Double Twin Domination Number of Some Special Types of Graphs

G.Mahadeven, S.Anuthiya

Abstract: ***Recently, In[6] the concept of Double Twin Domination number of a graph DTD (G) was introduced by G. Mahadevan et.al., DTwin (u, v) is sum of number of a $u-v$ paths of length less than or equal to four. The total number of vertices that dominates every pair of vertices $SDTwin\,(G) = \sum DTwin(u,v)$ for $u,v \in V\,(G)$. The Double Twin Domination number of G is defined as $DTD\,(G) = \frac{SDTwin(G)}{\binom{n}{2}}$. In this paper, we investigate this number for some special types of graphs.***

Keywords: ***Medium Domination Number, Extended Medium Domination Number, Double Twin Domination Number.***

I. INTRODUCTION

Domination theory place vital role in graph theory.The concept of Medium domination number was introduced by Duygu Vargor and Pinar Dundar in [1] with real life application to protect the pairs of vertices in a graph. In any connected simple graph G of order p, the medium domination number of G is defined as$\gamma_m(G) = \frac{TDT\,(G)}{\binom{n}{2}}$.In [4], G. Mahadevan et.al., introduced the concept of the Extended Medium Domination number of a graph. The total number of vertices that dominate every pair of vertices $ETDV\,(G) = \sum edom\,(u,v)foru,v \in V\,(G)$. The extended medium domination number of a graph G is defined as $(G) = \frac{ETDV\,(G)}{\binom{n}{2}}$. Motivated by the above G. Mahadevan et.al., [8] introduced the concept of Double Twin domination of a graph. $DTwin\,(u,v)$ is the sum of number of $u-v$ path of length one two three and four. The total number of vertices that dominate every pair of vertices $SDTwin\,(G) = \sum DTwin\,(u,v)$ for $u,v \in V\,(G)$. In any simple graph G of n number of vertices, the double Twin domination number of G is defined as $DTD(G) = \frac{SDTwin\,(G)}{\binom{n}{2}}$.

The Corona product$G_1 \odot G_2$ is defined as the graph G obtained by taking one copy of G_1 of order n and n copies of G_2 and then joining the i[th] vertex of G_1 to every vertex in the i[th] copy of G_2. The Peacock head graph isobtained by joining n pendent edges to any one vertex of the cycle C_m and it is denoted by PC (n, m).

Revised Manuscript Received on December 5, 2019.

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$P_m\,(K_{1,n})$ is the graph obtain by pasting the root vertex of the pendant vertex of $Y_n = P_2 \times C_n$ is the Prism graph. Sunlet graph S_n is obtained by attaching pendent vertices to each verities of C_n. Web graph is obtain from prism by attaching pendentvertex to each vertex of the outer cycle and is denoted by W_n.

Notation1.1DTwin (G) – Double Twin Domination number of a graph.

SDTwin (G) – Sum of Double Twin Domination number of a graph.

DTD (G)–Double twin Total Domination number of a graph.

Definition1.2 Let G = (V, E) be a graph, where V, E be the vertex set and edge set respectively. DTwin (u, v) is sum of number of $u-v$ path of length one, two, three, and four. The total number of vertices that dominate every pair of vertices. $SDTwin\,(G) = \sum DTwin\,(u,v)$ for u, v ∈ V (G). In any simple graph G of n number of vertices the double twin domination number of G is defined as $DTD(G) = \frac{SDTwin\,(G)}{\binom{n}{2}}$.

Illustration.

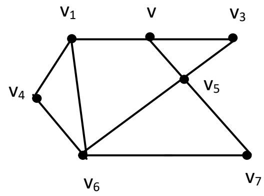

Figure1.1

From the above figure, DTwin(v_1, v_2) = 5; DTwin(v_1, v_3) = 6; DTwin(v_1, v_4) = 3; DTwin(v_1, v_5) = 6; DTwin(v_1, v_6) = 5; DTwin(v_1, v_7) = 6; DTwin(v_2, v_3) = 3; DTwin(v_2, v_4) = 5; DTwin(v_2, v_5) = 5; DTwin(v_2, v_6) = 5; DTwin(v_2, v_7) = 5; DTwin(v_3, v_4) = 7; DTwin(v_3, v_5) = 3; DTwin(v_3, v_6) = 6; DTwin(v_3, v_7) = 4; DTwin(v_4, v_5) = 6; DTwin(v_4, v_6) = 3; DTwin(v_4, v_7) = 4; DTwin(v_5, v_6) = 5; DTwin(v_5, v_7) = 3; DTwin(v_6, v_7) = 3.$SDTwin(G) = 98$;

$DTD(G) = \frac{SDTwin\,(G)}{\binom{n}{2}} = \frac{98}{\binom{7}{2}} = \frac{98}{21}$.

1.1Preliminaries

Theorem 1.1.1 [8]For any cycle graphC_m, $SDTwin(C_m) = 4m$.

Theorem 1.1.2 [8]For any graph$K_{1,m}$, $SDTwin(K_{1,m}) = \left[\frac{m(m+1)}{2}\right]$.

Theorem 1.1.3 [8]For any path P_m, SDTwin (P_m) = 4m – 10.

In [7], the authors obtained this number for many standard classes of graphs like path, cycle, wheel graph, complete graph, star graph, Cartesian product of path and Corona product of path. In continuation of that, in this paper we focus to obtain this number for many interesting special types of graphs.

II. DOUBLE TWIN DOMINATION NUMBER OF A SOME SPECIAL TYPE OF GRAPH

Notation : $(x_i, y_{i\mp n})$ is denotes the distance of vertices from (x_i, y_{i+n}) and (x_i, y_{i-n}) where $n = 1,2,3$

Theorem2.1 If $G = C_n \odot K_{1,m}$,then $DTD(G) = \frac{5m^2n+13nm+8n}{2\binom{n+nm}{2}}$ where $n > 4, m > 1$.

Proof. Consider the graph $C_n \odot K_{1,m}$. Let $\{a_1, a_2, \dots a_n\}$ be the vertices of the cycle C_n. $\{a_{i1}, a_{i2}, \dots a_{im}\}$ be the vertices of the star graph$K_{1,m}$, where i =1,2,…n. Now attach the root vertex of the star graph $K_{1,m}$ to each vertex of the cycle C_n say a_i.

$SDTwin(G) = \sum DTwin(u, v)$for $u, v \in V(G)$.

For any cycle graphC_n,$SDTwin(C_n) = 4n$

For any graph$K_{1,m}$,

$$SDTwin(n\ copy\ of\ K_{1,m}) = n\left[\frac{m(m+1)}{2}\right]$$

$DTwin(a_i, a_{(i+1)j}) = 1$; for$i = 1\ to\ n-1$; $j = 1\ to\ m$; Therefor$\sum_{i=1}^{n-1}(a_i, a_{(i+1)j}) = m(n-$ *1.DTwin an,a1j=m.*

$DTwin(a_i, a_{(i-1)j}) = 1$; for $i = 2\ to\ n$; $j = 1\ to\ m$; Therefore$\sum_{i=2}^{n}(a_i, a_{(i-1)j}) = m(n-1)$.$DTwin\ (a_1, a_{nj}) = m$.

$DTwin(a_i, a_{(i+2)j}) = 1$; for $i = 1\ to\ n-2$; $j = 1\ to\ m$; Therefore $\sum_{i=1}^{n-2}(a_i, a_{(i+2)j}) = m(n-2)$.

$DTwin\ (a_{n-1}, a_{1j}) = m, DTwin\ (a_n, a_{2j}) = m$.

$DTwin(a_i, a_{(i-2)j}) = 1$; for $i = 3\ to\ n$; $j = 1\ to\ m$; Therefore$\sum_{i=3}^{n}(a_i, a_{(i-2)j}) =$ *m(n−2).DTwin a1,an−1j=m,DTwin a2,anj=m.*

$DTwin(a_i, a_{(i+3)j}) = 1$; for $i = 1\ to\ n-3$; $j = 1\ to\ m$; Therefore$\sum_{i=1}^{n-3}(a_i, a_{(i+3)j})m(n-3)$.

$DTwin\ (a_{(n-2)}, a_{1j}) = m, DTwin\ (a_{(n-1)}, a_{2j}) = m$,

$DTwin\ (a_n, a_{3j}) = m$.

$DTwin(a_i, a_{(i-3)j}) = 1$; for $i = 4\ to\ n$; $j = 1\ to\ m$; Therefore $\sum_{i=4}^{n}(a_i, a_{(i-3)j}) = m(n-3)$.

$DTwin\ (a_1, a_{(n-2)j}) = m$,

$DTwin\ (a_2, a_{(n-1)j}) = m, DTwin\ (a_3, a_{nj}) = m$.

$DTwin(a_{ij}, a_{(i+1)k}) = 1$; for $i = 1\ to\ n-1$; $j = 1\ to\ m; k = 1\ to\ m$; Therefore $\sum_{i=1}^{n-1}(a_{ij}, a_{(i+1)k}) = m^2(n-1)$.

$DTwin\ (a_{nk}, a_{1k}) = m^2$.

$DTwin(a_{ij}, a_{(i+2)k}) = 1$; for $i = 1\ to\ n-2$; $j = 1\ to\ m$; $k = 1\ to\ m$; Therefore $\sum_{i=1}^{n-1}(a_{ij}, a_{(i+2)k}) = m^2(n-2)$.

$DTwin\ (a_{(n-1)j}, a_{1k}) = m^2, DTwin\ (a_{nj}, a_{2k}) = m^2$.

$$SDTwin(G) = 4n + n\left[\frac{m(m+1)}{2}\right] + m(n-1) + m + m(n-2) + m + m + m(n-3) + m + m + m + m(n-1) + m + m(n-2) + m + m + m(n-3) + m + m + m + m^2(n-1) + m^2 + m^2(n-2) + m^2 + m^2.$$

$SDTwin(G) = \frac{5m^2n+13nm+8n}{2}$.

$DTD(G) = \frac{5m^2n+13nm+8n}{2\binom{n+nm}{2}}$.

Illustration. For the graph $C_5 \odot K_{1,2}$

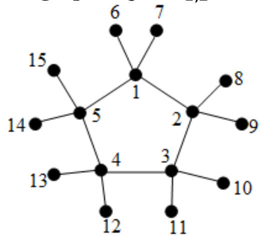

Figure 2.1

DTwin (1, 2) = 2; DTwin (1, 3) = 2;……..By considering all the various possible cases as in example….., it can be verified that $SDTwin(G) = 135; DTD(G) = \frac{135}{105} = \frac{9}{7}$.

Theorem2.2 If G = $PC\ (n, m)$ where $n > 1$ and $m \geq 5$, then$DTD\ (G) = \frac{n(n+13)+8m}{2\binom{n+m}{2}}$.

Proof. Let ($a_1, a_2 \dots a_i \dots . a_m$) be the vertices of the cycle C_m. ($b_1, b_2 \dots . b_n$) be the pendent vertices of the star $K_{1,n}$. Now attach the root vertex of the star $K_{1,n}$ to any vertex of the cycle C_m say a_1.

$SDTwin\ (G) = \sum DTwin(u, v)$for u, v ∈ V (G).

For any cycle C_m, $SDTwin\ (C_m) = 4m$ for any m.

For any star $K_{1,n}$, $SDTwin\ (K_{1,n}) = \frac{n(n+1)}{2}$

$DTwin(a_2, b_i) = 1$; for $i = 1\ to\ n$;Therefore$\sum_{i=1}^{n} DTwin(a_2, b_i) = n$.

$DTwin(a_3, b_i) = 1$; for $i = 1\ to\ n$; Therefore $\sum_{i=1}^{n} DTwin(a_3, b_i) = n$.

$DTwin(a_4, b_i) = 1$; for $i = 1\ to\ n$; Therefore $\sum_{i=1}^{n} DTwin(a_4, b_i) = n$.

$DTwin(a_m, b_i) = 1$;for $i = 1\ to\ n$; Therefore $\sum_{i=1}^{n} DTwin(a_m, b_i) = n$.

$DTwin(a_{m-1}, b_i) = 1$; for $i = 1\ to\ n$; Therefore $\sum_{i=1}^{n} DTwin(a_{m-1}, b_i) = n$.

$DTwin(a_{m-2}, b_i) = 1$; for $i = 1\ to\ n$; Therefore $\sum_{i=1}^{n} DTwin(a_{m-2}, b_i) = n$.

$SDTwin\ (G) =$ 4m + $\frac{n(n+1)}{2}$ + $6n$;

$$SDTwin(G) = \frac{n(n+13)+8m}{2}.$$

$$DTD\ (G) = \frac{n(n+13)+8m}{2\binom{n+m}{2}}.$$

Illustration. For the graph $PC\ (5, 6)$

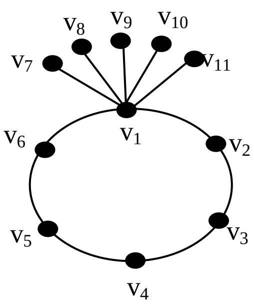

Figure2.2

DTwin (v_1, v_2) = 1; DTwin (v_1, v_3) = 2;……..By considering all the various possible cases as in example….., it can be verified that

$SDTwin\ (G) = 69;\ DTD\ (G)\ = \frac{69}{55}$.

Theorem2.3 If G =$P_m\,(K_{1,n})$ where $n\ >\ 2, m\ >\ 3$, then $DTD\ (G) = \frac{n(n+5)+4(2m-5)}{2\binom{n+m}{2}}$.

Proof. Let $P_m\,(K_{1,n})$ be a graph obtained by attaching the root vertex of $(K_{1,n})$ to the end vertex of the Path P_m. Let $(K_{1,n})$ be a star with l pendent vertices and one root vertex, P_m be a path with m vertices.

Let (a_1, a_2 ,…….a_m) be the vertices of the path P_m and (b_1 , b_2 ,…….b_n) be the pendent vertices of the star $K_{1, n}$. Attach the pendent vertex a_1 of the path P_m to the root vertex of $(K_{1,n})$.

$SDTwin\ (G)\ = \sum DTwin(u, v)$for u, v∈V (G).

For any path P_m,$SDTwin\ (Pm)\ =\ 4n - 10$ for any m. For any strar $K_{1, n}$, $SDTwin(K_{1,n}) = \left[\frac{n(n+1)}{2}\right]$

$DTwin(a_2, b_i) = 1;$ for $i\ =\ 1\ to\ n;$ Therefore $\sum_{i=1}^{n} DTwin(a_2, b_i) = n.$

$DTwin(a_3, b_i) = 1;$ for $i\ =\ 1\ to\ n;$ Therefore $\sum_{i=1}^{n} DTwin(a_3, b_i) = n.$

$DTwin(a_4, b_i) = 1;$ for $i\ =\ 1\ to\ n;$ Therefore $\sum_{i=1}^{n} DTwin(a_4, b_i) = n.$

$SDTwin\ (G)\ =\ 4m - 10\ +\ \frac{n(n+1)}{2} + 3n.$

$SDTwin(G) = \frac{n(n+7)+4(2m-5)}{2}.$

$DTD\ (G)\ = \frac{n(n+7)+4(2m-5)}{2\binom{n+m}{2}}.$

Illustration.For the graph $P_6(K_{1, 5})$,

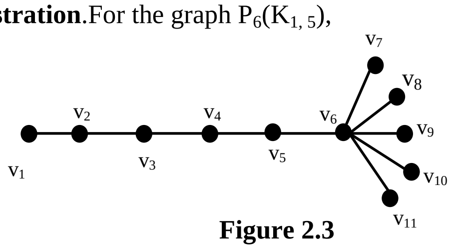

Figure 2.3

DTwin (v_1, v_2) = 1; DTwin (v_1, v_3) = 1;……..By considering all the various possible cases as in example….., it can be verified that

$SDTwin\ (G) = 37, DTD\ (G) = \frac{44}{55}$.

Observation2.4 If G = Y_3,then $DTD(Y_3) = 84$.

Observation2.5 If G = Y_4, then $DTD(Y_4) = 140$

Observation2.6 If G = Y_5, then $DTD(Y_5) = 200$

Theorem 2.7 If G = Y_n for $n > 6$, then $DTD(G)\ =\ \frac{41n}{\binom{2n}{2}}$

Proof. Consider the prism graph Y_n with 2n vertices.

Let the inner cycle vertices are {x_1, x_2, x_3,…x_n} and the outer cycle vertices are {y_1,y_2,y_3,…,y_n}.

$SDTwin\ (G)\ = \sum DTwin(u, v)$for $u, v \in V\ (G)$.

$DTwin(x_i, x_{i+1}) = 2;$ for $i\ =\ 1\ to\ n-1;$ Therefore $\sum_{i=1}^{n-1} DTwin(x_i, x_{i+1}) = 2(n-1)$.DTwin ($x_n$, x_1) = 2.

$DTwin(x_i, x_{i+2}) = 4;$ for $i\ =\ 1\ to\ n-2;$ Therefore $\sum_{i=1}^{n-2} DTwin(x_i, x_{i+2}) = 4(n-2)$.DTwin ($x_n$, x_2) = 4; DTwin (x_{n-1}, x_1) = 4.

$DTwin(x_i, x_{i+3}) = 1;$ for $i\ =\ 1\ to\ n-3;$ Therefore $\sum_{i=1}^{n-3} DTwin(x_i, x_{i+3}) = (n-3)$.DTwin ($x_n$, x_3) = 1; DTwin (x_{n-1}, x_2) =1; DTwin (x_{n-2}, x_1) =1.

$DTwin(x_i, x_{i+4}) = 1;$ for $i\ =\ 1\ to\ n-4$; Therefore $\sum_{i=1}^{n-4} DTwin(x_i, x_{i+4}) = (n-4).$

DTwin(x_n,x_4) = 1; DTwin(x_{n-1},x_3) = 1; DTwin(x_{n-2},x_2)= 1; DTwin(x_{n-3},x_1)=1.

$DTwin(y_i, y_{i+1}) = 2;$ for $i\ =\ 1\ to\ n-1$; Therefore $\sum_{i=1}^{n-1} DTwin(y_i, y_{i+1}) = 2(n-1)$.DTwin ($y_n$,$y_1$) = 2.

$DTwin(y_i, y_{i+2}) = 4;$ for $i\ =\ 1\ to\ n-2$; Therefore $\sum_{i=1}^{n-2} DTwin(y_i, y_{i+2}) = 4(n-2).$ DTwin(y_n,y_2) = 4; DTwin(y_{n-1},y_1) = 4.

$DTwin(y_i, y_{i+3}) = 1;$ for $i\ =\ 1\ to\ n-3$; Therefore $\sum_{i=1}^{n-3} DTwin(y_i, y_{i+3}) = (n-3).$ DTwin(y_n,y_3) = 1; DTwin(y_{n-1},y_2) = 1; DTwin(y_{n-2},y_1)=1.

$DTwin(y_i, y_{i+4}) = 1;$ for $i\ =\ 1\ to\ n-4$; Therefore $\sum_{i=1}^{n-4} DTwin(y_i, y_{i+4}) = (n-4).$ DTwin(y_n,y_4) = 1; DTwin(y_{n-1},y_3) = 1; DTwin(y_{n-2},y_2)=1; DTwin(y_{n-3},y_1)=1.

$DTwin(x_i, y_i) = 3;$ for $i\ =\ 1\ to\ n$; Therefore $\sum_{i=1}^{n} DTwin(x_i, y_i) = 3n.$

$DTwin(x_i, y_{i\mp1}) = 4;$ for $i\ =\ 1\ to\ n-1;$ Therefore $\sum_{i=1}^{n-1} DTwin(x_i, y_{i\mp1}) = 8(n-1).$ DTwin(x_n,y_1) = 4; DTwin(y_n,x_1) = 4.

$DTwin(x_i, y_{i\pm2}) = 3;$ for $i\ =\ 1\ to\ n-2;$ Therefore $\sum_{i=1}^{n-2} DTwin(x_i, y_{i\pm2}) = 6(n-2).$ DTwin(x_n,y_2) = 3; DTwin(y_1,x_{n-1}) = 3. DTwin(x_2,y_n) =3; DTwin(yn-1,x_1) = 3.

$DTwin\ (x_i, y_{i\mp3}) = 4$; for $i\ =\ 1\ to\ n-3$; Therefore $\sum_{i=1}^{n-3} DTwin(x_i, y_{i\mp3}) = 8(n-3).$ DTwin(x_n,y_3) = 4; DTwin(y_2,x_{n-1}) = 4; DTwin(y_1,x_{n-2}) = 4.DTwin(x_3,y_n) = 4; DTwin(y_{n-1},x_2) =4; DTwin(y_{n-2},x_1) = 4.

$$SDTwin\ (G)\ =\ 2(n-1) + 2 + 4(n-2) + 8 + (n-3) + 3 + (n-4) + 4 + 2(n-1) + 2 + 4(n-2) + 8 + (n-3)\ + 3\ + (n-4)\ + 4\quad + 3n + 8(n-1) + 8 + 6(n-2) + 12 + 8(n-3) + 24.$$

$SDTwin(G) =\ 41n.$

$DTD(G)\ =\ \frac{41n}{\binom{2n}{2}}.$

Illustration. For the graph Y_{10}

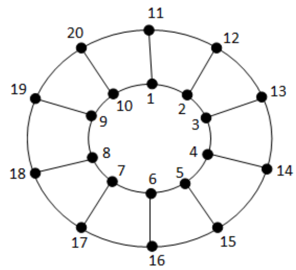

Figure 2.4

DTwin (1, 2) = 2; DTwin (1, 3) = 4;……..By considering all the various possible cases as in example….., it can be verified that

$SDTwin\ (G)\ =\ 410;\ DTD\ (G)\ = \frac{410}{190}\ = \frac{41}{19}.$

Observation2.8 If $G\ =\ S_3$, then $DTD(S_3) = 39.$

Observation2.9 If $G\ =\ S_4$, then $DTD(S_4) = 48.$

Theorem2.10 If $G\ =\ S_n$,for $n\ \geq 5$, then $DTD\ (G)\ = \frac{13n}{\binom{2n}{2}}.$

Proof. Consider the sunlet graph S_n with 2n vertices.
Let S_n be a graph obtained by attaching the product vertex to all the vertices of the cycle C_n. Let the $\{x_1, x_2, x_3 \ldots x_n\}$ be the vertices of the cycle C_n and $\{y_1, y_2, y_3 \ldots y_n\}$ be the pendent vertices.
Now attach the pendent vertex of x_1 to y_1, x_2 to y_2, … x_n to y_n respectively.
$SDTwin\ (G) = \sum DTwin(u,v)$ for $u, v \in V\ (G)$.
$DTwin(x_i, x_{i+1}) = 1$; for $i = 1\ to\ n-1$; Therefore $\sum_{i=1}^{n-1} DTwin(x_i,x_{i+1}) = (n-1)$.DTwin $(x_n, x_1) = 1$.
$DTwin(x_i, x_{i+2}) = 1$; for $i = 1\ to\ n-2$; Therefore $\sum_{i=1}^{n-2} DTwin(x_i,x_{i+2}) = (n-2)$.
DTwin $(x_n, x_2) = 1$; DTwin $(x_{n-1}, x_1) = 1$.
$DTwin(x_i, x_{i+3}) = 1$; for $i = 1\ to\ n-3$; Therefore $\sum_{i=1}^{n-3} DTwin(x_i,x_{i+3}) = (n-3)$.
DTwin $(x_n, x_3) = 1$; DTwin $(x_{n-1}, x_2) = 1$;
DTwin $(x_{n-2}, x_1) = 1$.
$DTwin(x_i, x_{i+4}) = 1$; for $i = 1\ to\ n-4$; Therefore $\sum_{i=1}^{n-4} DTwin(x_i,x_{i+4}) = (n-4)$.
DTwin $(x_n, x_4) = 1$; DTwin $(x_{n-1}, x_3) = 1$;
DTwin $(x_{n-2}, x_2) = 1$; DTwin $(x_{n-3}, x_1) = 1$.
$DTwin(y_i, y_{i+1}) = 1$;for $i = 1\ to\ n-1$; Therefore $\sum_{i=1}^{n-1} DTwin(y_i,y_{i+1}) = (n-1)$.DTwin $(y_n, y_1) = 1$.
$DTwin(y_i, y_{i+2}) = 1$; for $i = 1\ to\ n-2$; Therefore $\sum_{i=1}^{n-2} DTwin(y_i,y_{i+2}) = (n-2)$.
DTwin $(y_n, y_2) = 1$; DTwin $(y_{n-1}, y_1) = 1$.
$DTwin(x_i, y_i) = 1$; for $i = 1\ to\ n$; Therefore $\sum_{i=1}^{n} DTwin(x_i,y_i) = n$.
$DTwin(x_i, y_{i\mp1}) = 1$; for $i = 1\ to\ n-1$; Therefore $\sum_{i=1}^{n-1} DTwin(x_i,y_{i\mp1}) = 2(n-1)$.
DTwin $(x_n, y_1) = 1$; DTwin $(y_n, x_1) = 1$.
$DTwin(x_i, y_{i\mp2}) = 1$; for $i = 1\ to\ n-2$; Therefore $\sum_{i=1}^{n-2} DTwin(x_i,y_{i\mp2}) = 2(n-2)$.
DTwin (x_n, y_2) = DTwin$(y_1, x_{n-1}) = 1$.
DTwin(x_2, y_n) = DTwin$(yn_{-1}, x_1) = 1$.
$DTwin(x_i, y_{i\mp3}) = 1$; for $i = 1\ to\ n-3$; Therefore $\sum_{i=1}^{n-3} DTwin(x_i,y_{i\mp3}) = 2(n-3)$.
DTwin $(x_n, y_3) = 1$; DTwin $(y_2, x_{n-1}) = 1$;
DTwin $(y_1, x_{n-2}) = 1$; DTwin $(x_3, y_n) = 1$;
DTwin $(y_{n-1}, x_2) = 1$; DTwin $(y_{n-2}, x_1) = 1$.

$$SDTwin\ (G) = (n-1) + 1 + (n-2) + 2 + (n-3) + 3 + (n-4) + 4 + (n-1) + 1 + (n-2) + 2 + (n-3) + 3 + (n-4) + 4 + n + 2(n-1) + 2 + 2(n-2) + 4 + 2(n-3) + 6.$$

$$SDTwin\ (G) = n + n + n + n + n + n + n + 2n + 2n + 2n = 13n$$

$DTD\ (G) = \frac{13n}{\binom{2n}{2}}$.

Illustration.forthe graph S_{10}

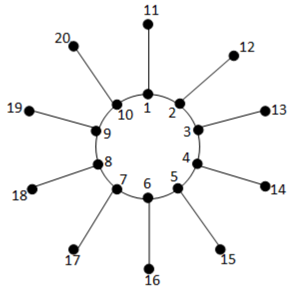

Figure 2.5

DTwin (1, 2) = 1; DTwin (1, 3) = 1;……..By considering all the various possible cases as in example….., it can be verified that

$SDTwin\ (G) = 130$; $DTD\ (G) = \frac{130}{190} = \frac{13}{19}$.

Observation 2.11 If $G = W_3$, then $DTD(W_3) = 134$.
Observation 2.12 If $G = W_4$, then $DTD(W_4) = 220$.
Observation 2.13 If $G = W_5$, then $DTD(W_5) = 320$.
Theorem2.14 If $G = W_n$, for $n \geq 6$, then $DTD\ (G) = \frac{63n}{\binom{3n}{2}}$.

Proof. Consider the web graph W_n with 3n vertices.
Let the inner cycle vertices are $\{x_1,x_2,x_3,\ldots x_n\}$ and the outer cycle vertices are $\{y_1,y_2,y_3,\ldots,y_n\}$ and $\{z_1,z_2,z_3,\ldots z_n\}$ be the pendent vertices.
Now attaching the pendent vertices of y_1 to z_1, y_2 to z_2, y_3 to z_3 … y_n to z_n respectively.
$SDTwin\ (G) = \sum DTwin(u,v)$ for u, v ∈ V(G).
$DTwin(x_i, x_{i+1}) = 2$; for $i = 1\ to\ n-1$; Therefore $\sum_{i=1}^{n-1} DTwin(x_i,x_{i+1}) = 2(n-1). DTwin(x_n,x_1) = 2$.
$DTwin\ (x_i, x_{i+2}) = 4$; for $i = 1\ to\ n-2$; Therefore $\sum_{i=1}^{n-2} DTwin(x_i,x_{i+2}) = 4(n-2)$.
$DTwin(x_n,x_2) = 4, DTwin(x_{n-1},x_1) = 4$.
$DTwin\ (x_i, x_{i+3}) = 4$; for $i = 1\ to\ n-3$; Therefore $\sum_{i=1}^{n-3} DTwin(x_i,x_{i+2}) = 4(n-3)$.
$DTwin(x_n,x_3) = 4, DTwin(x_{n-1},x_2) = 4$,
$DTwin(x_{n-2},x_1) = 4$.
$DTwin\ (x_i, x_{i+4}) = 1$; for $i = 1\ to\ n-4$; Therefore $\sum_{i=1}^{n-4} DTwin(x_i,x_{i+4}) = (n-4)$.
$DTwin(x_n,x_4) = 1, DTwin(x_{n-1},x_3) = 1$,
$DTwin(x_{n-2},x_2) = 1,\ DTwin(x_{n-3},x_1) = 1$.
$DTwin\ (y_i, y_{i+1}) = 2$; for $i = 1\ to\ n-1$; Therefore $\sum_{i=1}^{n-1} DTwin(y_i,y) = 2(n-1). DTwin(y_n,y_1) = 2$.
$DTwin\ (y_i, y_{i+2}) = 4$; for $i = 1\ to\ n-2$; Therefore $\sum_{i=1}^{n-2} DTwin(y_i,y_{i+2}) = 4(n-2)$.
$DTwin(y_n,y_2) = 4, DTwin(y_{n-1},y_1) = 4$.
$DTwin\ (y_i, y_{i+3}) = 4$; for $i = 1\ to\ n-3$; Therefore $\sum_{i=1}^{n-3} DTwin(y_i,y_{i+2}) = 4(n-3)$.
$DTwin(y_n,y_3) = 4, DTwin\ (y_{n-1},y_2) = 4$,
$DTwin(y_{n-2},y_1) = 4$.
$DTwin\ (y_i, y_{i+4}) = 1$; for $i = 1\ to\ n-4$; Therefore $\sum_{i=1}^{n-4} DTwin(y_i,y_{i+4}) = (n-4)$.
$DTwin\ (y_n,y_4) = 1, DTwin\ (y_{n-1},y_3) = 1$,
$DTwin\ (y_{n-2},y_2) = 1, DTwin\ (y_{n-3},y_1) = 1$.
$DTwin(x_i,y_i) = 3$; for $i = 1\ to\ n$; Therefore $\sum_{i=1}^{n} DTwin(x_i,y_i) = 3n$.
$DTwin(x_i, y_{i+1}) = 4$; for $i = 1\ to\ n-1$; Therefore $\sum_{i=1}^{n-1} DTwin(x_i,y_{i+1}) = 4(n-1)$.
$DTwin\ (x_n,y_1) = 4$.
$DTwin(x_i, y_{i+2}) = 3$; for $i = 1\ to\ n-2$; Therefore $\sum_{i=1}^{n-2} DTwin(x_i,y_{i+2}) = 3(n-2)$.
$DTwin\ (x_n,y_2) = 3,\ DTwin(x_{n-1},y_1) = 3$.
$DTwin(x_i, y_{i+3}) = 4$; for $i = 1\ to\ n-3$; Therefore $\sum_{i=1}^{n-3} DTwin(x_i,y_{i+3}) = 4(n-3)$.
$DTwin(x_n,y_3) = 4, DTwin(x_{n-1},y_2) = 4$,
$DTwin(x_{n-2},y_3) = 4$.
$DTwin(x_i, y_{i-1}) = 4$; for $i = 2\ to\ n$; Therefore

Retrieval Number: D10241284S519/2019©BEIESP
DOI:10.35940/ijrte.D1024.1284S519

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$\sum_{i=2}^{n} DTwin(x_i, y_{i-1}) = 4(n-1). DTwin(x_n, y_1) = 4.$
$DTwin(x_i, y_{i-2}) = 3;$ for $i = 3$ to n; Therefore $\sum_{i=3}^{n} DTwin(x_i, y_{i-2}) = 3(n-2).$
$DTwin(x_2, y_n) = 3, DTwin(x_1, y_{n-1}) = 3.$
$DTwin(x_i, y_{i-3}) = 4;$ for $i = 4$ to n; Therefore $\sum_{i=3}^{n} DTwin(x_i, y_{i-3}) = 4(n-3).$
$DTwin(x_3, y_n) = 4, DTwin(x_2, y_{n-1}) = 4,$
$DTwin(x_1, y_{n-2}) = 4.$
$DTwin(x_i, z_i) = 3;$ for $i = 1$ to n; Therefore $\sum_{i=1}^{n} DTwin(x_i, z_i) = 3n.$
$DTwin(x_i, z_{i+1}) = 2;$ for $i = 1$ to $n-1$; Therefore $\sum_{i=1}^{n-1} DTwin\ (x_i, z_{i+1}) = 2(n-1). DTwin(x_n, z_1) = 2.$
$DTwin(x_i, z_{i+2}) = 3;$ for $i = 1$ to $n-2$; Therefore $\sum_{i=1}^{n-2} DTwin\ (x_i, z_{i+2}) = 3(n-2).$
$DTwin(x_n, z_2) = 3, DTwin(x_{n-1}, z_1) = 3.$
$DTwin(x_i, z_{i-1}) = 2;$ for $i = 2$ to n; Therefore $\sum_{i=2}^{n} DTwin\ (x_i, z_{i-1}) = 2(n-1). DTwin(x_1, z_n) = 2$
$DTwin(x_i, z_{i-2}) = 3;$ for $i = 3$ to n; Therefore $\sum_{i=3}^{n} DTwin\ (x_i, z_{i-2}) = 3(n-2).$
$DTwin(x_2, z_n) = 3, DTwin(x_1, z_{n-1}) = 3.$
$DTwin\ (y_i, z_i) = 1;$ for $i = 1$ to n; Therefore $\sum_{i=1}^{n} DTwin(y_i, z_i) = n.$
$DTwin\ (y_i, z_{i+1}) = 1;$ for $i = 1$ to $n-1$; Therefore $\sum_{i=1}^{n-1} DTwin(y_i, z_{i+1}) = (n-1). DTwin(y_n, z_1) = 1.$
$DTwin\ (y_i, z_{i+2}) = 1;$ for $i = 1$ to $n-2$; Therefore $\sum_{i=1}^{n-2} DTwin(y_i, z_{i+2}) = (n-2).$
$DTwin(y_n, z_2) = 1, DTwin(y_{n-1}, z_1) = 1.$
$DTwin\ (y_i, z_{i+3}) = 1;$ for $i = 1$ to $n-3$; Therefore $\sum_{i=1}^{n-3} DTwin(y_i, z_{i+3}) = (n-3).$
$DTwin(y_n, z_3) = 1, DTwin(y_{n-1}, z_2) = 1,$
$DTwin(y_{n-1}, z_2) = 1.$
$DTwin\ (y_i, z_{i-1}) = 1;$ for $i = 2$ to n; Therefore $\sum_{i=2}^{n} DTwin(y_i, z_{i-1}) = (n-1). DTwin(y_1, z_n) = 1.$
$DTwin\ (y_i, z_{i-2}) = 1;$ for $i = 3$ to n; Therefore $\sum_{i=3}^{n} DTwin(y_i, z_{i-2}) = (n-2).$
$DTwin(y_2, z_n) = 1, DTwin(y_1, z_{n-1}) = 1.$
$DTwin\ (y_i, z_{i-3}) = 1;$ for $i = 4$ to n; Therefore $\sum_{i=4}^{n} DTwin(y_i, z_{i-3}) = (n-3).$
$DTwin(y_3, z_n) = 1, DTwin(y_2, z_{n-1}) = 1,$
$DTwin(y_1, z_{n-2}) = 1.$
$DTwin(z_i, z_{i+1}) = 1;$ for $i = 1$ to $n-1$; Therefore $\sum_{i=1}^{n-1} DTwin(z_i, z_{i+1}) = (n-1). DTwin(z_n, z_1) = 1.$
$DTwin(z_i, z_{i+2}) = 1;$ for $i = 1$ to $n-2$; Therefore $\sum_{i=1}^{n-2} DTwin(z_i, z_{i+2}) = (n-2).$
$DTwin(z_n, z_2) = 1, DTwin(z_{n-1}, z_1) = 1.$

$$SDTwin\ (G) = 2(n-1) + 2 + 4(n-2) + 8 + n-3 + 3 + n-4 + 4 + 2(n-1) + 2 + 4(n-2) + 8 + n-3 + 3 + n-4 + 4 + 3n + 4(n-1) + 4 + 3(n-2) + 6 + 4(n-3) + 12 + 4(n-1) + 4 + 3(n-2) + 4(n-2) + 12 + 3n + 2(n-1) + 2 + 3(n-2) + 6 + n + n-1 + 1 + n-2 + 2 + n-3 + 3 + 2(n-1) + 2 + 3(n-2) + 6 + n-1 + 1 + n-2 + 2 + n-3 + 3 + n-1 + 1 + n-2 + 2.$$

$SDTwin\ (G) = 63n.$

$DTD\ (G) = \frac{63n}{\binom{3n}{2}}.$

Ill ustration. For the graph W_{10}

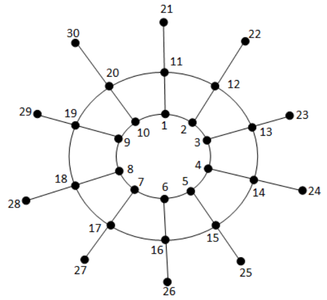

Figure 2.6

DTwin (1, 2) = 2; DTwin (1, 3) = 4;……..By considering all the various possible cases as in example….., it can be verified that $SDTwin\ (G) = 130;\ DTD\ (G) = \frac{630}{190} = \frac{63}{19}.$

III. CONCLUSION

In this paper,we investigated this parameter for Prism graph, Sun-let graph, Web graph, Peacock graph,$P_m(K_{1,n})$ and$C_n \odot K_{1,m}$. The authors investigate this number for many product related graph like strong product, tensor product, lexicographic product, semi product, corona product and some special types graphs which will be reported in the sub sequent papers.

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