Effects of Spring and Damper Elements in Aircraft Landing Gear Dynamics

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Abstract: In this study, a typical Aircraft Landing Gear with shock absorber was modeled as a Mass-Spring-Damper System. Basic components of the system were explained. The equations of the model were presented. Aircraft Landing Gear was also modeled in Matlab/Simulink for a given set of aircraft parameters. A case study for an Aircraft Landing Gear was solved and results were presented. Results included the variation of spring \( k_1 \) and damping coefficient \( b \) in a given interval to show their effects on the impact force and displacement of landing gear as main outputs to consider. Effect of damping coefficient \( b \) on impact force was found to be highest \((3.76\%)\), spring coefficient \( k_1 \) effect is moderate \((2.29\%)\), and spring coefficient \( k_2 \) is lower \((0.97\%)\), for a change of \( \pm 10\% \) of coefficients.

Keywords: Aircraft landing gear, mass-spring-damper system, shock absorber, system modeling.

I. INTRODUCTION

Shock absorbers are used in many fields from automotive to aviation today. There are designed for each specific application for a set of customer and design requirements. Although their main function is to absorb the impact energy of moving objects, they have basically two main components which are spring and damper. Shock Absorbers reduce the effects of travelling over rough ground for a vehicle on road or helps an aircraft to land more smoothly.


II. MATERIAL AND METHOD

An aircraft landing gear can be modelled as a mass-spring-damper system. Fig. 1 shows a simplified model of landing gear. In this model shock absorber was modelled as spring and damper, tire was modelled as spring and mass of aircraft was added to the model as a portion per landing gear.

A. Behavior of Springs

In general, springs can be classified as linear and non-linear springs as shown in Fig. 2. Linear spring is a straight line in force versus displacement plot. In order words, there is a constant slope of the line which is a spring constant and can be described as follows:

\[ F = kx(1) \]

where \( F \) is force and \( x \) is the displacement and \( k \) is the spring constant.

For non-linear springs as depicted in Fig. 2, the slope of the curve is not constant. For non-linear springs, equation of the curve is defined based on its ‘force vs. displacement’ behavior.
B. Behavior of Dampers
Dampers dissipate kinetic energy supplied to it. Typically with friction, it converts mechanical energy to heat. In other words, a damper resists motion by friction. If there is no motion (speed), there is no force to resist. Dampers can be simple hydraulic devices in a piston/cylinder arrangement so that hydraulic fluid can go from one side of the piston to the other side through a hole (orifice) in a contained cylinder to resist the motion and absorb mechanical shocks by converting kinetic energy to heat. They can be classified as linear and non-linear dampers as shown in Fig. 3. Linear damper is a straight line in force versus speed plot. It can be described as follows:

\[ F = bV \] (2)

where \( F \) is force and \( V \) is the velocity or speed and \( b \) is the damping coefficient.

For non-linear springs as depicted in Fig. 3, the slope of the curve is not constant. For non-linear springs, equation of the curve is defined based on its "force vs. displacement" behavior.

\[ F = ma \] (3)

\[ F_1 = m_1 \ddot{x}_1 = W_1 - F_{spring} - F_{damping} \] (4)

\[ F_1 = m_1 \ddot{x}_1 = W_1 - k_1 \dot{x} - b \dot{x} \] (5)

\[ x = x_1 - x_2 \] (6)

\[ m_1 \ddot{x}_1 + k_1 (x_1 - x_2) + b (\ddot{x}_1 - \ddot{x}_2) = W_1 \] (7)

\[ W_1 = m_1 g - L \] (8)

\[ m_1 \ddot{x}_1 + k_1 (x_1 - x_2) + b (\ddot{x}_1 - \ddot{x}_2) = m_1 g - L \] (9)

where:
- \( b \): shock absorber damping coefficient
- \( F_1 \): net force on \( m_1 \)
- \( F_2 \): ground reaction force
- \( g \): gravity
- \( k_1 \): shock absorber spring constant
- \( k_2 \): tire spring constant
- \( L \): lift force acting on aircraft
- \( m_1 \): aircraft mass (per landing gear)
- \( m_2 \): sum of wheel, tire and axle mass
- \( V \): aircraft vertical speed (descent velocity)
- \( W_1 \): aircraft weight minus lift force
- \( x_1 \): displacement of aircraft
- \( x_2 \): displacement of wheel

As a next step, \( F_2 \) force acting on the mass \( m_2 \) (wheel and tire mass) need to be formulated. Again, Newton’s second law of motion:

\[ F_2 = m_2 \ddot{x}_2 = F_{spring, shock absorb} + F_{damping} - F_{spring, tire} \] (10)

\[ m_2 \ddot{x}_2 = k_1 x + b \dot{x} - k_2 x_2 \] (11)

\[ x = x_1 - x_2 \] (12)

\[ m_1 \ddot{x}_1 + k_1 (x_1 - x_2) + b (\ddot{x}_1 - \ddot{x}_2) = W_1 \] (13)

\[ m_2 \ddot{x}_2 - k_1 (x_1 - x_2) - b (\ddot{x}_1 - \ddot{x}_2) + k_2 x_2 = 0 \] (14)

III. FORMULATION OF THE DYNAMIC PROBLEM

The objective of the dynamic problem is to find the resultant force acting on aircraft and displacements. A typical model can be used to write the equations of the motion as shown in Fig. 4. In order to find \( F_1 \) force acting on the mass \( m_1 \) (aircraft mass per landing gear), we can write Newton’s second law of motion:

\[ F = ma \] (3)

\[ F_1 = m_1 \ddot{x}_1 = W_1 - F_{spring} - F_{damping} \] (4)

\[ F_1 = m_1 \ddot{x}_1 = W_1 - k_1 \dot{x} - b \dot{x} \] (5)

\[ x = x_1 - x_2 \] (6)

\[ m_1 \ddot{x}_1 + k_1 (x_1 - x_2) + b (\ddot{x}_1 - \ddot{x}_2) = W_1 \] (7)

\[ W_1 = m_1 g - L \] (8)

\[ m_1 \ddot{x}_1 + k_1 (x_1 - x_2) + b (\ddot{x}_1 - \ddot{x}_2) = m_1 g - L \] (9)
IV. RESULTS AND DISCUSSION

The computer model was constructed in Matlab/Simulink as shown in Fig. 5. Matlab/Simulink solves the equation of motions simultaneously. The constructed model has some icons and associated parameters, therefore there are numerical values to be entered in the Matlab/Simulink model. The results are plotted by the help of graph icons which are shown in blue color in Fig. 5 on the right. Those five blue icons displays the results of vertical velocity change of aircraft ($V_1$), position or displacement of shock absorber ($x_1$), vertical acceleration of the aircraft ($a_1$) and as a multiplier of gravity ($g_1$), and most importantly $F_1$, Force acting on the mass $m_1$ (aircraft mass per landing gear).

C. Validation Case

For the validation purposes, numerical values to be input were chosen from Ref [3] as $b$: 500 lb.sec/in; $k_1$: 2800 lb/in; $k_2$: 12500 lb/in; $m_1$: 103.6 lb.sec^2/in; $m_2$: 0; $V$: 120 in/sec; $W_1$: 0 (weight minus lift force).

The results of the validation case matches with the results of [3]. The results of this study are presented in Fig. 6-9. $F_1$ force acting on aircraft was calculated and is show in Fig. 6. The maximum value of $F_1$ is equal to 56450 lb and it is around 0.13 seconds after landing. Minimizing $F_1$ force is important in the design of landing gears. Position or displacement of shock absorber ($x_1$) is shown in Fig. 7 and it starts from zero value at time zero and then it increases as the $F_1$ force increases, then reaches to a maximum value of 17.13 inches. This max value is important because the shock absorber has a design limit of displacement which should not be exceed. Fig. 8 shows vertical velocity of aircraft ($V_1$) which has an initial value of 120 in/s as input, then it decreases towards zero as landing occurs. Vertical acceleration of the aircraft in ($a_1$) starts from zero and it reaches 13.83 as maximum value (Fig. 9). Similarly vertical acceleration divided by gravity ($g$) reaches $13.83/9.81= 1.4$ as maximum value.
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D. Sensitivity Study

After the validation case, it is worthy to investigate the effect of some input design parameters to see the result of output parameters. The input design parameters fed to the Simulink model were changed from baseline values ±10% each, to see the effects on the results. The input design parameters considered for this purpose are as follows:

- \( b \) : shock absorber damping coefficient
- \( k_1 \) : shock absorber spring constant
- \( k_2 \) : tire spring constant

Table I shows the summary of this study with input and output parameters. The selected input parameters were changed for ±10% values, meaning 2 run cases for each. It makes 6 cases total for 3 parameters and also a baseline case for reference.

In order to evaluate the results in Table I, Fig. 10-12 was drawn for illustrative purposes. The effect of shock absorber damping coefficient (b) on results is shown in Fig.10. Shock absorber damping coefficient is proportional to the force and acceleration of aircraft, however it is inversely proportional to the displacement of the shock absorber, meaning that it has to absorb the impact energy in a shorter displacement. Shock absorber spring constant \( (k_1) \) and tire spring constant \( (k_2) \) have similar effects on results of force and displacement (Fig. 11 and Fig. 12).

Table I: Sensitivity Study of Selected Design Parameters

<table>
<thead>
<tr>
<th>Case Number</th>
<th>Input Design Parameters</th>
<th>Output Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( b ) (lb.sec/in)</td>
<td>( k_1 ) (lb/in)</td>
</tr>
<tr>
<td>Baseline</td>
<td>500</td>
<td>2800</td>
</tr>
<tr>
<td>1</td>
<td>450 -10%</td>
<td>2800</td>
</tr>
<tr>
<td>2</td>
<td>550 10%</td>
<td>2800</td>
</tr>
<tr>
<td>3</td>
<td>500 2520 -10%</td>
<td>12500</td>
</tr>
<tr>
<td>4</td>
<td>500 3080 10%</td>
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<tr>
<td>5</td>
<td>500 11250 -10%</td>
<td>12500</td>
</tr>
<tr>
<td>6</td>
<td>500 13750 10%</td>
<td>12500</td>
</tr>
</tbody>
</table>
V. CONCLUSION

In this study, a model of landing gear and shock absorber using related spring-mass-damper elements was constructed. The results are shown and discussed for given cases. The below points can be concluded from the study:

1) Landing gears can be modeled as spring-mass-damper systems
2) Dynamic equations of motions can be derived and solved simultaneously
3) Matlab/Simulink is a good tool to model and solve similar problems
4) $F_1$, force acting on the aircraft is heavily dependent on design parameters to be selected such as shock absorber damping coefficient (b), shock absorber spring constant ($k_1$) and tire spring constant ($k_2$)
5) As shock absorber damping coefficient (b) decreases, impact force $F_1$ also decreases and displacement of shock absorber ($x_1$) increases. Effect of damping coefficient (b) on impact force was found to be 3.76% for a change of ±10% in b.
6) If shock absorber spring constant ($k_1$) increases, impact force $F_1$ increases and displacement of shock absorber ($x_1$) decreases. Effect of spring constant ($k_1$) on impact force was found to be 2.29% for a change of ±10% in $k_1$.
7) Tire spring constant ($k_2$) increases, impact force $F_1$ increases and displacement of shock absorber ($x_1$) decreases. Effect of spring constant ($k_2$) on impact force was found to be 0.97% for a change of ±10% in $k_2$.

REFERENCES