Intuitionistic Fuzzy Soft Cubic Sets

V. Amarendra Babu, V. Siva Naga Malleswari

Abstract—We propose the definition of Intuitionistic fuzzy soft cubic set(IFSCS) and define few operations on IFSCS and deliberated its properties.

Keywords: Intuitionistic fuzzy soft set (IFSS), Interval-valued Intuitionistic fuzzy soft set (IVIFSS).

I. INTRODUCTION

In 1999, the conception of a soft set was designed by Molodstov[2]. Zadeh[6] deals with the conceit of a fuzzy set in 1965 and also introduces the theory of interval valued fuzzy sets[7].K.Attansov[1][3][4] suggested the conceit of intuitionistic fuzzy set and also introduces the conceit of interval valued intuitionistic fuzzy sets. The perception of a fuzzy soft set was initiated by Maji et al [8] and he enlarged the idea of soft sets to the intuitionistic fuzzy system [9]. The definition of the cubic set was initiated by Jun et al[5].

Here we propose the definition of IFSCS with the consolidation of interval-valued intuitionistic fuzzy set and intuitionistic fuzzy set. We define some operations on IFSCS and deliberated its properties.

II. PRELIMINARIES

Definition 2.1[3]:

A mapping given by S: X→P (χ), then a pair (S, X) is called a soft set over χ. Then a soft set on χ is characterized by a class of subsets of χ.

Definition 2.2:

A soft fuzzy set is defined by S= $\{(t_\theta,\mu(t_\theta)):t_\theta \in \chi,\theta \in P\}$ on χ. Here the mapping $\mu:\chi \to [0,1]P$ represents the association and the level of association of $t_\theta$in S is given by $\mu(t_\theta)$.

Definition 2.3: A set E= $\{<t, \mathrm{I}(t),\mu(t)>/t \in \chi\}$ is said to be a cubic set, where I is an IVFS and μ indicates fuzzy set. The cubic set is symbolized by(I,μ).

Definition2.4: An IFS $\mathcal{I}$ is characterised by $\mathcal{I} = \{<t: T(t), F(t) >: t \in \chi\}$. Here T: $\chi \to [0,1]$ represents the level of association and F:$\chi \to [0,1]$ represents the level of non-association of $t \in \chi$ to the set $\mathcal{I}$. Here χ indicates the universal set.

Definition 2.5: Let P(χ) symbolizes the class of all IFSs of χ and F: $\xi_1 \to$P (χ). The set (F,ξ_1) is defined as an IFSS on χ.Here χ indicates the universal set, ξ indicates the class of parameters and $\xi_1 \subset \xi$. P (χ) indicates the class of all the IFSs of χ.

Definition 2.6: An IVIFS I in χ is constituted by association function I_T and non association function I_F. I_T (t), I_F (t) ⊆[0, 1] for $t \in \chi$,whereχ is a universal set.

III. INTUITIONISTIC FUZZY SOFT CUBIC SET (IFSCS)

Definition 3.1: The pair ($\widehat{\mathcal{M}}$,ξ) is said to be an IFSCS on $\chi$and $\widehat{\mathcal{M}}: \xi \to IC(\chi)$.

Here ($\widehat{\mathcal{M}}$,ξ) = {$\widehat{\mathcal{M}}$ (θ_i) = $\{<\rho,\widehat{\mathcal{M}}_{\theta_i}(\rho), \mu_{\theta_i}(\rho)>: \rho \in \chi,\theta_i \in \xi\}$, where

$\widehat{\mathcal{M}}_{\theta_i}(\rho) = \{<\rho, [\widehat{\mathcal{M}}_{\theta_i}{}^{-T}(\rho), \widehat{\mathcal{M}}_{\theta_i}{}^{+T}(\rho)], [\widehat{\mathcal{M}}_{\theta_i}{}^{-F}(\rho), \widehat{\mathcal{M}}_{\theta_i}{}^{+F}(\rho)] >/ \rho \in \chi\}$is an IVIFSS and $\mu_{\theta_i}(\rho) = \{<\rho, \mu_{\theta_i}{}^{T}(\rho), \mu_{\theta_i}{}^{F}(\rho) >/ \rho \in \chi\}$ is an IFSS and IC(χ) indicates the collection of all IFSCSs, ξ indicates the set of parameters.

Definition 3.2: An IFSCS ($\widehat{\mathcal{M}}$,ξ) in χ is said to be

1) Truth internal (shortly, T-internal) if

$(\forall\rho \in \chi, \theta_i \in \xi), (\widehat{\mathcal{M}}_{\theta_i}{}^{-T}(\rho) \leq \mu_{\theta_i}{}^{T}(\rho) \leq \widehat{\mathcal{M}}_{\theta_i}{}^{+T}(\rho))$

2) Falsity internal (shortly, F-internal) if

$(\forall\rho \in \chi, \theta_i \in \xi), (\widehat{\mathcal{M}}_{\theta_i}{}^{-F}(\rho) \leq \mu_{\theta_i}{}^{F}(\rho) \leq \widehat{\mathcal{M}}_{\theta_i}{}^{+F}(\rho))$

If IFSCS in χ satisfies (1) and (2) then ($\widehat{\mathcal{M}}$,ξ) is an internal IFSCS in χ.

Definition 3.3: An IFSCS ($\widehat{\mathcal{M}}$,ξ) in χ is said to be

1) Truth external (shortly, T-external) if

$(\forall\rho \in \chi , \theta_i \in \xi), (\mu_{\theta_i}{}^{T}(\rho) \notin (\widehat{\mathcal{M}}_{\theta_i}{}^{-T}(\rho), \widehat{\mathcal{M}}_{\theta_i}{}^{+T}(\rho))$

2) Falsity external (shortly, F-external) if

$(\forall\rho \in \chi , \theta_i \in \xi), (\mu_{\theta_i}{}^{F}(\rho) \notin (\widehat{\mathcal{M}}_{\theta_i}{}^{-F}(\rho), \widehat{\mathcal{M}}_{\theta_i}{}^{+F}(\rho))$

If IFSCS in χ satisfies (1) and (2) then ($\widehat{\mathcal{M}}$,ξ) is an external IFSCS in χ.

Definition 3.4:

If ($\widehat{\mathcal{M}}$,ξ_1)={$\widehat{\mathcal{M}}$(θ_i)={<x,$\widehat{\mathcal{M}}_{\theta_i}(\rho), \lambda_{\theta_i}(\rho)$>: $\rho \in \chi, \theta_i \in \xi_1$},

($\widehat{\mathcal{N}}$,ξ_2)={$\widehat{\mathcal{N}}$(θ_i)={< ρ ,$\widehat{\mathcal{N}}_{\theta_i}(\rho), \mu_{\theta_i}(\rho)$>: $\rho \in \chi$, $\theta_i \in \xi_2$} are two IFSCS s in χ and ξ_1 and ξ_2 are any two subsets of ξ, then

($\widehat{\mathcal{M}}$, ξ_1)= ($\widehat{\mathcal{N}}$, ξ_2) if and only if

i. ξ_1 =ξ_2 and

ii. $\widehat{\mathcal{M}}$ (θ_i)= $\widehat{\mathcal{N}}$ (θ_i) ⇔ $\widehat{\mathcal{M}}_{\theta_i}(\rho) = \widehat{\mathcal{N}}_{\theta_i}(\rho)$ and $\lambda_{\theta_i}(\rho) = \mu_{\theta_i}(\rho) \forall\rho \in \chi$ corresponding to each $\theta_i \in \xi_1$.

Definition 3.5: An IFSCS ($\widehat{\mathcal{M}}$,ξ_1) is called a subset of an IFSCS ($\widehat{\mathcal{N}}$,ξ_2) if $\xi_1 \subseteq \xi_2$ and $\widehat{\mathcal{M}}$ (θ) ⊆ $\widehat{\mathcal{N}}$(θ) if and only if $\widehat{\mathcal{M}}_\theta(\rho) \subseteq \widehat{\mathcal{N}}_\theta(\rho)$ and $\lambda_\theta(\rho) \subseteq \mu_\theta(\rho)$ for all $\rho \in \chi$. We denote it by ($\widehat{\mathcal{M}}$,ξ_1) ⊆ ($\widehat{\mathcal{N}}$,ξ_2).

$\widehat{\mathcal{M}}_\theta(\rho) \subseteq \widehat{\mathcal{N}}_\theta(\rho) \Longrightarrow \widehat{\mathcal{M}}_\theta^{-T}(\rho) \leq \widehat{\mathcal{N}}_\theta^{-T}(\rho)$

$\widehat{\mathcal{M}}_\theta^{+T}(\rho) \leq \widehat{\mathcal{N}}_\theta^{+T}(\rho)$

$\widehat{\mathcal{M}}_\theta^{-F}(\rho) \geq \widehat{\mathcal{N}}_\theta^{-F}(\rho)$

$\widehat{\mathcal{M}}_\theta^{+F}(\rho) \geq \widehat{\mathcal{N}}_\theta^{+F}(\rho)$

Revised Version Manuscript Received on 18 January, 2019.

Dr V. AmarendraBabu, Department of Mathematics, AcharyaNagarjuna University, Nagarjuna Nagar, AndhraPradesh, India. (Email: amarendravelisela@ymail.com)

V.Siva Naga Malleswari, Assistant Professor, PVPSIT, AndhraPradesh, India. (Email: vsnm.maths@gmail.com)

$\lambda_\theta(\rho) \subseteq \mu_\theta(\rho) \implies \lambda_\theta^T(\rho) \leq \mu_\theta^T(\rho)$
$\lambda_\theta^F(\rho) \geq \mu_\theta^F(\rho)$

Definition 3.6: An IFSCS $(\widehat{\mathcal{M}},\xi)^c$ is said to be the complement of an IFSCS $(\widehat{\mathcal{M}},\xi)$ if $(\widehat{\mathcal{M}},\xi)^c=(\widehat{\mathcal{M}}^c, \urcorner\xi)$ where $\widehat{\mathcal{M}}^c: \urcorner\xi \rightarrow \widehat{\mathcal{M}}(\chi)$.

i.e $(\widehat{\mathcal{M}},\xi)^c= \{\langle\rho,\widehat{\mathcal{M}}_\theta^c(\rho),\lambda_\theta^c(\rho)\rangle:\rho \in \chi, \theta \in \xi\}$
$= \{\langle\rho, 1-\widehat{\mathcal{M}}_\theta^T(\rho), 1-\widehat{\mathcal{M}}_\theta^F(\rho), 1-\lambda_\theta^T(\rho), 1-\lambda\theta F(\rho):\rho\in\chi, \theta\in\xi\}$

$=\{\langle\rho, 1-\widehat{\mathcal{M}}_\theta^{+T}(\rho), 1-\widehat{\mathcal{M}}_\theta^{-T}(\rho), 1-\widehat{\mathcal{M}}_\theta^{-F}(\rho), 1-\widehat{\mathcal{M}}_\theta^{+F}(\rho), 1-\lambda_\theta^T(\rho), 1-\lambda_\theta^F(\rho)\rangle: \rho \in \chi, \theta \in \xi\}$

Remark 3.7: $(\widehat{\mathcal{M}},\xi)^c= \chi - \widehat{\mathcal{M}}(\theta)$ for all $\theta \in \xi$.

Definition3.8: If
$(\widehat{\mathcal{M}},\xi_1)=\{\langle\rho, \widehat{\mathcal{M}}_\theta(\rho), \lambda_\theta(\rho) \mid \rho \in \chi, \theta \in \xi_1\rangle\}$,
$(\widehat{\mathcal{N}},\xi_2)=\langle\rho, \widehat{\mathcal{M}}_\theta(\rho), \lambda_\theta(\rho) \mid \rho \in \chi, \theta \in \xi_2\rangle\}$ are two IFSCSs on (χ,ξ) then $(\widehat{\mathcal{M}},\xi_1) \cup (\widehat{\mathcal{N}}, \xi_2)$=(H, C) and C=$\xi_1 \cup \xi_2$, for all $\theta \in$ C

$$H(e) = \begin{cases} \widehat{\mathcal{M}}(\theta) \ if \ \theta \in \xi_1 - \xi_2 \\ \widehat{\mathcal{N}}(\theta) if \ \theta \in \xi_2 - \xi_1 \\ \widehat{\mathcal{M}}(\theta) \cup \widehat{\mathcal{N}}(\theta) \ if \ \theta \in \xi_1 \cap \xi_2 \end{cases}$$

Where $H(\theta)$ ={ ρ,[max(inf $\widehat{\mathcal{M}}_\theta(\rho)$,inf $\widehat{\mathcal{N}}_\theta(\rho)$), max(sup $\widehat{\mathcal{M}}_\theta(\rho)$,sup $\widehat{\mathcal{N}}_\theta(\rho)$)],
,[min(inf $\widehat{\mathcal{M}}_\theta(\rho)$,inf $\widehat{\mathcal{N}}_\theta(\rho)$), min(sup $\widehat{\mathcal{M}}_\theta(\rho)$,sup $\widehat{\mathcal{N}}_\theta(\rho)$)],max $(\lambda_\theta(\rho), \mu_\theta(\rho))/\rho \in \chi\}$

Definition 3.9:
If $(\widehat{\mathcal{M}}, \xi_1)$= $\{\langle\rho, \widehat{\mathcal{M}}_\theta(\rho), \lambda_\theta(\rho) \mid \rho \in \chi, \theta \in \xi_1\rangle\}$ and $(\widehat{\mathcal{N}},\xi_2) = \{\langle\rho, \widehat{\mathcal{M}}_\theta(\rho), \lambda_\theta(\rho) \mid \rho \in \chi, \theta \in \xi_2\rangle\}$ are tow IFSCSs on (χ, ξ) and $(\widehat{\mathcal{M}}, \xi_1) \cap (\widehat{\mathcal{N}}, \xi_2)$=(H, C)where C=$\xi_1 \cap \xi_2$, for all $\theta \in$ C.

$H(\theta) = \widehat{\mathcal{M}}(\theta) \cap \widehat{\mathcal{N}}(\theta)$

Where H (θ) ={t,[min(inf $\widehat{\mathcal{M}}_\theta(\rho)$,inf $\widehat{\mathcal{N}}_\theta(\rho)$), min(sup $\widehat{\mathcal{M}}_\theta(\rho)$,sup $\widehat{\mathcal{N}}_\theta(\rho)$)],
,[max(inf $\widehat{\mathcal{M}}_\theta(\rho)$,inf $\widehat{\mathcal{N}}_\theta(\rho)$), max(sup $\widehat{\mathcal{M}}_\theta(\rho)$,sup $\widehat{\mathcal{N}}_\theta(\rho)$)],min $(\lambda_\theta(\rho), \mu_\theta(\rho))/\rho \in \chi\}$

Definition 3.10: If $(\widehat{\mathcal{M}},\xi_1)$ and $(\widehat{\mathcal{N}},\xi_2)$ are two IFSCSs then $(\widehat{\mathcal{M}},\xi_1)$ AND $(\widehat{\mathcal{N}},\xi_2)$ is an IFSCS symbolized by $(\widehat{\mathcal{M}}, \xi_1)\wedge(\widehat{\mathcal{N}}, \xi_2)$=(Q, ξ_1 Xξ_2) where

$$Q^T(a_i, b_i)(q) = \{\min\left(\widehat{\mathcal{M}}^T(a)(q), \widehat{\mathcal{M}}^T(b)(q)\right), \min\left(\lambda^T(a)(q), \lambda^T(b)(q)\right)\}$$

$$Q^F(a_i, b_i)(q) = \{\min\left(\widehat{\mathcal{M}}^F(a)(q), \widehat{\mathcal{M}}^F(b)(q)\right), \min\left(\lambda^F(a)(q), \lambda^F(b)(q)\right)\}$$

For all $(a_i, b_i) \in \xi_1$ Xξ_2

Definition 3.11: If $(\widehat{\mathcal{M}},\xi_1)$ and $(\widehat{\mathcal{N}},\xi_2)$ are two IFSCSs then $(\widehat{\mathcal{M}},\xi_1)$ OR $(\widehat{\mathcal{N}},\xi_2)$ is an IFSCS symbolized by $(\widehat{\mathcal{M}}, \xi_1)\vee(\widehat{\mathcal{N}}, \xi_2)$=(Q, ξ_1 Xξ_2) where

$$Q^T(a_i, b_i)(q) = \{\max\left(\widehat{\mathcal{M}}^T(a)(q), \widehat{\mathcal{M}}^T(b)(q)\right), \max\left(\lambda^T(a)(q), \lambda^T(b)(q)\right)\}$$

$$Q^F(a_i, b_i)(q) = \{\max\left(\widehat{\mathcal{M}}^F(a)(q), \widehat{\mathcal{M}}^F(b)(q)\right), \max\left(\lambda^F(a)(q), \lambda^F(b)(q)\right)\}$$

For all $(a_i, b_i) \in \xi_1$Xξ_2

IV. SOME RESULTS ON INTUITIONISTIC FUZZY SOFT CUBIC SETS

Proposition 4.1: If $(\widehat{\mathcal{M}}, \xi)$ and $(\widehat{\mathcal{N}}, \xi)$ are two IFSCS on χ then

(i)$(\widehat{\mathcal{M}}, \xi) \subseteq (\widehat{\mathcal{N}}, \xi)$ iff $(\widehat{\mathcal{M}}, \xi) \cap (\widehat{\mathcal{N}}, \xi) = (\widehat{\mathcal{M}}, \xi)$

(ii) $(\widehat{\mathcal{M}}, \xi) \subseteq (\widehat{\mathcal{N}}, \xi)$ iff $(\widehat{\mathcal{M}}, \xi) \cup (\widehat{\mathcal{N}}, \xi) = (\widehat{\mathcal{N}}, \xi)$

Proof:

(i)Suppose that $(\widehat{\mathcal{M}}, \xi) \subseteq (\widehat{\mathcal{N}}, \xi)$ then $\widehat{\mathcal{M}}(\theta) \subseteq \widehat{\mathcal{N}}(\theta)$ for all $\theta \in \xi$.

Let $(\widehat{\mathcal{M}}, \xi) \cap (\widehat{\mathcal{N}}, \xi)$=(H, ξ)

Then H $(\theta) = \widehat{\mathcal{M}}(\theta) \cap \widehat{\mathcal{N}}(\theta) = \widehat{\mathcal{M}}(\theta)$ for every $\theta \in \xi$.

Hence (H, ξ) = $(\widehat{\mathcal{M}}, \xi)$.

Consider $(\widehat{\mathcal{M}}, \xi) \cap (\widehat{\mathcal{N}}, \xi) = (\widehat{\mathcal{M}}, \xi)$

Then $\widehat{\mathcal{M}}(\theta) = \widehat{\mathcal{M}}(\theta) \cap \widehat{\mathcal{N}}(\theta)$ for every $\theta \in \xi$

Then $\widehat{\mathcal{M}}(\theta) \subseteq \widehat{\mathcal{N}}(\theta)$ for all $\theta \in \xi$

Hence $(\widehat{\mathcal{M}}, \xi) \subseteq (\widehat{\mathcal{N}}, \xi)$.

(ii) Suppose that $(\widehat{\mathcal{M}}, \xi) \subseteq (\widehat{\mathcal{N}}, \xi)$ then $\widehat{\mathcal{M}}(\theta) \subseteq \widehat{\mathcal{N}}(\theta)$ for all $\theta \in \xi$.

Let $(\widehat{\mathcal{M}}, \xi) \cup (\widehat{\mathcal{N}}, \xi)$ = (H, ξ)

Then H$(\theta) = \widehat{\mathcal{M}}(\theta) \cup \widehat{\mathcal{N}}(\theta) = \widehat{\mathcal{N}}(\theta)$ for all $\theta \in \xi$.

Then (H, ξ) = $(\widehat{\mathcal{N}}, \xi)$.

Suppose that $(\widehat{\mathcal{M}}, \xi) \cup (\widehat{\mathcal{N}}, \xi) = (\widehat{\mathcal{N}}, \xi)$

Then $\widehat{\mathcal{N}}(\theta) = \widehat{\mathcal{M}}(\theta) \cup \widehat{\mathcal{N}}(\theta)$ for all $\theta \in \xi$

Then $\widehat{\mathcal{M}}(\theta) \subseteq \widehat{\mathcal{N}}(\theta)$ for all $\theta \in \xi$

Hence $(\widehat{\mathcal{M}}, \xi) \subseteq (\widehat{\mathcal{N}}, \xi)$.

Proposition 4.2: Let χ be an initial universe set and ξ be a set of parameters. If $(\widehat{\mathcal{M}}, \xi)$, $(\widehat{\mathcal{N}}, \xi)$, $(\widehat{H}, \xi)$ and $(\widehat{K}, \xi)$ are IFSCSs on χ.Then

(i)If $(\widehat{\mathcal{M}}, \xi) \cap (\widehat{\mathcal{N}}, \xi) = \emptyset$ then $(\widehat{\mathcal{M}}, \xi) \subseteq (\widehat{\mathcal{N}}, \xi)^c$

(ii) If $(\widehat{\mathcal{M}}, \xi) \subseteq (\widehat{\mathcal{N}}, \xi)$ and $(\widehat{\mathcal{N}}, \xi) \subseteq (\widehat{H}, \xi)$ then $(\widehat{\mathcal{M}}, \xi) \subseteq (\widehat{H}, \xi)$.

(iii) If $(\widehat{\mathcal{M}}, \xi) \subseteq (\widehat{\mathcal{N}}, \xi)$ and $(\widehat{H}, \xi) \subseteq (\widehat{K}, \xi)$

(iv)

(v)

then $(\widehat{\mathcal{M}}, \xi) \cap \left(\widehat{H}, \xi\right) \subseteq (\widehat{\mathcal{N}}, \xi) \cap (\widehat{K}, \xi)$.

(vi) If $(\widehat{\mathcal{M}}, \xi) \subseteq (\widehat{\mathcal{N}}, \xi)$ iff $(\widehat{\mathcal{N}}, \xi)^c \subseteq (\widehat{\mathcal{M}}, \xi)^c$

Proof:

(i)Suppose $(\widehat{\mathcal{M}}, \xi) \cap (\widehat{\mathcal{N}}, \xi) = \emptyset$ then $\widehat{\mathcal{M}}(\theta) \cap \widehat{\mathcal{N}}(\theta) = \emptyset$

So $\widehat{\mathcal{M}}(\theta) \subseteq \chi \backslash \widehat{\mathcal{N}}(\theta) = \widehat{\mathcal{N}}^c(\theta)$ for all $\theta \in \xi$

Then we have $(\widehat{\mathcal{M}}, \xi) \subseteq (\widehat{\mathcal{N}}, \xi)^c$

(ii) Suppose $(\widehat{\mathcal{M}}, \xi) \subseteq (\widehat{\mathcal{N}}, \xi)$ and $(\widehat{\mathcal{N}}, \xi) \subseteq (\widehat{H}, \xi)$

Then $\widehat{\mathcal{M}}(\theta) \subseteq \widehat{\mathcal{N}}(\theta)$ and $\widehat{\mathcal{N}}(\theta) \subseteq \widehat{H}(\theta)$

Then $\widehat{\mathcal{M}}(\theta) \subseteq \widehat{\mathcal{N}}(\theta) \subseteq \widehat{H}(\theta)$ for all $\theta \in \xi$

Then $(\widehat{\mathcal{M}}, \xi) \subseteq (\widehat{H}, \xi)$.

(iii) Suppose $(\widehat{\mathcal{M}}, \xi) \subseteq (\widehat{\mathcal{N}}, \xi)$ and $(\widehat{H}, \xi) \subseteq (\widehat{K}, \xi)$

Then $\widehat{\mathcal{M}}(\theta) \subseteq \widehat{\mathcal{N}}(\theta)$ and $\widehat{H}(\theta) \subseteq \widehat{K}(\theta)$ for every $\theta \in \xi$

Then $(\widehat{\mathcal{M}}, \xi) \cap \left(\widehat{H}, \xi\right) \subseteq (\widehat{\mathcal{N}}, \xi) \cap (\widehat{K}, \xi)$.

(iv) Suppose $(\widehat{\mathcal{M}}, \xi) \subseteq (\widehat{\mathcal{N}}, \xi)$

iff$\widehat{\mathcal{M}}(\theta) \subseteq \widehat{\mathcal{N}}(\theta)$

iff $(\widehat{\mathcal{N}}(\theta))^c \subseteq (\widehat{\mathcal{M}}(\theta))^c$

iff$\widehat{\mathcal{N}}^c(\theta) \subseteq \widehat{\mathcal{M}}^c(\theta)$ for all $\theta \in \xi$

iff$(\widehat{\mathcal{N}}, \xi)^c \subseteq (\widehat{\mathcal{M}}, \xi)^c$

Definition 4.3: If $\{(\widehat{\mathcal{M}}_i,\xi) i \in$ I } is a calss of IFSCS on χ then

(i)$\cup_{i\in I}(\widehat{\mathcal{M}}_i, \xi) = (\widehat{H}, \xi)$ Where $\widehat{H}(\theta) = \cup_{i\in I} \widehat{\mathcal{M}}_i(\theta)$ for $\theta \in \xi$.

(ii) $\cap_{i\in I}(\widehat{\mathcal{M}}_i, \xi) = (\widehat{K}, \xi)$ Where $\widehat{K}(\theta) = \cap_{i\in I} \widehat{\mathcal{M}}_i(\theta)$ for $\in \xi$.

Proposition 4.4: If $\{(\widehat{\mathcal{M}}_{i,}\xi)/i \in \mathrm{I}\}$ is a class of IFSCS on χ, then

(i) $(\cup_{i\in I}(\widehat{\mathcal{M}}_i, \xi))^c = \cap_{i\in I}(\widehat{\mathcal{M}}_i, \xi)^c$

(ii) $(\cap_{i\in I}(\widehat{\mathcal{M}}_i, \xi))^c = \cup_{i\in I}(\widehat{\mathcal{M}}_i, \xi)^c$

Proof:

(i)Let $(\cup_{i\in I}(\widehat{\mathcal{M}}_i, \xi))= (\widehat{H}, \xi)$

Then $(\cup_{i\in I}(\widehat{\mathcal{M}}_i, \xi))^c = (\widehat{H}, \xi)^c$

Since $H^c(\theta) = \chi\backslash\widehat{H}(\theta) = \chi\backslash \cup_{i\in I} \widehat{\mathcal{M}}_i(\theta) = \cap_{i\in I}(\chi\backslash\widehat{\mathcal{M}}_i(\theta))$ for all $\theta \in \xi$ .... (1)

On the other hand, $\cap_{i\in I}(\widehat{\mathcal{M}}_i, \xi)^c = (K, \xi)$

Then $K(\theta) = \cap_{i\in I} \widehat{\mathcal{M}}_i{}^c(\theta) = \cap_{i\in I}(\chi\backslash \widehat{\mathcal{M}}_i(\theta))$ for all $\in \xi$ .... (2)

From (1) and (2), $(\cup_{i\in I}(\widehat{\mathcal{M}}_i, \xi))^c = \cap_{i\in I}(\widehat{\mathcal{M}}_i, \xi)^c$.

(ii) Let $(\cap_{i\in I}(\widehat{\mathcal{M}}_i, \xi)) = (\widehat{H}, \xi)$ implies $\cap_{i\in I}(\widehat{\mathcal{M}}, \xi)^c = (\widehat{H}, \xi)^c$

Since $\widehat{H}^c(\theta) = \chi\backslash\widehat{H}(\theta) = \chi\backslash \cap_{i\in I} \widehat{\mathcal{M}}_i(\theta) = \cup_{i\in I} \chi\backslash\widehat{\mathcal{M}}_i(\theta)$ for all $\theta \in \xi$ .... (1)

On the other hand, $\cup_{i\in I}(\widehat{\mathcal{M}}_i, \xi)^c = (K, \xi)$

Then $K(\theta) = \cup_{i\in I} \widehat{\mathcal{M}}_i{}^c(\theta) = \cup_{i\in I}(\chi\backslash \widehat{\mathcal{M}}_i(\theta))$ for all $\in \xi$ .... (2)

(iii) From (1) and (2), $(\cap_{i\in I}(\widehat{\mathcal{M}}_i, \xi))^c = \cup_{i\in I}(\widehat{\mathcal{M}}_i, \xi)^c$

Proposition 4.5: Show that

(i) $(\emptyset, \xi)^c = (\chi, \xi)$

(ii) $(\chi, \xi)^c = (\emptyset, \xi)$

Proof:

(i)Let $(\emptyset, \xi) = (\widehat{\mathcal{M}}, \xi)$ then for all $\theta \in \xi$,

$\widehat{\mathcal{M}}(\theta)=\{\left(\rho, \widehat{\mathcal{M}}_\theta^T(\rho), \widehat{\mathcal{M}}_\theta^F(\rho)\right), \lambda_\theta^T(\rho), \lambda_\theta^F(\rho)/ \rho \in \chi\}$

$=\{\rho, ([0,0], [0,0]), (0, 0)/\rho \in \chi\}$

$(\emptyset, \xi)^c=(\widehat{\mathcal{M}}, \xi)^c$

$=\{\left(\rho, \widehat{\mathcal{M}}_\theta^T(\rho), \widehat{\mathcal{M}}_\theta^F(\rho)\right), \lambda_\theta^T(\rho), \lambda_\theta^F(\rho)/ \rho \in \chi\}^c$

$= \{\left(\rho, 1-\widehat{\mathcal{M}}_\theta^T(\rho), 1-\widehat{\mathcal{M}}_\theta^F(\rho)\right), 1-\lambda_\theta^T(\rho), 1-\lambda_\theta^F(\rho)/ \rho \in \chi\}$

$= \{\rho, [1-\widehat{\mathcal{M}}_\theta^{+T}, 1-\widehat{\mathcal{M}}_\theta^{-T}](\rho), [1-\widehat{\mathcal{M}}_\theta^{-F}, 1-$ *Mθ+Fρ, 1−λθTρ,1−λθFρ/ρ∈𝕌}*

$=\{(\rho, ([1,1], [1,1]), (1,1))/ \rho \in \chi\}$

Thus $(\emptyset, \xi)^c = (\chi, \xi)$.

(ii) Let $(\chi, \xi) = (\widehat{\mathcal{M}}, \xi)$ then for all $\theta \in \xi$,

$\widehat{\mathcal{M}}(\theta)=\{\left(\rho, \widehat{\mathcal{M}}_\theta^T(\rho), \widehat{\mathcal{M}}_\theta^F(\rho)\right), \lambda_\theta^T(\rho), \lambda_\theta^F(\rho)/ \rho \in \chi\}$

$=\{\rho, ([1,1][1,1]), (1, 1)/\rho \in \chi\}$

$(\chi, \xi)^c=(\widehat{\mathcal{M}}, \xi)^c$

$=\{\left(\rho, \widehat{\mathcal{M}}_\theta^T(\rho), \widehat{\mathcal{M}}_\theta^F(\rho)\right), \lambda_\theta^T(\rho), \lambda_\theta^F(\rho)/ \rho \in \chi\}^c$

$= \{\left(\rho, 1-\widehat{\mathcal{M}}_\theta^T(\rho), 1-\widehat{\mathcal{M}}_\theta^F(\rho)\right), 1-\lambda_\theta^T(\rho), 1-\lambda_\theta^F(\rho)/ \rho \in \chi\}$

$= \{\rho, [1-\widehat{\mathcal{M}}_\theta^{+T}, 1-\widehat{\mathcal{M}}_\theta^{-T}](\rho), [1-\widehat{\mathcal{M}}_\theta^{-F}, 1-$ *Mθ+Fρ, 1−λθTρ,1−λθFρ/ρ∈χ}*

$=\{(\rho, ([0,0], [0,0]), (0,0))/ \rho \in \chi\}$

Thus $(\chi, \xi)^c = (\emptyset, \xi)$.

Proposition 4.6: If $(\widehat{\mathcal{M}}, \xi)$ is an IFSCS then

(i) $(\widehat{\mathcal{M}}, \xi) \cup (\emptyset, \xi) = (\widehat{\mathcal{M}}, \xi)$.

(ii) $(\widehat{\mathcal{M}}, \xi) \cup (\chi, \xi) = (\chi, \xi)$.

Proof:

(i) $(\widehat{\mathcal{M}}, \xi) = \{\left(\rho, \widehat{\mathcal{M}}_\theta^T(\rho), \widehat{\mathcal{M}}_\theta^F(\rho)\right), \lambda_\theta^T(\rho), \lambda_\theta^F(\rho)/ \rho \in \chi, \theta \in \xi\}$

$(\emptyset, \xi) = \{(\rho, ([0,0], [0,0]), (0,0))/ \rho \in \chi\}$

$(\widehat{\mathcal{M}}, \xi) \cup (\emptyset, \xi)$

$= \{(\rho, \max(\widehat{\mathcal{M}}_\theta^T(\rho), 0), \max(\widehat{\mathcal{M}}_\theta^T(\rho), 0), \max(\lambda_\theta^T(\rho), 0), \max(\lambda_\theta^F(\rho), 0) / \rho \in \chi, \theta \in \xi\}$

$= \{\left(\rho, \widehat{\mathcal{M}}_\theta^T(\rho), \widehat{\mathcal{M}}_\theta^F(\rho)\right), \lambda_\theta^T(\rho), \lambda_\theta^F(\rho)/ \rho \in \chi, \theta \in \xi\}$

$= (\widehat{\mathcal{M}}, \xi)$.

Thus $(\widehat{\mathcal{M}}, \xi) \cup (\emptyset, \xi) = (\widehat{\mathcal{M}}, \xi)$.

(ii) $(\widehat{\mathcal{M}}, \xi) = \{\left(\rho, \widehat{\mathcal{M}}_\theta^T(\rho), \widehat{\mathcal{M}}_\theta^F(\rho)\right), \lambda_\theta^T(\rho), \lambda_\theta^F(\rho)/ \rho \in \chi, \theta \in \xi\}$

$(\chi, \xi) = \{(\rho, ([1,1], [1,1]), (1,1))/ \rho \in \chi\}$

$(\widehat{\mathcal{M}}, \xi) \cup (\chi, \xi)$

$= \{(\rho, \max(\widehat{\mathcal{M}}_\theta^T(\rho), 1), \max(\widehat{\mathcal{M}}_\theta^T(\rho), 1), \max(\lambda_\theta^T(\rho), 1), \max(\lambda_\theta^F(\rho), 1) / \rho \in \chi, \theta \in \xi\}$

$= \{(\rho, ([1,1], [1,1]), (1,1))/ \rho \in \chi, \theta \in \xi\}$

$= (\chi, \xi)$.

Thus $(\widehat{\mathcal{M}}, \xi) \cup (\chi, \xi) = (\chi, \xi)$.

Proposition 4.7: If $(\widehat{\mathcal{M}}, \xi)$ is an IFSCS then

(i) $(\widehat{\mathcal{M}}, \xi) \cap (\emptyset, \xi) = (\emptyset, \xi)$.

(ii) $(\widehat{\mathcal{M}}, \xi) \cap (\chi, \xi) = (\mathcal{M}, \xi)$.

Proof:

(i) $(\widehat{\mathcal{M}}, \xi) = \{\left(\rho, \widehat{\mathcal{M}}_\theta^T(\rho), \widehat{\mathcal{M}}_\theta^F(\rho)\right), \lambda_\theta^T(\rho), \lambda_\theta^F(\rho)/ \rho \in \chi, \theta \in \xi\}$

$(\emptyset, \xi) = \{(\rho, ([0,0], [0,0]), (0,0))/ \rho \in \chi\}$

$(\widehat{\mathcal{M}}, \xi) \cap (\emptyset, \xi)$

$= \{(\rho, \min(\widehat{\mathcal{M}}_\theta^T(\rho), 0), \min(\widehat{\mathcal{M}}_\theta^T(\rho), 0), \min(\lambda_\theta^T(\rho), 0), \min(\lambda_\theta^F(\rho), 0) / \rho \in \chi, \theta \in \xi\}$

$= \{(\rho, ([0,0], [0,0]), (0,0))/ \rho \in \chi\}$

$= (\emptyset, \xi)$.

Thus $(\widehat{\mathcal{M}}, \xi) \cap (\emptyset, \xi) = (\emptyset, \xi)$.

(ii) $(\widehat{\mathcal{M}}, \xi) = \{\left(\rho, \widehat{\mathcal{M}}_\theta^T(\rho), \widehat{\mathcal{M}}_\theta^F(\rho)\right), \lambda_\theta^T(\rho), \lambda_\theta^F(\rho)/ \rho \in \chi, \theta \in \xi\}$

$(\mathbb{U}, \xi) = \{(\rho, ([1,1], [1,1]), (1,1))/ \rho \in \chi\}$

$(\widehat{\mathcal{M}}, \xi) \cap (\chi, \xi)$

$= \{(\rho, \min(\widehat{\mathcal{M}}_\theta^T(\rho), 1), \min(\widehat{\mathcal{M}}_\theta^T(\rho), 1), \min(\lambda_\theta^T(\rho), 1), \min(\lambda_\theta^F(\rho), 1) / \rho \in \chi, \theta \in \xi\}$

$= \{\left(\rho, \widehat{\mathcal{M}}_\theta^T(\rho), \widehat{\mathcal{M}}_\theta^F(\rho)\right), \lambda_\theta^T(\rho), \lambda_\theta^F(\rho)/ \rho \in \chi, \theta \in \xi\}$

$= (\widehat{\mathcal{M}}, \xi)$.

Thus $(\widehat{\mathcal{M}}, \xi) \cap (\chi, \xi) = (\widehat{\mathcal{M}}, \xi)$.

Proposition 4.8: If $(\widehat{\mathcal{M}}, \xi_1)$ is an IFSCS and $\xi_1, \xi_2 \subseteq \xi$, then

(i) $(\widehat{\mathcal{M}}, \xi_1) \cup (\emptyset, \xi_2) = (\widehat{\mathcal{M}}, \xi_1)$ iff $\xi_2 \subseteq \xi_1$,

(ii) $(\widehat{\mathcal{M}}, \xi_1) \cup (\chi, \xi_2) = (\chi, E_2)$ iff $\xi_1 \subseteq \xi_2$,

Proof: $(\widehat{\mathcal{M}}, \xi_1) = \{\left(\rho, \widehat{\mathcal{M}}_\theta^T(\rho), \widehat{\mathcal{M}}_\theta^F(\rho)\right), \lambda_\theta^T(\rho), \lambda_\theta^F(\rho)/ \rho \in \chi, \theta \in \xi_1\}$

Also let $(\emptyset, \xi_2) = (\widehat{\mathcal{N}}, \xi_2)$, then

$\widehat{\mathcal{N}}(\theta)= \{(\rho, ([0,0], [0,0]), (0,0))/ \rho \in \chi\}$

Let $(\widehat{\mathcal{M}}, \xi_1) \cup (\emptyset, \xi_2) = (\widehat{\mathcal{M}}, \xi_1) \cup (\widehat{\mathcal{N}}, \xi_2)$ = (H, C) and $C=\xi_1 \cup \xi_2$

$$H(\theta) = \begin{cases} \widehat{\mathcal{M}}(\theta) \; if \; \theta \in \xi_1 - \xi_2 \\ \widehat{\mathcal{N}}(\theta) \; if \; \theta \in \xi_2 - \xi_1 \\ \widehat{\mathcal{M}}(\theta) \cap \widehat{\mathcal{N}}(\theta) \; if \; \theta \in \xi_1 \cap \xi_2 \end{cases}$$

$$H(\theta) = \begin{cases} \{(\rho, \left(\widehat{\mathcal{M}}_\theta^T(\rho), \widehat{\mathcal{M}}_\theta^F(\rho)\right), \lambda_\theta^T(\rho), \lambda_\theta^F(\rho)/ \rho \in \chi\} \; if \theta \in \xi_1 - \xi_2 \\ \{(\rho, \left(\widehat{\mathcal{N}}_\theta^T(\rho), \widehat{\mathcal{N}}_\theta^F(\rho)\right), \mu_\theta^T(\rho), \mu_\theta^F(\rho)/ \rho \in \chi\} \; if \theta \in \xi_2 - \xi_1 \\ \{(\rho, max\left(\widehat{\mathcal{M}}_\theta(\rho), \widehat{\mathcal{N}}_\theta(\rho)\right), \max(\lambda_\theta(\rho), \mu_\theta(\rho)/\rho \in \chi\} if \; \theta \in \xi_1 \cap \xi_2 \end{cases}$$

$$H(\theta) = \begin{cases} \{(\rho, \left(\widehat{\mathcal{M}}_\theta^T(\rho), \widehat{\mathcal{M}}_\theta^F(\rho)\right), \lambda_\theta^T(\rho), \lambda_\theta^F(\rho)/ \rho \in \chi\} \; if \theta \in \xi_1 - \xi_2 \\ \{(\rho, ([0,0], [0,0]), (0,0))/ \rho \in \chi\} \; if \theta \in \xi_2 - \xi_1 \\ \{(\rho, max(\widehat{\mathcal{M}}_\theta(\rho), 0), \max(\lambda_\theta(\rho), 0), \max(\mu_\theta(\rho), 0) /\rho \in \chi\} if \; \theta \in \xi_1 \cap \xi_2 \end{cases}$$

$Let \; E_2 \subseteq E_1$, then

$$H(\theta) = \begin{cases} \{(\rho, \left(\widehat{\mathcal{M}}_\theta^T(\rho), \widehat{\mathcal{M}}_\theta^F(\rho)\right), \lambda_\theta^T(\rho), \lambda_\theta^F(\rho)/ \rho \in \mathbb{U}\} \; if \theta \in \xi_1 - \xi_2 \\ \{(\rho, \left(\widehat{\mathcal{M}}_\theta^T(\rho), \widehat{\mathcal{M}}_\theta^F(\rho)\right), \lambda_\theta^T(\rho), \lambda_\theta^F(\rho)/ \rho \in \mathbb{U}\} \; if \theta \in \xi_1 \cap \xi_2 \end{cases}$$

$= \widehat{\mathcal{M}}$(e) for all $\theta \in \; \xi_1$.

Conversely, let $(\widehat{\mathcal{M}}, \xi_1) \cup (\emptyset, \xi_2) = (\widehat{\mathcal{M}}, \xi_1)$

Then $\xi_1 \cup \xi_2 = \xi_1$ implies $\xi_2 \subseteq \xi_1$.

(i) $(\widehat{\mathcal{M}}, \xi_1) = \{\left(\rho, \widehat{\mathcal{M}}_\theta^T(\rho), \widehat{\mathcal{M}}_\theta^F(\rho)\right), \lambda_\theta^T(\rho), \lambda_\theta^F(\rho)/ \rho \in \chi, \theta \in \; \xi_1\}$

Also let $(\chi, E_2) = (\widehat{\mathcal{N}}, \xi_2)$, then

$\widehat{\mathcal{N}}(\theta)= \{(\rho, ([1,1], [1,1]), (1,1))/ \rho \in \chi\}$

Let $(\widehat{\mathcal{M}}, \xi_1) \cup (\chi, \xi_2) = (\widehat{\mathcal{M}}, \xi_1) \cup (\widehat{\mathcal{N}}, \xi_2)$ = (H, C) and $C=\xi_1 \cup \xi_2$

$$H(\theta) = \begin{cases} \widehat{\mathcal{M}}(\theta) \; if \; \theta \in \xi_1 - \xi_2 \\ \widehat{\mathcal{N}}(\theta) \; if \; \theta \in \xi_2 - \xi_1 \\ \widehat{\mathcal{M}}(\theta) \cup \widehat{\mathcal{N}}(\theta) \; if \; \theta \in \xi_1 \cap \xi_2 \end{cases}$$

$$H(\theta) = \begin{cases} \{(\rho, \left(\widehat{\mathcal{M}}_\theta^T(\rho), \widehat{\mathcal{M}}_\theta^F(\rho)\right), \lambda_\theta^T(\rho), \lambda_\theta^F(\rho)/ \rho \in \chi\} \; if \theta \in \xi_1 - \xi_2 \\ \{(\rho, \left(\widehat{\mathcal{N}}_\theta^T(\rho), \widehat{\mathcal{N}}_\theta^F(\rho)\right), \mu_\theta^T(\rho), \mu_\theta^F(\rho)/ \rho \in \chi\} \; if \theta \in \xi_2 - \xi_1 \\ \{(\rho, max\left(\widehat{\mathcal{M}}_\theta(\rho), \widehat{\mathcal{N}}_\theta(\rho)\right), \max(\lambda_\theta(\rho), \mu_\theta(\rho)/\rho \in \chi\} if \; \theta \in \xi_1 \cap \xi_2 \end{cases}$$

$$H(\theta) = \begin{cases} \{(\rho, \left(\widehat{\mathcal{M}}_\theta^T(\rho), \widehat{\mathcal{M}}_\theta^F(\rho)\right), \lambda_\theta^T(\rho), \lambda_\theta^F(\rho)/ \rho \in \chi\} \; if \theta \in \xi_1 - \xi_2 \\ \{(\rho, ([1,1], [1,1]), (1,1))/\rho \in \chi\} \; if \theta \in \xi_2 - \xi_1 \\ \{(\rho, max(A_\theta(\rho), 1), \max(\lambda_\theta(\rho), 1), \max(\mu_\theta(\rho), 1) /\rho \in \chi\} if \; \theta \in \xi_1 \cap \xi_2 \end{cases}$$

$Let \; \xi_1 \subseteq \xi_2$, then

$$H(\theta) = \begin{cases} \{(t, (1,1), (1,1)): \rho \in \chi\} if \theta \in \xi_1 - \xi_2 \\ \{(t, (1,1), (1,1)): \rho \in \chi\} if \theta \in \xi_1 \cap \xi_2 \end{cases}$$

$= \widehat{\mathcal{M}}$(e) For all $\theta \in \; \xi_1$.

Conversely, let $(\widehat{\mathcal{M}}, \xi_1) \cup (\chi, \xi_2) = (\chi, \xi_2)$

Then $\xi_2 = \xi_1 \cup \xi_2$ implies $\xi_1 \subseteq \xi_2$.

Proposition 4.9: If $(\mathcal{M}, \xi_1)$ is an IFSCS and $\xi_1, \xi_2 \subseteq \xi$, then

(i) $(\widehat{\mathcal{M}}, \xi_1) \cap (\emptyset, \xi_2) = (\emptyset, \xi_1 \cap \xi_2)$.

(ii) $(\widehat{\mathcal{M}}, \xi_1) \cap (\chi, \xi_2) = (\widehat{\mathcal{M}}, \xi_1 \cap \xi_2)$.

Proof:

(i) $(\widehat{\mathcal{M}}, \xi_1) = \{\left(\rho, \widehat{\mathcal{M}}_\theta^T(\rho), \widehat{\mathcal{M}}_\theta^F(\rho)\right), \lambda_\theta^T(\rho), \lambda_\theta^F(\rho)/ \rho \in \chi, \theta \in \; \xi_1\}$

Also let $(\emptyset, \xi_2) = (\widehat{\mathcal{N}}, \xi_2)$, then

$\mathcal{N}(\theta)= \{(\rho, ([0,0], [0,0]), (0,0))/ \rho \in \chi\}$

Let $(\widehat{\mathcal{M}}, \xi_1) \cap (\emptyset, \xi_2) = (\widehat{\mathcal{M}}, \xi_1) \cap (\widehat{\mathcal{N}}, \xi_2)$ = (H, C) where $C=\xi_1 \cap \xi_2$ for all $\theta \in C$

$H(\theta) = \{(\rho, min\left(\widehat{\mathcal{M}}_\theta(\rho), \widehat{\mathcal{N}}_\theta(\rho)\right), \min(\lambda_\theta(\rho), \mu_\theta(\rho)/\rho \in \chi\}$

$= \{(\rho, \min(\widehat{\mathcal{M}}_\theta^T(\rho), 0), \min(\widehat{\mathcal{M}}_\theta^F(\rho), 0), \min(\lambda_\theta^T(\rho), 0), \min(\lambda_\theta^F(\rho), 0) /\rho \in \chi\}$

$= \{(\rho, ([0,0], [0,0]), (0,0))/ \rho \in \chi\}$

$= (\widehat{\mathcal{N}}, C) = (\emptyset, C)$

Hence $(\widehat{\mathcal{M}}, \xi_1) \cap (\emptyset, \xi_2) = (\emptyset, \xi_1 \cap \xi_2)$.

(ii) $(\widehat{\mathcal{M}}, \xi_1) = \{\left(\rho, \widehat{\mathcal{M}}_\theta^T(\rho), \widehat{\mathcal{M}}_\theta^F(\rho)\right), \lambda_\theta^T(\rho), \lambda_\theta^F(\rho)/ \rho \in \chi, \theta \in \; \xi_1\}$

Also let $(\chi, \xi_2) = (\widehat{\mathcal{N}}, \xi_2)$, then

$\widehat{\mathcal{N}}$(e)$= \{(\rho, ([1,1], [1,1]), (1,1))/ \rho \in \chi\}$

Let $(\widehat{\mathcal{M}}, \xi_1) \cap (\chi, \xi_2) = (\widehat{\mathcal{M}}, \xi_1) \cap (\widehat{\mathcal{N}}, \xi_2)$ = (H, C) and $C=\xi_1 \cap \xi_2$

$H(\theta) = \{(\rho, min\left(\widehat{\mathcal{M}}_\theta(\rho), \widehat{\mathcal{N}}_\theta(\rho)\right), \min(\lambda_\theta(\rho), \mu_\theta(\rho)/\rho \in \chi\}$

$= \{(\rho, \min(\widehat{\mathcal{M}}_\theta^T(\rho), 1), \min(\widehat{\mathcal{M}}_\theta^F(\rho), 1), \min(\lambda_\theta^T(\rho), 1), \min(\lambda_\theta^F(\rho), 1) /\rho \in \chi\}$

$= \left(\rho, \widehat{\mathcal{M}}_\theta^T(\rho), \widehat{\mathcal{M}}_\theta^F(\rho)\right), \lambda_\theta^T(\rho), \lambda_\theta^F(\rho)/ \rho \in \chi$ for all $\theta \in C$

$= (\widehat{\mathcal{M}}, C)$

Thus $(\widehat{\mathcal{M}}, \xi_1) \cap (\chi, \xi_2) = (\widehat{\mathcal{M}}, \xi_1 \cap \xi_2)$.

Proposition 4.10: If $(\widehat{\mathcal{M}}, \xi_1)$ and $(\widehat{\mathcal{N}}, \xi_2)$ are two IFSCSs and $\xi_1, \xi_2 \subseteq \xi$ then $((\widehat{\mathcal{M}}, \xi_1) \cup (\widehat{\mathcal{N}}, \xi_2))^c \subseteq (\widehat{\mathcal{M}}, \xi_1)^c \cup (\widehat{\mathcal{N}}, \xi_2)^c$

Proof:

Let $(\widehat{\mathcal{M}}, \xi_1) \cup (\widehat{\mathcal{N}}, \xi_2)$=(H, C) where $C = \xi_1 \cup \xi_2$ and for all $\theta \in C$

$$H(e) = \begin{cases} \left\{\left(\rho, \widehat{\mathcal{M}}_\theta^T(\rho), \widehat{\mathcal{M}}_\theta^F(\rho)\right), \lambda_\theta^T(\rho), \lambda_\theta^F(\rho): \rho \in \chi)\right\} if \; \theta \in \xi_1 - \xi_2 \\ \left\{\left(\rho, \widehat{\mathcal{N}}_\theta^T(\rho), \widehat{\mathcal{N}}_\theta^F(\rho)\right), \mu_\theta^T(\rho), \mu_\theta^F(\rho): \rho \in \chi)\right\} if \; \theta \in \xi_2 - \xi_1 \\ \{(\rho, \max\left(\widehat{\mathcal{M}}_\theta(\rho), \widehat{\mathcal{N}}_\theta(\rho)\right), \max(\lambda_\theta(\rho), \mu_\theta(\rho): \rho \in \chi\} \; if \; \theta \in \xi_1 \cap \xi_2 \end{cases}$$

Thus $((\widehat{\mathcal{M}}, \xi_1) \cup (\widehat{\mathcal{N}}, \xi_2))^c = (H, C)^c$ where $C = \xi_1 \cup \xi_2$ and for all $\theta \in C$

$$(H(\theta))^c = \begin{cases} \left(\widehat{\mathcal{M}}(\theta)\right)^c if \; \theta \in \xi_1 - \xi_2 \\ \left(\widehat{\mathcal{N}}(\theta)\right)^c if \; \theta \in \xi_2 - \xi_1 \\ \left(\widehat{\mathcal{M}}(\theta) \cup \widehat{\mathcal{N}}(\theta)\right)^c \; if \; \theta \in \xi_1 \cap \xi_2 \end{cases}$$

$$= \begin{cases} \left\{\left(\rho, 1 - \widehat{\mathcal{M}}_\theta^T(\rho), 1 - \widehat{\mathcal{M}}_\theta^F(\rho)\right), 1 - \lambda_\theta^T(\rho), 1 - \lambda_\theta^F(\rho): \rho \in \chi)\right\} if \; \theta \in \xi_1 - \xi_2 \\ \left\{\left(\rho, 1 - \widehat{\mathcal{N}}_\theta^T(\rho), 1 - \widehat{\mathcal{N}}_\theta^F(\rho)\right), 1 - \mu_\theta^T(\rho), 1 - \mu_\theta^F(\rho): \rho \in \chi)\right\} if \; \theta \in \xi_2 - \xi_1 \\ \{(\rho, 1 - \max\left(\widehat{\mathcal{M}}_\theta^T(\rho), \widehat{\mathcal{N}}_\theta^T(\rho)\right), 1 - \max\left(\widehat{\mathcal{M}}_\theta^F(\rho), \widehat{\mathcal{N}}_\theta^F(\rho)\right), 1 - \max\left(\lambda_\theta^T(\rho), \lambda_\theta^T(\rho), 1 - \max\left(\mu_\theta^F(\rho), \mu_\theta^F(\rho)\right)\right)\} \\ if \; \theta \in \xi_1 \cap \xi_2 \end{cases}$$

Again $(\widehat{\mathcal{M}}, \xi_1)^c \cup (\widehat{\mathcal{N}}, \xi_2)^c = (I, J)$ say $J = \xi_1 \cup \xi_2$ and for all $\theta \in J$

$$I(\theta) = \begin{cases} \left(\widehat{\mathcal{M}}(\theta)\right)^c if \; \theta \in \xi_1 - \xi_2 \\ \left(\widehat{\mathcal{N}}(\theta)\right)^c if \; \theta \in \xi_2 - \xi_1 \\ \left(\widehat{\mathcal{M}}(\theta) \cup \widehat{\mathcal{N}}(\theta)\right)^c \; if \; \theta \in \xi_1 \cap \xi_2 \end{cases}$$

$$= \begin{cases} \left\{\left(\rho, 1 - \widehat{\mathcal{M}}_\theta^T(\rho), 1 - \widehat{\mathcal{M}}_\theta^F(\rho)\right), 1 - \lambda_\theta^T(\rho), 1 - \lambda_\theta^F(\rho): \rho \in \chi)\right\} if \; \theta \in \xi_1 - \xi_2 \\ \left\{\left(\rho, 1 - \widehat{\mathcal{N}}_\theta^T(\rho), 1 - \widehat{\mathcal{N}}_\theta^F(\rho)\right), 1 - \mu_\theta^T(\rho), 1 - \mu_\theta^F(\rho): \rho \in \chi)\right\} if \; \theta \in \xi_2 - \xi_1 \\ \{(\rho, 1 - \max\left(\widehat{\mathcal{M}}_\theta^T(\rho), \widehat{\mathcal{N}}_\theta^T(\rho)\right), 1 - \max\left(\widehat{\mathcal{M}}_\theta^F(\rho), \widehat{\mathcal{N}}_\theta^F(\rho)\right), 1 - \max(\lambda_\theta^T(\rho), \lambda_\theta^T(\rho), 1 - \max(\mu_\theta^F(\rho), \mu_\theta^F(\rho)))\} \\ if \; \theta \in \xi_1 \cap \xi_2 \end{cases}$$

Published By:
Blue Eyes Intelligence Engineering
& Sciences Publication

$C \subseteq J$ for all $\theta \in J$. $\left(H(\theta)\right)^c \subseteq I(\theta)$.

Thus $((\widehat{\mathcal{M}},\xi_1) \cup (\widehat{\mathcal{N}},\xi_2))^c \subseteq (\widehat{\mathcal{M}},\xi_1)^c \cup (\widehat{\mathcal{N}},\xi_2)^c$

Proposition 4.11: If $\left(\widehat{\mathcal{M}},\xi\right)$ and $\left(\widehat{\mathcal{N}},\xi\right)$ are two IFSCSs then

(i)$(\left(\widehat{\mathcal{M}},\xi\right) \cup \left(\widehat{\mathcal{N}},\xi\right))^c = (\widehat{\mathcal{M}},\xi)^c \cap (\widehat{\mathcal{N}},\xi)^c$

(ii) $(\left(\widehat{\mathcal{M}},\xi\right) \cap \left(\widehat{\mathcal{N}},\xi\right))^c = (\widehat{\mathcal{M}},\xi)^c \cup (\widehat{\mathcal{N}},\xi)^c$

Proof:

(i)Consider $\left(\widehat{\mathcal{M}},\xi\right) \cup \left(\widehat{\mathcal{N}},\xi\right) = (H,\xi)$ and

$H(\theta) = \widehat{\mathcal{M}}(\theta) \cup \widehat{\mathcal{N}}(\theta) = \{(\rho, \max(\widehat{\mathcal{M}}_\theta^T(\rho), \widehat{\mathcal{N}}_\theta^T(\rho)), \max(\widehat{\mathcal{M}}_\theta^F(\rho), \widehat{\mathcal{N}}_\theta^F(\rho))\}$

Thus $(\left(\widehat{\mathcal{M}},\xi\right) \cup \left(\widehat{\mathcal{N}},\xi\right))^c = (H,\xi)^c$

$(H(\theta))^c = (\widehat{\mathcal{M}}(\theta) \cup \widehat{\mathcal{N}}(\theta))^c$

$= \{(\rho, \max(\widehat{\mathcal{M}}_\theta^T(\rho), \widehat{\mathcal{N}}_\theta^T(\rho)), \max(\widehat{\mathcal{M}}_\theta^F(\rho), \widehat{\mathcal{N}}_\theta^F(\rho))\}^c$

$= \{(\rho, 1 - max(\widehat{\mathcal{M}}_\theta^T(\rho), \widehat{\mathcal{N}}_\theta^T(\rho)), 1 - \max(\widehat{\mathcal{M}}_\theta^F(\rho), \widehat{\mathcal{N}}_\theta^F(\rho))\}$

Again $(\widehat{\mathcal{M}},\xi)^c \cap (\widehat{\mathcal{N}},\xi)^c = (I,\xi)$

$I(\theta) = \left(\widehat{\mathcal{M}}(\theta)\right)^c \cap \left(\widehat{\mathcal{N}}(\theta)\right)^c$

$= \{(\rho, \min(1 - \widehat{\mathcal{M}}_\theta^T(\rho), 1 - \widehat{\mathcal{N}}_\theta^T(\rho)), \min(1 - \widehat{\mathcal{M}}_\theta^F(\rho), 1 - \widehat{\mathcal{N}}_\theta^F(\rho))\}^c$

$= \{(\rho, 1 - \max(\widehat{\mathcal{M}}_\theta^T(\rho), \widehat{\mathcal{N}}_\theta^T(\rho)), 1 - \max(\widehat{\mathcal{M}}_\theta^F(\rho), \widehat{\mathcal{N}}_\theta^F(\rho))\}^c$

Thus$(\left(\widehat{\mathcal{M}},\xi\right) \cup \left(\widehat{\mathcal{N}},\xi\right))^c = (\widehat{\mathcal{M}},\xi)^c \cap (\widehat{\mathcal{N}},\xi)^c$.

(ii) Let $\left(\widehat{\mathcal{M}},\xi\right) \cap \left(\widehat{\mathcal{N}},\xi\right) = (H,\xi)$

$H(\theta) = \widehat{\mathcal{M}}(\theta) \cap \widehat{\mathcal{N}}(\theta) = \{(\rho, \min(\widehat{\mathcal{M}}_\theta^T(\rho), \widehat{\mathcal{N}}_\theta^T(\rho)), \min(\widehat{\mathcal{M}}_\theta^F(\rho), \widehat{\mathcal{N}}_\theta^F(\rho))\}$

Thus $(\left(\widehat{\mathcal{M}},\xi\right) \cap \left(\widehat{\mathcal{N}},\xi\right))^c = (H,\xi)^c$

$(H(\theta))^c = (\widehat{\mathcal{M}}(\theta) \cap \widehat{\mathcal{N}}(\theta))^c$

$= \{(\rho, \min(\widehat{\mathcal{M}}_\theta^T(\rho), \widehat{\mathcal{N}}_\theta^T(\rho)), \min(\widehat{\mathcal{M}}_\theta^F(\rho), \widehat{\mathcal{N}}_\theta^F(\rho))\}^c$

$= \{(\rho, 1 - min(\widehat{\mathcal{M}}_\theta^T(\rho), \widehat{\mathcal{N}}_\theta^T(\rho)), 1 - \min(\widehat{\mathcal{M}}(\rho), \widehat{\mathcal{N}}_\theta^F(\rho))\}$

Again $(\widehat{\mathcal{M}},\xi)^c \cup (\widehat{\mathcal{N}},\xi)^c = (I,\xi)$

$I(\theta) = (\widehat{\mathcal{M}}(\theta))^c \cup (\widehat{\mathcal{N}}(\theta))^c$

$= \{(\rho, \max(1 - \widehat{\mathcal{M}}_\theta^T(\rho), 1 - \widehat{\mathcal{N}}_\theta^T(\rho)), \max(1 - \widehat{\mathcal{M}}_\theta^F(\rho), 1 - \widehat{\mathcal{N}}_\theta^F(\rho))\}^c$

$= \{(\rho, 1 - \min(\widehat{\mathcal{M}}_\theta^T(\rho), \widehat{\mathcal{N}}_\theta^T(\rho)), 1 - \min(\widehat{\mathcal{M}}_\theta^F(\rho), \widehat{\mathcal{N}}_\theta^F(\rho))\}^c$

Thus$(\left(\widehat{\mathcal{M}},\xi\right) \cap (\mathcal{N},\xi))^c = (\widehat{\mathcal{M}},\xi)^c \cup (\widehat{\mathcal{N}},\xi)^c$.

Proposition 4.12: If $\left(\widehat{\mathcal{M}},\xi_1\right)$ and $\left(\widehat{\mathcal{N}},\xi_2\right)$ are two IFSCSs and $\xi_1,\xi_2 \subseteq \xi$,then

(i)$(\left(\widehat{\mathcal{M}},\xi_1\right) \wedge (\mathcal{N},\xi_2))^c = (\widehat{\mathcal{M}},\xi_1)^c \vee (\widehat{\mathcal{N}},\xi_2)^c$

(ii) $(\left(\widehat{\mathcal{M}},\xi_1\right) \vee (\mathcal{N},\xi_2))^c = (\widehat{\mathcal{M}},\xi_1)^c \wedge (\widehat{\mathcal{N}},\xi_2)^c$

Proof:

(i)Let $\left(\widehat{\mathcal{M}},\xi_1\right) \wedge \left(\widehat{\mathcal{N}},\xi_2\right) = (H,\xi_1 X \xi_2)$ and

$H(m,n) = \{(\rho, \mathrm{Min}(\widehat{\mathcal{M}}_\theta^T(\rho), \widehat{\mathcal{N}}_\theta^T(\rho)), \mathrm{Min}(\widehat{\mathcal{M}}_\theta^F(\rho), \widehat{\mathcal{N}}_\theta^F(\rho))\}$ for all $m \in \xi_1, n \in \xi_2$

$((v,\xi_1) \wedge \left(\widehat{\mathcal{N}},\xi_2\right))^c = (H,\xi_1 X \xi_2)^c \; for\ all\ (a,b) \in \xi_1 X \xi_2$

$(H(m,n))^c = \{(\rho, 1 - \min(\widehat{\mathcal{M}}_\theta^T(\rho), \widehat{\mathcal{N}}_\theta^T(\rho)), 1 - \min(\mathcal{M}\theta F\rho, \mathcal{N}\theta F\rho)\}c$for all $m\in\xi 1, n\in\xi 2$

$Let\ (\widehat{\mathcal{M}},\xi_1)^c \vee (\widehat{\mathcal{N}},\xi_2)^c = (R,\xi_1 X \xi_2)$and

$R(m,n) = \{(\rho, \mathrm{Max}(1 - \widehat{\mathcal{M}}_\theta^T(\rho), 1 - \widehat{\mathcal{N}}_\theta^T(\rho)), \mathrm{Max}(1 - \widehat{\mathcal{M}}_\theta^F(\rho), 1 - \widehat{\mathcal{N}}_\theta^F(\rho)\}\}$ for all $m \in \xi_1, n \in \xi_2$

$= \{(\rho, 1 - \min\left(\widehat{\mathcal{M}}_\theta^T(\rho), \widehat{\mathcal{N}}_\theta^T(\rho)\right), 1 - \min\left(\widehat{\mathcal{M}}_\theta^F(\rho), \widehat{\mathcal{N}}_\theta^F(\rho)\right)\}$ for all $m \in \xi_1, n \in \xi_2$

Thus$(\left(\widehat{\mathcal{M}},\xi_1\right) \wedge (\mathcal{N},\xi_2))^c = (\widehat{\mathcal{M}},\xi_1)^c \vee (\widehat{\mathcal{N}},\xi_2)^c$.

(ii) Let $\left(\widehat{\mathcal{M}},\xi_1\right) \vee \left(\widehat{\mathcal{N}},\xi_2\right) = (H,\xi_1 X \xi_2)$ where

$H(m,n) = \{(\rho, \max(\widehat{\mathcal{M}}_\theta^T(\rho), \widehat{\mathcal{N}}_\theta^T(\rho)), \max(\widehat{\mathcal{M}}_\theta^F(\rho), \widehat{\mathcal{N}}_\theta^F(\rho))\}$ for all $m \in \xi_1, n \in \xi_2$

$(\left(\widehat{\mathcal{M}},\xi_1\right) \vee \left(\widehat{\mathcal{N}},\xi_2\right))^c = (H,\xi_1 X \xi_2)^c \; for\ all\ (a,b) \in \xi_1 X \xi_2$

$(H(m,n))^c = \{(\rho, 1 - \max(\widehat{\mathcal{M}}_\theta^T(\rho), \widehat{\mathcal{N}}_\theta^T(\rho)), 1 - \max(\mathcal{M}\theta F\rho, \mathcal{N}\theta F\rho)\}c$for allm $\in \xi_1, n \in \xi_2$

$Let\ (\widehat{\mathcal{M}},\xi_1)^c \wedge (\widehat{\mathcal{N}},\xi_2)^c = (R,\xi_1 X \xi_2)$ Where

R(m,n)= $\{(\rho, \min(1 - \widehat{\mathcal{M}}_\theta^T(\rho), 1 - \widehat{\mathcal{N}}_\theta^T(\rho)), \min(1 - \mathcal{M}\theta F\rho, 1 - \mathcal{N}\theta F\rho\}\}$forall $m\in\xi 1, n\in\xi 2$

$= \{(\rho, 1 - \max\left(\widehat{\mathcal{M}}_\theta^T(\rho), \widehat{\mathcal{N}}_\theta^T(\rho)\right), 1 - \max\left(\widehat{\mathcal{M}}_\theta^F(\rho), \widehat{\mathcal{N}}_\theta^F(\rho)\right)\}$ for all $m \in \xi_1, n \in \xi_2$

Thus $(\left(\widehat{\mathcal{M}},\xi_1\right) \vee \left(\widehat{\mathcal{N}},\xi_2\right))^c = (\widehat{\mathcal{M}},\xi_1)^c \wedge (\widehat{\mathcal{N}},\xi_2)^c$

V. CONCLUSION

The coneitof an IFSCS is proposed and some operations are defined on IFSCS. Some properties are deliberated regarding the conceit of IFSCS. We expect the future work on topological spaces upon this conceit.

REFERENCES

1. K.A.Atansov, G.Gargov, Interval valued intuitionistic fuzzy sets, Fuzzy sets and systems31 (1989), 343-349, North holand.
2. D. Molodtsov, Soft set theory-First results, Computers Math. Applic. 37 (4/5), 19-31, (1999).
3. K. Atanassov and S. Stoeva, Intuitionistic fuzzy sets, in: Polish Symp.
4. K. Atanassov, Intuitionistic fuzzy sets, in: V. Sgurev, Ed., and VII ITKR's Session, Sofia, June 1983 (Central Sci. and Techn. Library, Bulg. Academy of Sciences, 1984).
5. Y.B.Jun, C.S.Kim and K.O.Yang, Cubic sets, Annals of fuzzy mathematics and Information-4(1) (2012)8398.
6. L.A.Zadeh, Fuzzy sets, Inform. Control 8 (1965) 338-353.
7. L.Zadeh, The concept of a linguistic variable and its application to approximate reasoning-I, Inform.Sci. 8(1975), 199-249.
8. P.K.Maji, R.Biswas, A.R.Roy, Fuzzy soft sets, J.Fuzzy Math. 9(3) (2001), 589-602.
9. P.K.Maji, R.Biswas, A.R.Roy, Intuitionistic Fuzzy soft sets, J.Fuzzy Math. 9(3) (2001), 677-692.