb{R}^3 : \max(|S_t|, |I_t|, |R_t| \le \lambda)\}$$

Consider a mapping $E(A) = (E_1(A), E_2(A), E_3(A))$ and

$$\begin{aligned} E_1(A) &= \mu-\beta S(\kappa)I(\kappa)-\mu S(\kappa)-\omega S(\kappa)+\varepsilon R(\kappa) \\ E_2(A) &= \beta S(\kappa)I(\kappa)-\mu I(\kappa)-\gamma I(\kappa) \\ E_3(A) &= \omega S(\kappa)+\gamma I(\kappa)-\mu R(\kappa)-\varepsilon R(\kappa) \end{aligned}$$

For any $A, \bar{A} \in \Omega$, it follows from the above equation that,

$$\begin{aligned} &\|E(A)-E(\bar{A})\| = |E_1(A)-E_1(\bar{A})|+|E_2(A)-E_2(\bar{A})|+|E_3(A)-E_3(\bar{A})| \\ &= |\mu-\beta S_\kappa I_\kappa-(\mu+\omega)S_\kappa+\varepsilon R_\kappa-\mu+\beta\bar{S}_\kappa\bar{I}_\kappa+(\mu+\omega)\bar{S}_\kappa-\varepsilon\bar{R}_\kappa| \\ &+|\beta S_\kappa I_\kappa-(\mu+\gamma)I_\kappa-\beta\bar{S}_\kappa\bar{I}_\kappa+(\mu+\gamma)\bar{I}_\kappa| \\ &+|\omega S_\kappa+\gamma I_\kappa-(\mu+\varepsilon)R_\kappa-\omega\bar{S}_\kappa-\gamma I_\kappa+(\mu+\varepsilon)\bar{R}_\kappa| \\ &= |-\beta(S_\kappa I_\kappa-\bar{S}_\kappa\bar{I}_\kappa)-\mu(S_\kappa-\bar{S}_\kappa)-\omega(S_\kappa-\bar{S}_\kappa)+\varepsilon(R_\kappa-\bar{R}_\kappa)| \\ &+|\beta(S_\kappa I_\kappa-\bar{S}_\kappa\bar{I}_\kappa)-\mu(I_\kappa-\bar{I}_\kappa)-\gamma(I_\kappa-\bar{I}_\kappa)| \\ &+|\omega(S_\kappa-\bar{S}_\kappa)+\gamma(I_\kappa-\bar{I}_\kappa)-\mu(R_\kappa-\bar{R}_\kappa)-\varepsilon(R_\kappa-\bar{R}_\kappa)| \end{aligned}$$

where $\Delta = \max\{\mu+2[\beta\lambda+\omega], \mu+2[\beta\lambda+\gamma], (\mu+2\varepsilon)\}$

Thus, E(A) satisfies the Lipschitz condition leading to the existence and uniqueness of solution for fractional order system (2).

IV. EXISTENCE OF EQUILIBRIUM STATES AND LOCAL STABILITY

This segment investigates the stability of the equilibrium states of system (3). The equilibrium states are determined from equation (3). WFE $= \left(\frac{\mu+\varepsilon}{\mu+\varepsilon+\omega}, 0, \frac{\omega}{\mu+\varepsilon+\omega}\right)$ for the condition $I = 0$, and

$$\text{EE} = \left(\frac{\mu+\gamma}{\beta}, \frac{\beta(\mu+\varepsilon)-(\mu+\gamma)(\mu+\varepsilon+\omega)}{\beta(\mu+\varepsilon+\gamma)}, \frac{\gamma\beta+(\omega-\gamma)(\mu+\gamma)}{\beta(\mu+\varepsilon+\gamma)}\right)$$

when $I > 0$. Let $R_0 = \frac{\beta(\mu+\varepsilon)}{(\mu+\gamma)(\mu+\varepsilon+\omega)}$ is the Reproduction Number. Obviously the EE state is significant only if $R_0 > 1$ and the Jacobian of system (3) is given in (4).

$$J(S,I,R) = \begin{bmatrix} 1-A[\beta I+\mu+\omega] & -A\beta S & A\varepsilon \\ A\beta I & 1+A[\beta S-\mu-\gamma] & 0 \\ A\omega & A\gamma & 1-A[\mu+\varepsilon] \end{bmatrix} \quad (4)$$

Here A is $\frac{h^{\alpha}}{\Gamma(1+\alpha)}$. The local stability of the equilibrium state is analyzed from the eigen values Θ, $|J(S,I,R)-I\Theta| = 0$.

A. Stability of WFE State

The Jacobian matrix of the WFE state of system (3) is

$$J(WFE) = \begin{bmatrix} 1-A[\mu+\omega] & -\frac{A\beta(\mu+\varepsilon)}{\mu+\varepsilon+\omega} & A\varepsilon \\ 0 & 1+A\left[\frac{\beta(\mu+\varepsilon)}{\mu+\varepsilon+\omega}-\mu-\gamma\right] & 0 \\ A\omega & A\gamma & 1-A[\mu+\varepsilon] \end{bmatrix}$$

Now the eigen values are determined by third order characteristic polynomial equation

$\rho(\Theta) = \Theta^3 + S_1\Theta^2 + S_2\Theta + S_3 = 0$

- $S_1 = A[2\mu+\varepsilon+\omega-(\mu+\gamma)(R_0-1)]-3$;
- $S_2 = [1+A(\mu+\gamma)(R_0-1)][2-A(2\mu+\varepsilon+\omega)] + (1+A\mu)[1-A(\mu+\varepsilon+\gamma)]$;
- $S_3 = [1+A(\mu+\gamma)(R_0-1)](A\mu-1)[1-A(\mu+\varepsilon+\omega)]$.

With the aid of Routh - Hurwitz criteria for the characteristic polynomial $\rho(\Theta)$, if $S_1 > 0; S_3 > 0$ and $S_1S_2 > S_3$ then the WFE state is locally stable. Also the characteristic polynomial has a negative real root if $R_0 < 1$ and a positive real root for $R_0 > 1$. From the above study, we hold the following theorems.

THEOREM 2.

The WFE state is locally asymptotically stable (LAS) when $R_0 < 1$, and for $R_0 > 1$ it is unstable (saddle state)

THEOREM 3.

The WFE state is globally asymptotically stable (GAS) when $R_0 \le 1$.

B. Stability of EE State

Here, we analyze the local stability of EE state. The Jacobian for the same of system (3) is

$$J(EE)=\begin{bmatrix}1-A[B+\mu+\omega] & -A(\mu+\gamma) & A\varepsilon\\ AB & 1 & 0\\ A\omega & A\gamma & C\end{bmatrix}$$

where $B=\dfrac{\beta(\mu+\varepsilon)-(\mu+\varepsilon+\omega)(\mu+\gamma)}{\mu+\varepsilon+\gamma};C=1-A(\mu+\varepsilon)$. To compute the eigen values, we determine the third order characteristic polynomial equation

$$\rho(\Phi)=\Phi^3+u_0\Phi^2+u_1\Phi+u_2=0$$

- $u_0=A\left[2\mu+\varepsilon+\omega-(\mu+\gamma)(R_0-1)\right]-3$;
- $u_1=\left[1+A(\mu+\gamma)(R_0-1)\right]\left[2-A(2\mu+\varepsilon+\omega)\right]+(1+A\mu)\left[1-A(\mu+\varepsilon+\gamma)\right]$;
- $u_2=\left[1+A(\mu+\gamma)(R_0-1)\right](A\mu-1)\left[1-A(\mu+\varepsilon+\omega)\right]$

Applying Routh - Hurwitz criteria for $\rho(\Phi)$. It is clearly that $u_0>0;u_2>0$ and $u_0u_1>u_2$. Hence the EE state is locally stable. Also the roots of the characteristic polynomial is negative real part. From the above result, we propose the following theorem.

THEOREM 4.

The EE state is locally asymptotically stable (sink) if $R_0>1$.

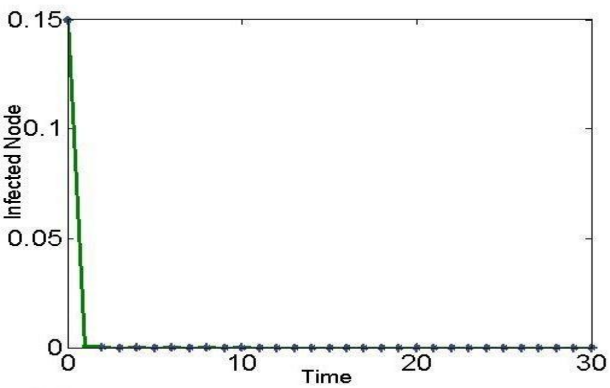
V. NUMERICAL PERFORMANCE AND SIMULATIONS

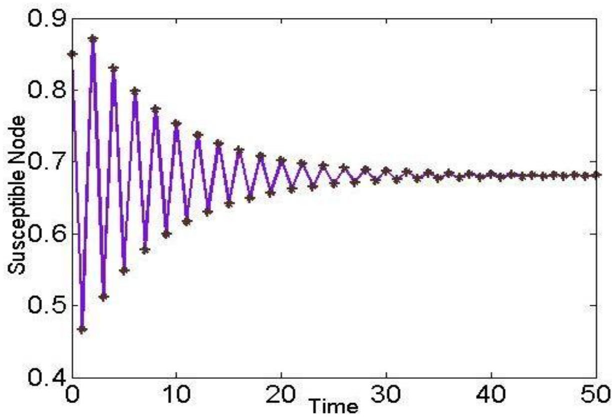
In this section, above theoretical analysis is verified and supported with appropriate examples. Also Numerical simulations manifest clearly interesting rich complex dynamics behaviors. We will also analyze the behavior of the susceptible, infected and recovered nodes in the WSN. To demonstrate the theoretical analysis, we put forward numerical examples of the model in the following three cases. In all the cases, the initial Susceptible, Infected and Recovered nodes assume the values $S(0)=0.85;I(0)=0.15$ and $R(0)=0$ respectively.

Table II. Parameter values

S.No	Cases	α	h	μ	β	ω	γ	ε	$R_0=\dfrac{\beta(\mu+\varepsilon)}{(\mu+\gamma)(\mu+\varepsilon+\omega)}$
1	(i)	0.9	0.75	1.45	0.45	0.75	0.19	0.15	$R_0=0.1868$
2	(ii)	0.9	0.75	0.8	1.9	0.76	0.34	0.34	$R_0=1$
3	(iii)	0.9	0.75	0.01	1.75	0.001	0.5	0.001	$R_0=3.1454$

Fig.2 assumes the case - (i) parameter values from the Table II with initial condition $(0.85,0.15,0)$ which yields the WFE state $(0.6809,0,0.3191)$. The epidemic threshold value is $R_0=0.1868$, which is less than one. Eigen values are $\Theta_1=-0.0150;\ \Theta_2=-0.5534$ and $\Theta_3=-0.0704$, and they have no positive real part. Hence from Theorem 2, WFE is LAS. Similarly from Table II, for case - (ii), parameters with the same initial values tend to WFE state $(0.6,0,0.4)$. Here $R_0=1$. Therefore,from Theorem 3, WFE is GAS. See Fig.3





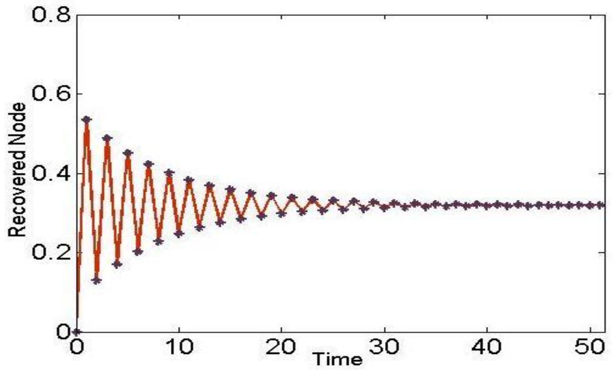


Fig. 2. WFE state with $R_0<1$

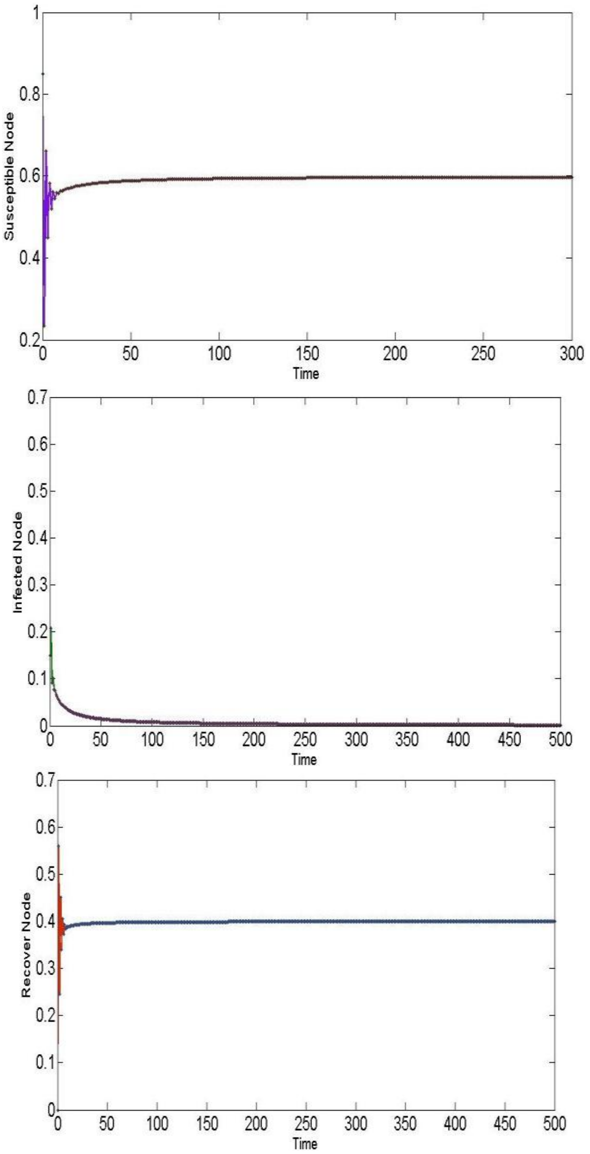
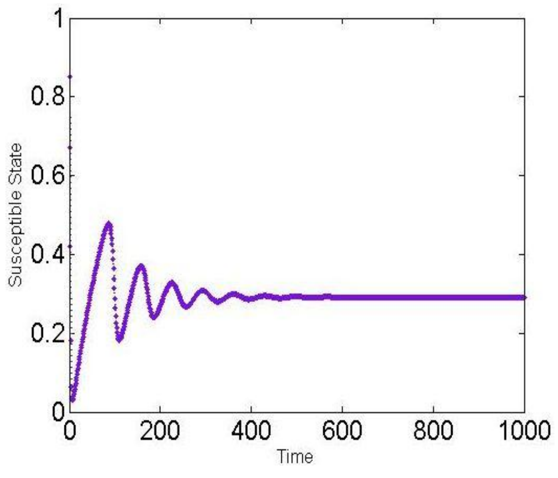
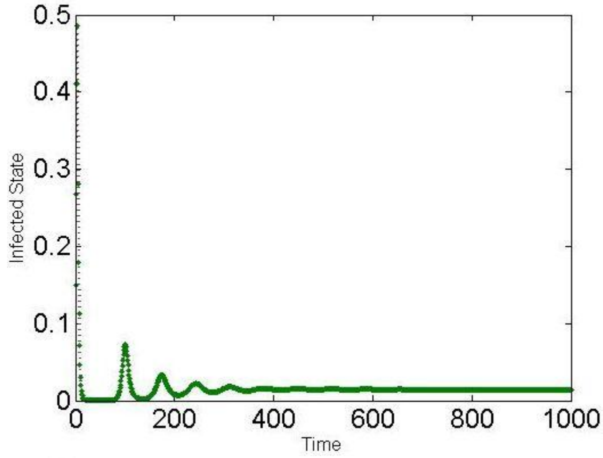


Fig. 3. WFE state with $R_0 = 1$

Fig.4, Considering the case - (iii) with numerical values for the parameters from Table II and initial state $(0.85, 0.15, 0)$, EE state is $(0.2914, 0.0147, 0.6939)$. The reproduction number is $R_0 = 3.1454$, which is greater than 1. Eigen values are $\Theta_1 = -0.9924$ and $\Theta_{2,3} = -0.9846 \pm i0.0601$ and all the eigen roots have negative real part. Hence by Theorem 4, EE **state is LAS.**





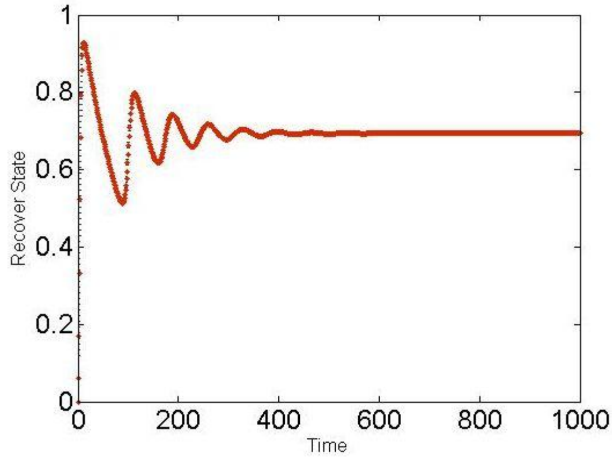
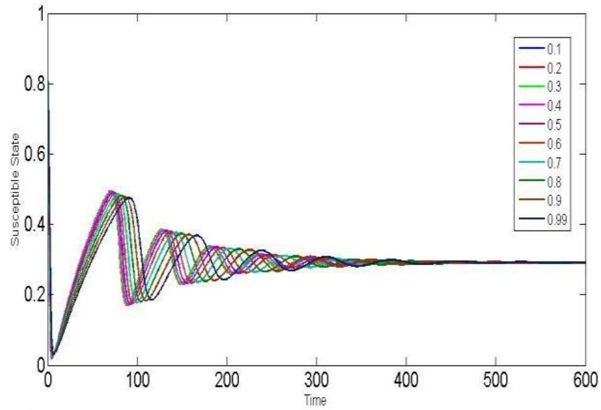
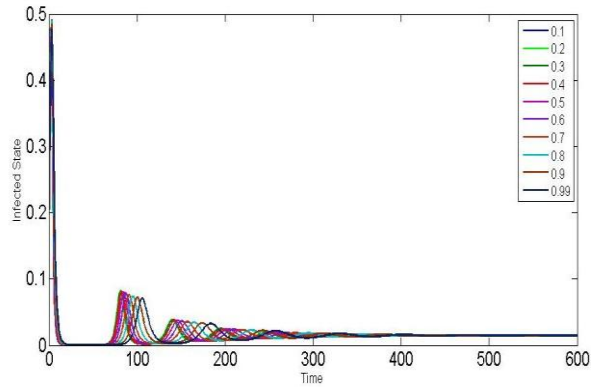


Fig. 4. EE state with $R_0 > 1$





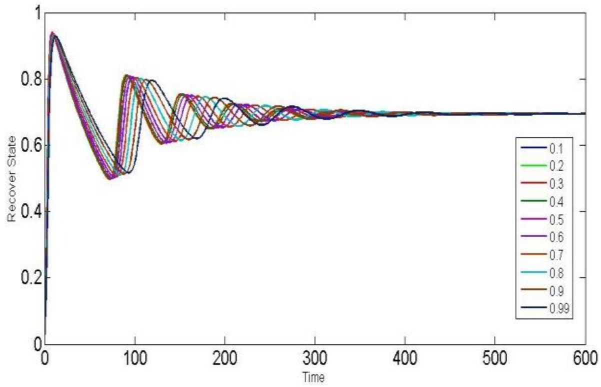


Fig. 4. EE state with different fractional order α

By varying the fractional order from 0.1 to 0.99 and fixing the other parameter values as in case - (iii). Fig.5 clearly shows that all the nodes are starting at the initial value $(0.85, 0.15, 0)$ and the nodes behavior are different for different fractional orders. But over the time period for the different fractional orders all the three nodes approach the same endemic equilibrium state.

VI. CONCLUSION

In this paper, we analysed a SIR model to study the dynamical behavior of worm propagation in WSNs. The reproduction number R_0 is computed to discuss the nature of worm propagation. The equilibrium states are obtained and local stability of these states are analyzed. When $R_0 \leq 1$, worms in WSNs are eliminated and the model (3) is stable at WFE. Similarly worms exit consistently and model reaches stablility at the EE state when $R_0 > 1$. Numerical performance verifiy the theoretical analysis.

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