

# SMC Design for Flexible Link Manipulator in Presence of External Disturbances

Naveen Kumar, Jyoti Ohri



**Abstract:** A flexible link manipulator (FLM) has become the globally research topic over last two decades. It has various advantages such as light weight, high speed with low inertia, large work space and consumes less energy comparatively. However, this flexibility make system more complex. The FLM performance is measured in terms of accuracy of trajectory tracking with minimum oscillations. But in case of FLM, due to flexibility, oscillation and accuracy in trajectory has been increased. To overcome this problem, a robust nonlinear observer based sliding mode controller (SMC) has been used in this paper. Moreover, dynamics modelling of FLM has been developed using Lagrange Method. To enhance the tracking of FLM, integral sliding mode controller (i-SMC) has been designed. The effectiveness of these controller has been tested in presence of disturbances at each state and result obtained are demonstrated

**Keywords :** SMC, Integral SMC, Flexible Link Manipulator, Disturbance and MATLAB

## I. INTRODUCTION

Robotic manipulator has key demands in industries where repetitive task are required to perform with high accuracy. These link are the primary stage of haptic devices where they provide user input to the virtual environment [1], [2]. The manipulators are basically mechanically combination of links to perform various tasks. These links are of different type i.e. rigid and flexible. Flexible link have better advantages over rigid one in terms of light weight, speed of operation, low inertia, low power consumption etc. In the past few decade, FLM has received great attention of researcher due to the said advantages [3]. FLM has disadvantages in terms of complex dynamic structure than rigid link. It makes more challenging task for control law designer where precise control action is always desirable. But due to the FLM flexibility, precise position tracking is very difficult task [4], [5]. So a robust control is required which can eliminate the oscillations while tracking the position for FLM.

The SMC control has basically two step of operation: reaching phase and sliding phase. In reaching phase, a system is forced towards a stable sliding manifold. In sliding phase, system is force to slide over manifold surface to reach an equilibrium point as shown in figure 1 [6]. The implementation of SMC controller is based on selection of sliding surface selection using which desired performance can be achieved. Next a control law is to design that drives state trajectories towards sliding surface manifold

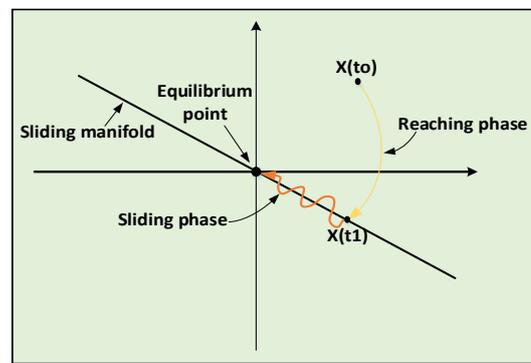


Figure 1: Sliding mode control basic working

The linear controller are designed for linearized system and SMC are used mainly for nonlinear system such as inverted pendulum, FLM etc. But the linear system are not suitable for the system where equilibrium point system state are far distant.

The recent advancement in technology has increased the application of flexible link manipulator in the field of robotics (example Lightweight Robot), where high precision and accuracy is required. In case of FLM, additional complexity has been added due to flexibility in the link. This make system dynamics more complex compare to rigid link. And hence, increase the difficult for control engineers to design a control law [7]. Further, flexibility in the link decay the preciseness of system tracking which leads more oscillations in tracking a trajectory. So, it is highly desired to have a control law which can abolish the system oscillations and enhance systems capability of tracking [8]. Different author have various control strategies in literature to follow the desired trajectories for FLM. H. Geniele, et al [9] have designed a control theory for end or top point control for FLM. Owais Khan, et al [10] have used SMC and linear quadratic regulator (LQR) for FLM and corresponding results shows effectiveness of control law. Ali Dehghani, et al [11] have presented self-tuning PID control using fuzzy logic for FLM.

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To evaluate the stability of the proposed controller, some uncertainty is assumed in system parameters. Organization of this paper is as follow: section 2 presents mathematical model of FLM. The conventional SMC and integral SMC based control law is designed in section 3. The simulation and results are displayed in section 4. The work is concluded in section 5.

## II. MATHEMATICAL MODEL OF FLM

The FLM is current are of research due to its advantages over rigid link in terms of light weight, low inertia, larger workspace and high speed of operations. The dynamic model of FLM has been developed using Lagrange Method. The strain gauge has been used to detect the deflection of tip of flexible link and clamped at one end near to base. The other end is free to move as shown in Figure 2. The flexible link is rotated by means of DC motor as actuator. The output of strain gauge is proportional to the link deflection.

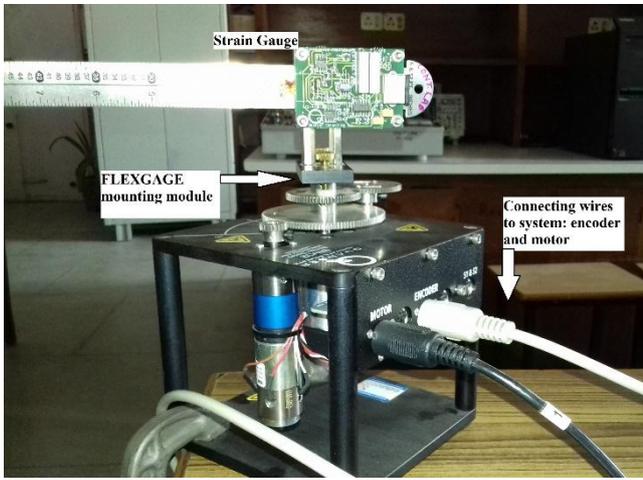


Figure 2: Strain Gauge and DC motor connection to FLM

The input to DC motor is voltage (V), the outputs are the arm deflection angle ( $\alpha$ ) and angular position ( $\theta$ ) [4], [12]. The symbolic representation for FLM is shown in Figure 3.

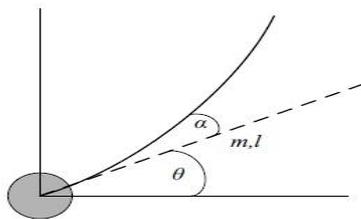


Figure 3: Systemic Flexible Link [12].

Euler-LaGrange method has been used to obtain the linearization dynamic model for FLM. Various notation used in following equations are tabulated in table I. The difference between total Kinetic Energy (KE) and total Potential Energy (PE) of the system is equal to Lagrangian function (L) as shown in (1).

$$L = KE - PE \quad (1)$$

The total KE is sum of KE of hub and KE of link.

$$KE_{total} = KE_{Hub} + KE_{Link} = T_{output} \quad (2)$$

$$T_{output} = \frac{1}{2}J_{eq}\dot{\theta}^2 + \frac{1}{2}J_{Link}(\dot{\theta} + \dot{\alpha})^2 \quad (3)$$

The total spring potential energy is

$$PE_{total} = PE_{spring} = \frac{1}{2}K_{stiff} \alpha^2 \quad (4)$$

Two Euler equations has been developed for two parameters  $\theta$  and  $\alpha$  as:

$$\frac{\delta}{\delta t} \left( \frac{\delta L}{\delta \dot{\theta}} \right) - \frac{\delta L}{\delta \theta} = T_{output} - B_{eq} \dot{\theta} \quad (5)$$

$$\frac{\delta}{\delta t} \left( \frac{\delta L}{\delta \dot{\alpha}} \right) - \frac{\delta L}{\delta \alpha} = 0 \quad (6)$$

By using (1), (5) and (6) we obtain,

$$J_{eq} \ddot{\theta} + J_{Link}(\ddot{\theta} + \ddot{\alpha}) = T_{output} - B_{eq} \dot{\theta} \quad (7)$$

$$J_{Link}(\ddot{\theta} + \ddot{\alpha}) + K_{stiff} \alpha = 0 \quad (8)$$

The rotation of the flexible link is done by means of DC Motor. It provides an equivalent torque to rotate the flexible link. The block diagram model of DC Motor is shown in Figure 4. The equivalent output torque on the flexible link from the DC Motor is given in (9) [12].

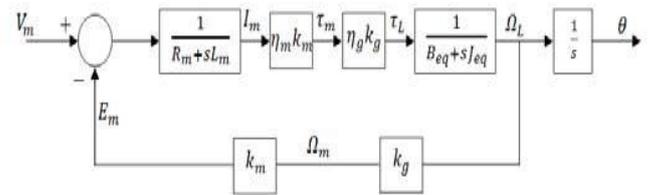


Figure 4: Block Diagram of DC motor.

$$T_{output} = \frac{\eta_m \eta_g K_t K_g (V_m - K_g K_m \dot{\theta})}{R_m} \quad (9)$$

The state space model for FLM using the equations (7), (8) and (9), is given in equation (11) and (12) choosing  $\dot{\theta}$ ,  $\dot{\alpha}$ ,  $\dot{\theta}$  and  $\dot{\alpha}$  as system states [12].

$$\begin{bmatrix} \dot{\theta} \\ \dot{\alpha} \\ \dot{\theta} \\ \dot{\alpha} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \frac{K_{stiff}}{J_{eq}} & -\frac{\eta_m \eta_g K_m K_t K_g^2 + B_{eq} R_m}{J_{eq} R_m} & 0 \\ 0 & -\frac{K_{stiff}(J_{eq} + J_{arm})}{J_{eq} J_{arm}} & \frac{\eta_m \eta_g K_m K_t K_g^2 + B_{eq} R_m}{J_{eq} R_m} & 0 \end{bmatrix} \begin{bmatrix} \theta \\ \alpha \\ \dot{\theta} \\ \dot{\alpha} \end{bmatrix} + \begin{bmatrix} 0 & 0 & \frac{\eta_m \eta_g K_m K_g}{J_{eq} R_m} & -\frac{\eta_m \eta_g K_m K_g}{J_{eq} R_m} \end{bmatrix}^T V \quad (10)$$

$$\theta = [1 \ 0 \ 0 \ 0] [\theta \ \alpha \ \dot{\theta} \ \dot{\alpha}]^T \quad (11)$$

The FLM state space model in equation (10) & (11), has been used to Sliding Mode control design in section 3.

Table 1: Parameters of FLM

Parameter	Variable	Value
Equivalent inertia of load (Kg m <sup>2</sup> )	$J_{eq}$	0.0026 kg.m <sup>2</sup>
Inertia of link (Kg m <sup>2</sup> )	$J_{Link}$	0.0038 kg.m <sup>2</sup>
Equivalent viscous friction (N.m.s/rad)	$B_{eq}$	0.004 N.m.s/rad
Gear box efficiency	$\eta_g$	0.9
Gear ratio	$K_g$	14:5
Motor torque constant (N.m/A)	$K_t$	0.00767 N.m/A

Armature Resistance ( $\Omega$ )	$R_m$	2.6 $\Omega$
Motor efficiency	$\eta_m$	0.69
Motor back emf constant (V.s/rad)	$K_m$	0.00767 V.s/rad
Mass of Link (kg)	m	0.065 kg
Link stiffness	$K_{stiff}$	1.3522
Length of Link (m)	$l$	0.42 m

The nomenclature of variables used for the state space model is given in following table.

**Table 2: Description of System State [13]**

Variable	States	Description
$x_1$	$\theta$	Angular position of link
$x_2$	$\alpha$	Angular position of link deviation
$x_3$	$\dot{\theta}$	Angular velocity of link
$x_4$	$\dot{\alpha}$	Angular velocity of Link deviation

State space model is given by,

$$\dot{x}_1 = x_3 \quad (12)$$

$$\dot{x}_2 = x_4 \quad (13)$$

$$\dot{x}_3 = Ax_2 - Bx_3 + Cu \quad (14)$$

$$\dot{x}_4 = -Dx_2 + Bx_3 - Cu \quad (15)$$

$$y = x_1 \quad (16)$$

where,  $u$  is the input and  $y$  is the output.

$$A = \frac{K_{stiff}}{J_{eq}}, \quad B = \frac{\eta_m \eta_g K_m K_t K_g^2 + B_{eq} R_m}{J_{eq} R_m}$$

$$C = \frac{\eta_m \eta_g K_m K_g}{J_{eq} R_m}, \quad D = \frac{K_{stiff}(J_{eq} + J_{arm})}{J_{eq} J_{arm}}$$

### III. SLIDING MODE CONTROLLER

Sliding mode control is a nonlinear method used to ensure the tracking of desired trajectories. The principle of operation of SMC is to keep the state variable of system on sliding manifold. Next, state variable of controlled system slides over the sliding surface towards the equilibrium point.

#### A. Sliding Mode Controller (SMC) Design

The error dynamics of FLM system is defined in equation (17),

$$e = y - y_d \quad (17)$$

Here,  $y$  is output trajectory and  $y_d$  is the desired trajectory of the system. Differentiating (17)

$$\dot{e} = \dot{y} - \dot{y}_d$$

$$y = x_1$$

$$\dot{e} = \dot{x}_1 - \dot{y}_d$$

$$\dot{x}_1 = x_3$$

$$\dot{e} = x_3 - \dot{y}_d \quad (18)$$

Again differentiating (19)

$$\ddot{e} = \dot{x}_3 - \ddot{y}_d \quad (19)$$

The equation for the sliding surface is given as

$$s_1 = \dot{e} + \lambda_1 e = 0 \quad (21)$$

where  $\dot{e}$  is the derivative of error

Differentiating (21), gives,

$$\dot{s}_1 = \ddot{e} + \lambda_1 \dot{e} = 0$$

$$s_1 = \dot{x}_3 - \ddot{y}_d + \lambda_1(x_3 - \dot{y}_d) = 0$$

$$s_1 = Ax_2 + Bx_3 + Cu - \ddot{y}_d + \lambda_1(x_3 - \dot{y}_d) = 0 \quad (22)$$

To obtain the stabilize dynamic system in equation (22), the SMC control law  $u$  is chosen as,

$$u = u_{eq} + u_c$$

$$u_{eq} = [C]^{-1}[-Ax_2 - Bx_3 + \ddot{y}_d - \lambda_1(x_3 - \dot{y}_d)]$$

$$u_c = [C]^{-1}[-K_1|s_1|^\alpha \text{sgn}(s_1)]$$

$$u = [C]^{-1}[-Ax_2 - Bx_3 + \ddot{y}_d - \lambda_1(x_3 - \dot{y}_d) - K_1|s_1|^\alpha \text{sgn}(s_1)] \quad (23)$$

Where  $K_1$  is designed to ensure sliding with  $0 < \alpha < 1$ .

#### B. Integral Sliding Mode Controller

The integral sliding surface for the system is selected as [14]

$$s_2 = \dot{e} + 2\lambda_2 e + \lambda_2^2 \int_0^t e(t) dt = 0 \quad (24)$$

Differentiating (24),

$$\dot{s}_2 = \ddot{e} + 2\lambda_2 \dot{e} + \lambda_2^2 e = 0$$

$$\dot{s}_2 = \dot{x}_3 - \ddot{y}_d + 2\lambda_2(x_3 - \dot{y}_d) + \lambda_2^2(x_1 - y_d) = 0$$

$$\dot{s}_2 = Ax_2 + Bx_3 + Cu - \ddot{y}_d + 2\lambda_2(x_3 - \dot{y}_d) + \lambda_2^2(x_1 - y_d) = 0 \quad (25)$$

Control law  $u$  to stabilize the dynamics of system in equation (25) is,

$$u = u_{eq} + u_c$$

$$u_{eq} = [C]^{-1}[-Ax_2 - Bx_3 + \ddot{y}_d - 2\lambda_2(x_3 - \dot{y}_d) - \lambda_2^2(x_1 - y_d)]$$

$$u_c = [C]^{-1}[-K_2|s_2|^\alpha \text{sgn}(s_2)]$$

$$u = [C]^{-1}[-Ax_2 - Bx_3 + \ddot{y}_d - 2\lambda_2(x_3 - \dot{y}_d) - K_2|s_2|^\alpha \text{sgn}(s_2) - \lambda_2^2 e] \quad (26)$$

Where  $K_2$  is designed to ensure sliding with  $0 < \alpha < 1$ .

### IV. SIMULATION AND RESULT

The FLM described in section 3, for the angular position control, have been simulated in this section. For the simulation, following parameters has been used in the closed loop control system,

$$K_1 = K_2 = 5, \lambda_1 = \lambda_2 = 2, \alpha = 0.5$$

The Flexible Link Manipulator system is used with following initial conditions

$$\text{Initial condition} = [0.1 \ 0 \ 0 \ 0]$$

Further, robustness of designed controller is validated by adding the following disturbances at each state of FLM.

$$dt1 = 0.02 \sin(t)$$

$$dt2 = 0.02 \sin(t)$$

$$dt3 = 0.02 \sin(t)$$

$$dt4 = 0.02 \sin(t) \quad (27)$$

The model is tested with dynamics characteristic designed control scheme with reference input.

The desired trajectory for the tracking of angle of Flexible link is chosen as,

$$y_d = 0.3 \sin(0.5t) + 0.2 \cos(0.5t) \quad (28)$$

$$\dot{y}_d = 0.15 \cos(0.5t) - 0.1 \sin(0.5t) \quad (29)$$

Here, desired objective of the designed control system is to examine the performance in terms of trajectory tracking and disturbance cancellation, i.e. tracking of tip position w.r.t the desired trajectory.

#### A. Conventional Sliding Mode Controller (C-SMC)

In previous section angular position control of FLM has been implemented using conventional sliding mode controller in the presence of disturbances. First, simulation has been performed using FLM angular tracking without disturbance with respect to reference trajectory is shown in Figure 5.



The corresponding error obtained is shown in Figure 6. 0

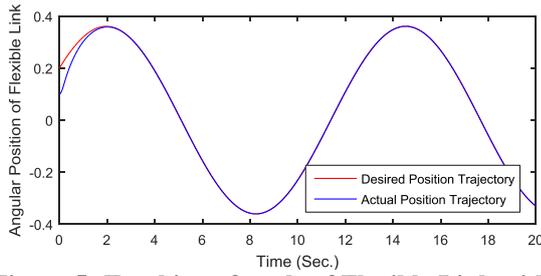


Figure 5: Tracking of angle of Flexible Link without disturbances

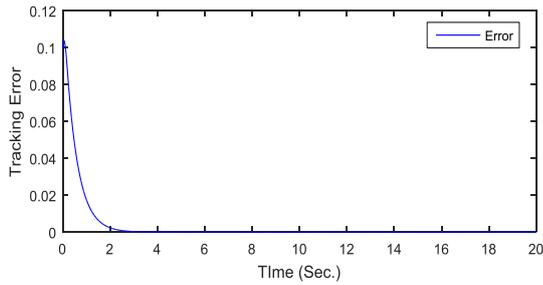


Figure 6: Tracking of error of Flexible Link without disturbances

The tracking of the angle of flexible link manipulator with disturbance with respect to reference trajectory is shown in Figure 7. The corresponding error obtained is shown in Figure 8.

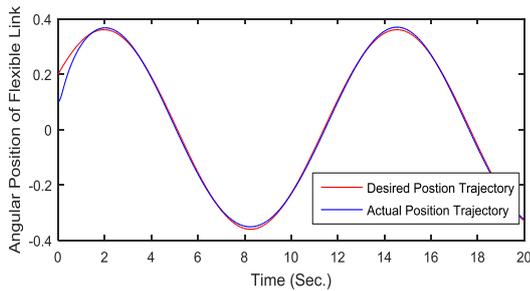


Figure 7: Tracking of angle of Flexible Link with disturbances

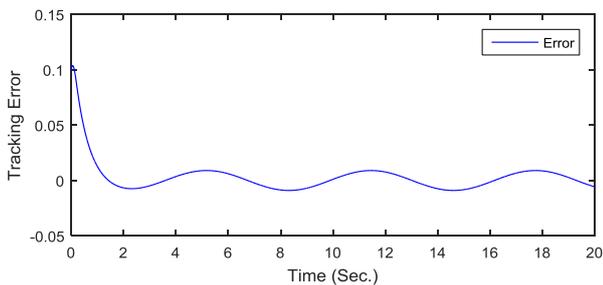


Figure 8: Tracking error of Flexible Link with disturbances

**B. Integral Sliding Mode Controller (I-SMC)**

Further Integral sliding model controller has been designed for FLM without disturbance. The tracking of the angle of flexible link manipulator without disturbance with respect to reference trajectory is shown in Figure 9. The corresponding error obtained is shown in Figure 10.

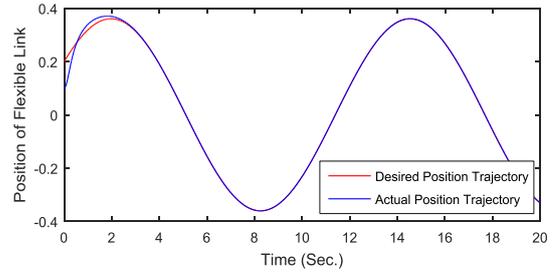


Figure 9: Tracking of angle of Flexible Link without disturbances using Integral SMC

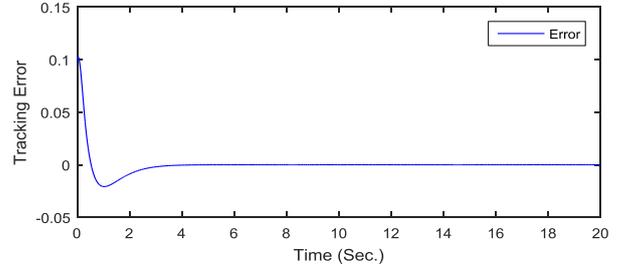


Figure 10: Tracking of error of Flexible Link without disturbances using Integral SMC

The tracking of the angle of flexible link manipulator with disturbance with respect to reference trajectory is shown in Figure 11. The corresponding error obtained is shown in Figure 12.

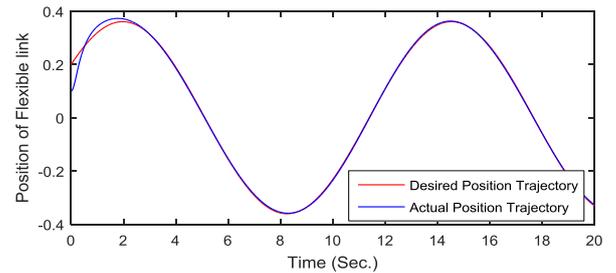


Figure 11: Tracking of angle of Flexible Link with disturbances using Integral SMC

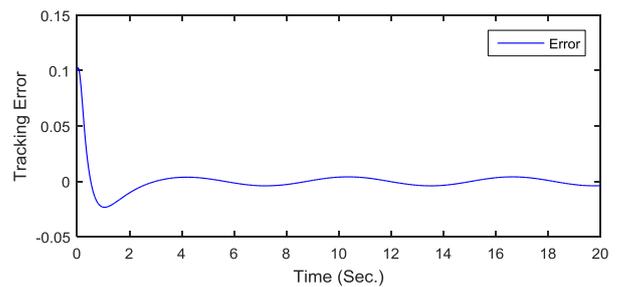


Figure 12: Tracking of error of Flexible Link with disturbances using Integral SMC

In the Figure 5 to Figure 12, tracking error with and without disturbance has been presented for FLM system using C-SMC and I-SMC respectively. These results shows that, tracking error has been approaching near to zero which validates the effectiveness of the control law. Moreover, the designed control law has shown its robustness in presence of disturbance. The comparative analysis has been shown in table 3 for rms (root means square) error.

This shows that I-SMC gives better performance for flexible link manipulator in with and without disturbance.

**Table 3: Flexible Link Tracking Error**

Controller Applied to Flexible Link Manipulator	Flexible Link Manipulator Position Tracking Error	
	Without Disturbance	With Disturbance
C-SMC	0.0099	0.0129
I-SMC	0.0082	0.0098

### V. CONCLUSION

This paper proposed two control strategies for flexible link manipulator: the conventional sliding mode control (C-SMC) and a novel integral sliding mode control (I-SMC). These control strategies have been implemented on dynamic model developed using Euler-LaGrange method for flexible link. The effectiveness of designed control law has been validated using simulation in MATLAB. The acquired result shows that tracking of flexible link has been improved. Moreover, robustness of designed control has been validated in presence of disturbance at each state of flexible link. Future of these control strategies is rested for hardware testing.

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