Bootstrapping DEA Efficiency Estimates of Indian Railways System

G. R. Kanmani, V. Prakash

Abstract: Railways and Roadways are the two means of transport over the land in India. But Railways played a vital role in integrating markets and increasing trade in terms of the economy and in terms of politics and also it forms the finances of Indian government.

Based on the above facts this study mainly focuses on the performance and efficiency evaluation of Indian Railways during 2016-2017. To attain this objective 16 zones of Indian Railways were considered and we applied DEA and Bootstrapped DEA methods. In this study CCR model reveals 11 zones are efficient and BCC model indicates 14 zones are efficient. Besides, this study reveals that bootstrapping efficiency scores are comparatively less than the original CRS and VRS efficiency score.

Keywords: Bootstrapping DEA, Constant returns to scale (CRS), Data Envelopment Analysis (DEA), Variable returns to scale (VRS).

I. INTRODUCTION

India’s public transport systems are amongst the most exploited systems in the world. Public transport system in India refers to Road transport, Rail transport, Air ways and Sea transports. Of these rail transport plays a major and crucial role in transporting commodities and persons from one place to various parts of the country. Indian Railways has rich legacy of values and norms that have helped it sustain its place in the country. Indian Railways run by government of India follow the tradition bound value system. The Indian Railways is not just a transport system, it is a culture in force and an integral expression of national solidarity. The nation moves ahead with it. In fact, the wheels of the railways symbolize an ever advancing, growing, developing nation. It reaches to the remote parts of the country. Thus The role of railways over the years has thus turned vital and become more crucial by changing the economic, social, demographic and environmental scenario. The rail transport in India began in the mid-nineteenth century. There are 17 Railway zones in India. Indian Railways is partite for administrative convenience into several regional railways. In beginning stage there were 9 zones and this structure had not changed much for four decades. But in recent 7 new zones were created, giving a total of 16. Kolkata metro is the 17th zone which is not included in this study. Efficiency and performance of railways depends on work culture of its various departments. Indian railway system is the most important channel of the country’s inland transport and it effectively forms the lifeline of the country. It contributes to economic growth and also promotes national integration through large scale movement of traffic, both freight and passenger. Intact railways constitute the backbone of surface transport system in India. Based on the above facts this study throws light on the performance of every aspect of Indian railways.

There are many efficiency measurement tools and among all Data Envelopment Analysis (DEA) is the most powerful tool for finding the efficiency measurement which has been widely and popularly used in recent years. Data Envelopment Analysis (DEA), is frequently called frontier analysis, was first put forward by Charnes, Cooper and Rhodes in 1978. It is a consonantly a new ‘data oriented’ approach for evaluating the performance of a set of peer entities called Decision Making Units (DMU), which convert multiple inputs into multiple outputs. These DEA applications have used DMUs in many forms to assess the performance of entities such as hospitals, US air force, wings, universities, cities, courts, business firms and others, including the performance of countries, regions etc.

DEA is used to calculate performance of Indian Railways among the 16 zones in India. So the author has considered 16 zones with 4 input and 3 outputs. Each zone is considered as decision making unit and the study is based on secondary data. It is collected from the “Indian Railways, Annual Statistical statements, Government of India, Ministry of Railways (Railway Board)". The structure of the paper is organized as follows, the review of relevant literature is described in section II. The methodology used for analysis is discussed detail in section III. Section IV describes the data structure. Section V deals with the empirical investigation based on the data structure. Finally the paper ends with conclusion on empirical investigation.

II. REVIEW OF LITERATURE

Based on the works of Farrell (1957), Charnes Cooper Rhodes (1978, 1979) developed the theory relating to DEA and it has emerged as an analytical tool for measuring the productive efficiency and this model admits to Constant Returns to Scale. The extension of CCR model is developed by Banker Charnes and Cooper (1984) which admits Variable Returns to Scale. B. Efron (1979) introduced a general method called the “Bootstrap" and shown to work satisfactorily on a variety of estimation problems. This article shows that Jackknife can be thought as a linear expansion method for approximating the bootstrap. The paper contains the construction of examples like variance of the sample median, error rates in a linear discriminant analysis, ratio estimation, estimating regression parameters etc., The elaborated models for...
DEA were evolved during 1980s.

Wong and Beasley (1990) for the first time developed a method for restricting weight flexibility in data envelopment analysis. Seiford and Thrall (1990) inflicted the mathematical programming approach for frontier estimation and examined the effect of model orientation on the efficiency frontier and the effect of convexity requirements on Returns to scale. They also made methodological extensions and proposed alternative models.

Lovell and Pastor (1995) modified the existing radial DEA model so that it merely satisfies both properties via invariant and translation invariant and introduced a new additive DEA models that satisfy both properties. Coelli (1996) developed a computer program to conduct DEA for the purpose of calculating technical efficiency scores in production. The works of Seiford (1996) includes an evolution map which illustrates the DEA growth, the timing of the major events and the interconnections and influences between the topics. Barr et al (1994) used DEA to predict bank failure. Nolan (1996) studied technical efficiency in 29 average size US bus systems using a DEA model.

Simar and Wilson (2000) proposed a general methodology for bootstrapping in frontier models by allowing for heterogeneity in the structure of efficiency. This is the extension work of the more restrictive method of Simar and Wilson (1998). This methodology has been illustrated through a numerical example with real data.

Evangelis Desli and Subhash Ray (2004), proposed a bootstrap procedure that empirically generates the conditional distribution of efficiency for each individual DMU given systematic factors that influence its efficiency. Instead of resampling directly from the pooled DEA scores the authors of this article regressed pooled DEA scores on a set of exploratory variable not included at the DEA stage and bootstrapped the residuals from this regression. For empirical investigation data from the US airline industry are utilized.

Basanta R. Dhungana et al (2004) measuring the economic inefficiency of Nepalese rice farms by applying data envelopment analysis. A sample of 76 nepalese rice farms are considered for this study and investigation reveals that there is a significant variations in the level of inefficiency across sample farms. It is observed that the variations in the sample farms may be due to the variations in the usage of resources such as seed, labour, fertilizers and mechanical power. Besides an application of Tobit regression indicates that the variation is also related to form specific attributes such as the farmers level of risk attitude, gender, age etc.,

John B Walden (2006) presented a method for examining the underlined statistical structure of statistical models using DEA method and readily available software. The bootstrapping technique presented here is one method for constructing a stochastic DEA models, and is easily implemented using the GAMS language. M.M. Movahedi et al (2007), evaluated the efficiency of Iranian railway using data envelopment analysis method. The analysis of the study involves the railway activities from 1971 to 2004, efficiency estimation of each year and comparison with other years.

He Xuan (2017) studied the change of high speed railway to the urban structure along the route by taking accessibility as an object. This study takes in to account two scenarios such as high speed rail and no high speed rail. In this article 4 different indexes are selected which reflects different aspects of accessibility. The researcher used data envelopment analysis to synthesize 4 reachability indexes and obtains a comprehensive accessibility index.

Milan Radijic et al (2018) evaluated the efficiency of 25 banks operating in serbia during the period 2005-2016. To avoid misvaluation of banks efficiency a statistically sound framework namely bootstrapped I-distance is suggested for determining weight bounds in the global assurance region DEA model.

Somayyeh Danesh Asgari et al (2018) presented a novel approach choosing right indicators of urban railway system through integrating of balanced score card and three stage data envelopment analysis. In this article DEA model served for the purpose of selecting the right indicators utilized efficiently by decision making units (subway stations).

### III. METHODOLOGY

#### A. MODEL SPECIFICATION

**Constant returns to scale (CRS) DEA model &Variable returns to scale (VRS) model**

Let there be 'n' DMUs each with 'm' inputs and 's' outputs. For DMU 'o' the basic CCR output maximization(input oriented model)which is also called as CRS model is outlined as follows:

\[
\text{Max } h_0 = \frac{\sum_{i=1}^{m} u_i x_{ij}}{\sum_{j=1}^{s} v_j y_{ij}}
\]

Subject to constraints:

\[
\frac{\sum_{i=1}^{m} u_i x_{ij}}{\sum_{j=1}^{s} v_j y_{ij}} \leq 1 \text{ for each unit 'j'}
\]

where \( u_i, v_j \geq 0 \)

The weights \( u_i \) and \( v_j \) are applied to outputs \( y_{ij} \) and inputs \( x_i \) that are chosen to maximize the efficiency score \( h_0 \) for DMU \( o \). The above is fractional programming problem. Generally the fractional programming problem is difficult to solve and it gives infinite number of solutions. Hence the above fractional programming problem can be converted into Linear programming problem by normalizing either the denominator or the numerator. Here normalizing the denominator that is \( \sum_{j=1}^{s} v_j x_{0j} = 1 \) and added this as an additional constraint. Also the original constraint is manipulated in order to convert the fractional programming to linear programming. These two steps results the following model:

\[
\text{Max } h_0 = \sum_{j=1}^{s} u_j y_{0j}
\]

Subject to constraints:

\[
\sum_{i=1}^{m} v_i x_{ij} = 1 \quad \text{and} \quad \sum_{i=1}^{m} u_i x_{ij} \leq 1
\]

\( u_i, v_j \geq 0 \)

The above program is executed once for each participating DMU resulting in the optimal weights being determined for each DMU. Each DMU selects input and output weight that maximizes the efficiency score. A DMU gets efficiency score of unity which is considered to be efficient and otherwise it is inefficient. For computational efficiency reasons the above linear program is converted to its dual and it is given as,

\[
\text{Min } \theta
\]

Subject to constraints:

\[
X^\lambda \leq X_0
\]

\[
Y^\lambda \geq Y_0
\]

\[
\lambda \geq 0
\]
Solving the above linear programming problem for each DMU we get efficiency score \( \theta \) and the DMU weights \( \lambda \). A DMU is said to be efficient if and only if,

(i) \( \theta = 1 \)

(ii) All slacks are equal to zero

Similarly CRS output oriented model is,

Max \( \phi \)

Subject to the constraints: \( Y \lambda \geq Y_0 \)

\( X \lambda \leq X_0 \)

\( \lambda \geq 0, \phi \geq 1 \)

Solving the above models DMU is said to be efficient if and only if,

(i) \( \phi = 1 \)

(ii) All slacks are equal to zero

Banker, Chambers and Cooper (1984) developed the BCC model by adjoining the convexity constraint \( \sum_{i=1}^{n} \lambda_i = 1 \) with CCR Model. It is also referred as VRS model. Input oriented and output oriented BCC model is presented here,

<table>
<thead>
<tr>
<th>INPUT ORIENTED</th>
<th>OUTPUT ORIENTED</th>
</tr>
</thead>
<tbody>
<tr>
<td>Min ( \theta )</td>
<td>Max ( \phi )</td>
</tr>
<tr>
<td>Subject to constraints:</td>
<td>Subject to constraints:</td>
</tr>
<tr>
<td>( Y \lambda \geq Y_0 )</td>
<td>( Y \lambda \geq \phi Y_0 )</td>
</tr>
<tr>
<td>( X \lambda \leq X_0 )</td>
<td>( X \lambda \leq X_0 )</td>
</tr>
<tr>
<td>( \sum \lambda = 1, \lambda \geq 0, \phi \geq 0 )</td>
<td>( \sum \lambda = 1, \lambda \geq 0 )</td>
</tr>
</tbody>
</table>

The basic DEA models discussed above involves computation of efficiency scores by solving one linear program for each DMU. DEA models are well known for its great features. But the DEA model which is deterministic in nature, still suffers from the fact that the model is sensitive to the sampling variation in obtaining the true frontier. This is due to the fact that the efficiency score is derived from the sampling variation in obtaining the true frontier. Thus, the issue Bootstrap DEA approach is applied and described in the following section.

### B. Bootstrapping DEA models

Simar and Wilson (1998) proposed to use bootstrapping technique (Efron and Tibshirani 1993) which might weaken the sensitivity of the sampling variation. The purpose of using the bootstrapping approach is,

1. To correct DEA estimator for a bias.
2. To estimate the confidence interval for those indices.

The algorithm proposed by Simar and Wilson (1998, 2000) can be summarised as follows:

1. Solve the original DEA model to obtain \( \theta_1, \ldots, \theta_n \).
2. Let \( \theta_{B1}, \ldots, \theta_{Bn} \) be a sample generated from \( \theta_1, \ldots, \theta_n \).
3. Smooth the sampled values using the following formula:

\[
\tilde{\theta}_i = \{ \theta_{Bi} + h_\phi \} \text{ if } \theta_{Bi} + h_\phi \geq 1 \text{ or } 2 - \theta_{Bi} - h_\phi \text{ if } \theta_{Bi} + h_\phi < 1 \}
\]

4. Obtain the final value \( \tilde{\theta} \) by adjusting the smoothed sample value using the following formula:

\[
\tilde{\theta}_i = \bar{B} + \frac{(\tilde{\theta} - \bar{B})}{(1 + h^2/\phi^2)^{2/4}} \text{ where } \bar{B} = \frac{1}{n} \sum_{i=1}^{n} \theta_{Bi}
\]

and \( \bar{\sigma}^2 = \frac{1}{n} \sum_{i=1}^{n} (\tilde{\theta}_i - \bar{\theta})^2 \)

5. Adjust the original outputs using the ratio \( \frac{\tilde{\theta}_i}{\tilde{\theta}} \)
6. Resolve the original DEA model using the adjusted outputs to obtain \( \tilde{\theta}_i \)
7. Repeat steps 2-6 B times to provide for B sets of estimates; i.e. each firm will have B estimates of theta. For this analysis, 1000 samples were generated for each vessel.

According to the equation mentioned in step 3, ‘\( h \)’ is a smoothing parameter, and ‘\( \varepsilon \)’ is a randomly drawn error term. The most difficult step in the procedure above is to find an appropriate value of \( \varepsilon \). ‘\( h \)’ is determined, by using the “normal reference rule”, as follows:

\[
h = (4/(p + q + 2))^{(1/(p + q + 4))} \text{ where } p \text{ equals the number of inputs, } q \text{ the number of outputs and } N \text{ the number of observations in the sample.}
\]

Once the number of desired samples is generated, the bias of the original estimate of theta is calculated as follows:

\[
\tilde{\theta}_k = \bar{\theta}_k - \text{bias } \tilde{\theta}_k = 2 \ast \bar{\theta}_k - B \ast \sum_{b=1}^{B} \tilde{\theta}_{kb}
\]

A bias corrected estimator of the true value of \( \theta(x, y) \), \( \tilde{\theta}_k \), can then be computed using the following formula(Simar and Wilson 2000b):

\[
\tilde{\theta}_k = \tilde{\theta}_k - \text{bias } \tilde{\theta}_k = 2 \ast \bar{\theta}_k - B \ast \sum_{b=1}^{B} \tilde{\theta}_{kb}
\]

It may be difficult to interpret the scores obtained in each DMU after performing the DEA analysis. Generally DEA efficiency scores are reported and used in summarizations with no corresponding measures of statistical reliability. So to compute statistics relating to the DEA individual scores the efficiency scores can be bootstrapped(Ferrier & Hirschberg 1997,1999).

### IV. DATA STRUCTURE

16 zones are considered as DMUs and it is characterized by 4 inputs and 3 outputs. The inputs are Number of employees, Number of Rolling stock, Train Kilometres (in 1000s) and passengers originating. The outputs are Passenger Kilometres (in 1000s), Passenger carried (in 1000s) and Tonne kilometres (in 1000s). Descriptive statistics on input and output variables are presented in Table I.

**Table 1: Descriptive Statistics of Input and Output variables**

<table>
<thead>
<tr>
<th>Variables</th>
<th>Count</th>
<th>Minimum</th>
<th>Maximum</th>
<th>Average</th>
<th>Standard deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of employees</td>
<td>16</td>
<td>38028</td>
<td>146732</td>
<td>77662</td>
<td>35666.4</td>
</tr>
<tr>
<td>Number of rolling stock</td>
<td>16</td>
<td>429533</td>
<td>1998279</td>
<td>1078253</td>
<td>78513.6</td>
</tr>
<tr>
<td>Train Kilometres</td>
<td>16</td>
<td>60906</td>
<td>268896</td>
<td>132917.4</td>
<td>21445.1</td>
</tr>
<tr>
<td>Passenger originating</td>
<td>16</td>
<td>957110</td>
<td>1757789</td>
<td>1479630</td>
<td>1233444.0</td>
</tr>
</tbody>
</table>
V. EMPIRICAL INVESTIGATION

To overcome the upward bias in efficiency estimation, bootstrap method proposed by Simar and Wilson (1998, 2000) is applied. The efficiency results from the original model and the bootstrap approach along with the confidence intervals for the efficiency is presented in Table II.

Table-II: Results of Bootstrapping CRS Efficiency scores and Confidence Interval

<table>
<thead>
<tr>
<th>DMUs</th>
<th>Efficiency Score</th>
<th>Bias Corrected</th>
<th>Bias</th>
<th>CI-Lower</th>
<th>CI-Upper</th>
<th>Variance</th>
<th>Rho (ρ)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Zone 1</td>
<td>1</td>
<td>0.93271</td>
<td>0.06729</td>
<td>0.79157</td>
<td>0.99381</td>
<td>0.00868</td>
<td>33.41093</td>
</tr>
<tr>
<td>Zone 2</td>
<td>0.911</td>
<td>0.87734</td>
<td>0.03566</td>
<td>0.8088</td>
<td>0.90953</td>
<td>0.01136</td>
<td>188.50614</td>
</tr>
<tr>
<td>Zone 3</td>
<td>1</td>
<td>0.96755</td>
<td>0.03245</td>
<td>0.92788</td>
<td>0.99772</td>
<td>0.00488</td>
<td>643.63152</td>
</tr>
<tr>
<td>Zone 4</td>
<td>1</td>
<td>0.93467</td>
<td>0.06533</td>
<td>0.79797</td>
<td>0.9985</td>
<td>0.00773</td>
<td>37.65017</td>
</tr>
<tr>
<td>Zone 5</td>
<td>1</td>
<td>0.94357</td>
<td>0.05643</td>
<td>0.86104</td>
<td>0.99794</td>
<td>0.00301</td>
<td>98.67516</td>
</tr>
<tr>
<td>Zone 6</td>
<td>1</td>
<td>0.94663</td>
<td>0.05337</td>
<td>0.86945</td>
<td>0.9978</td>
<td>0.00256</td>
<td>116.74163</td>
</tr>
<tr>
<td>Zone 7</td>
<td>1</td>
<td>0.93255</td>
<td>0.06745</td>
<td>0.78086</td>
<td>0.99787</td>
<td>0.00914</td>
<td>31.72657</td>
</tr>
<tr>
<td>Zone 8</td>
<td>0.70713</td>
<td>0.68165</td>
<td>0.02548</td>
<td>0.6393</td>
<td>0.70531</td>
<td>0.00044</td>
<td>351.84112</td>
</tr>
<tr>
<td>Zone 9</td>
<td>1</td>
<td>0.94899</td>
<td>0.05101</td>
<td>0.86777</td>
<td>0.99794</td>
<td>0.00212</td>
<td>141.19774</td>
</tr>
<tr>
<td>Zone 10</td>
<td></td>
<td>0.94502</td>
<td>0.05498</td>
<td>0.86981</td>
<td>0.99744</td>
<td>0.00277</td>
<td>107.45164</td>
</tr>
<tr>
<td>Zone 11</td>
<td></td>
<td>0.86561</td>
<td>0.02946</td>
<td>0.7857</td>
<td>0.86413</td>
<td>0.00066</td>
<td>353.96345</td>
</tr>
<tr>
<td>Zone 12</td>
<td></td>
<td>0.9725</td>
<td>0.03324</td>
<td>0.87424</td>
<td>0.97085</td>
<td>0.00098</td>
<td>301.19691</td>
</tr>
<tr>
<td>Zone 13</td>
<td></td>
<td>0.94861</td>
<td>0.05139</td>
<td>0.87814</td>
<td>0.9978</td>
<td>0.00195</td>
<td>154.18419</td>
</tr>
<tr>
<td>Zone 14</td>
<td></td>
<td>0.80326</td>
<td>0.02833</td>
<td>0.73996</td>
<td>0.80109</td>
<td>0.00307</td>
<td>547.63042</td>
</tr>
<tr>
<td>Zone 15</td>
<td></td>
<td>0.93303</td>
<td>0.06697</td>
<td>0.77789</td>
<td>0.99822</td>
<td>0.00956</td>
<td>30.35902</td>
</tr>
<tr>
<td>Zone 16</td>
<td></td>
<td>0.93175</td>
<td>0.06825</td>
<td>0.77967</td>
<td>0.99766</td>
<td>0.00993</td>
<td>29.1258</td>
</tr>
<tr>
<td>Mean</td>
<td>0.95371</td>
<td>0.84633</td>
<td>0.04906</td>
<td>0.81562</td>
<td>0.95175</td>
<td>0.00385</td>
<td>43.12578</td>
</tr>
</tbody>
</table>

It is observed that the mean efficiency score moves from 95% to 84% for original CRS efficiency and bootstrapping efficiency respectively and confidence intervals indicate the values of real efficiency scores lies between the interval 0.815 and 0.951. According to Table 2 rho(ρ) value is greater than 1 for all the zones and the bias correction is valid. If rho (ρ) is less than 1 then conventional DEA should be used. Further for VRS specification also bootstrap DEA is used which contains the results of original efficiency indicators, the bias and bias corrected efficiency score, confidence intervals etc., and is presented in Table III.

Table-III: Results of Bootstrapping VRS Efficiency scores and Confidence Interval

<table>
<thead>
<tr>
<th>DMUs</th>
<th>Efficiency Score</th>
<th>Bias Corrected</th>
<th>Bias</th>
<th>CI-Lower</th>
<th>CI-Upper</th>
<th>Variance</th>
<th>Rho (ρ)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Zone 1</td>
<td>1</td>
<td>0.98774</td>
<td>0.01226</td>
<td>0.92329</td>
<td>0.9996</td>
<td>0.00044</td>
<td>738.36451</td>
</tr>
<tr>
<td>Zone 2</td>
<td>0.91381</td>
<td>0.90904</td>
<td>0.00477</td>
<td>0.89751</td>
<td>0.91358</td>
<td>0.00004</td>
<td>7622.8558</td>
</tr>
<tr>
<td>Zone 3</td>
<td>1</td>
<td>0.98859</td>
<td>0.01141</td>
<td>0.93566</td>
<td>0.99968</td>
<td>0.00031</td>
<td>1064.8078</td>
</tr>
<tr>
<td>Zone 4</td>
<td>1</td>
<td>0.98817</td>
<td>0.01183</td>
<td>0.92412</td>
<td>0.99968</td>
<td>0.00039</td>
<td>839.33313</td>
</tr>
<tr>
<td>Zone 5</td>
<td>1</td>
<td>0.98794</td>
<td>0.01206</td>
<td>0.92437</td>
<td>0.99969</td>
<td>0.00041</td>
<td>801.46198</td>
</tr>
<tr>
<td>Zone 6</td>
<td>1</td>
<td>0.98759</td>
<td>0.01241</td>
<td>0.92176</td>
<td>0.99971</td>
<td>0.00046</td>
<td>700.72324</td>
</tr>
<tr>
<td>Zone 7</td>
<td>1</td>
<td>0.98785</td>
<td>0.01215</td>
<td>0.92227</td>
<td>0.99968</td>
<td>0.00044</td>
<td>740.73061</td>
</tr>
<tr>
<td>Zone 8</td>
<td>1</td>
<td>0.98816</td>
<td>0.01184</td>
<td>0.92314</td>
<td>0.99966</td>
<td>0.00042</td>
<td>783.72222</td>
</tr>
<tr>
<td>Zone 9</td>
<td>1</td>
<td>0.98815</td>
<td>0.01185</td>
<td>0.92311</td>
<td>0.99971</td>
<td>0.00041</td>
<td>801.53397</td>
</tr>
<tr>
<td>Zone 10</td>
<td></td>
<td>0.98815</td>
<td>0.01185</td>
<td>0.92339</td>
<td>0.99977</td>
<td>0.00042</td>
<td>782.9748</td>
</tr>
<tr>
<td>Zone 11</td>
<td></td>
<td>0.98752</td>
<td>0.01248</td>
<td>0.92257</td>
<td>0.99967</td>
<td>0.00045</td>
<td>720.5008</td>
</tr>
</tbody>
</table>
Here the scenario is same as in CRS assumption. The mean of the bias corrected efficiency score is slightly lower than mean of the original efficiency score, which indicates there is a moderate level of inefficiency among the zones not as much as CRS. The interval lies between 0.867 and 0.992 for real efficiency score. rho(p) value satisfies the condition for bias correction where (p=1) that is the variances are low for the estimates and the results of the study shows they are powerful individually as well as globally.

**CONCLUSION**

CCR (CRS) model reveals that 11 zones are efficient and BCC(VRS) model shows 14 zones are efficient among 16 zones. From CRS and VRS we may conclude that all the zones are performing well. In scale efficiency it results that South central zone is operating under IRS which suggests that Train kilometer of that zone can be increased. Eastern, North East frontier, South eastern, South western are operating under DRS suggests that number of employees and Train kilometres can be decreased and other 11 zones operates under CRS. Confidence interval of CRS indicates the value of real efficiency scores lies between 81% to 95% and in VRS it lies between 87% to 99%. Hence Bootstrapping DEA under CRS and VRS suggests that variance of the individual estimates are individually and globally robust and bias corrections are valid in this study.

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