Some Results on the Eccentric Sequence of Graphs

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Abstract: ***The distance d(u, v) from a vertex u of graph G to a vertex v is the length of a shortest u to v path. The degree of the vertex u is the number of vertices at distance one. The sequence of numbers of vertices having 0,1,2,3,... is called the degree sequence, which is the list of degrees of vertices of G arranged in non-decreasing order. The eccentricity e(a) of a is the distance of a farthest vertex from a. Let G be a connected graph. The Eccentric Sequence of G is the list of the eccentricities of its vertices arranged in non-decreasing order. In this paper, we characterize the eccentric sequence of some of the derived graphs namely the line graph of the integral graph , the eccentric sequence of Mycieleskian of a graph.***

Keywords: ***Degree Sequence of graphs, Eccentric Sequence of graphs, Line graph of a graph, Mycieleskian of a graph.***

I. INTRODUCTION

The idea of degrees in graphs has paved way for the study of various basic or formational properties of graphs and has therefore attracted the attention of many researchers. Based on the concept of degrees of graphs, degree sequence of graphs were developed. We define a non- decreasing sequence of non- negative integers to be a degree sequence of a graph. This non-negative sequence is called a graphical sequence of some graph G if G realizes the sequence. If two graphs have same degree sequence then they are said to be degree equivalent. Realizability of any sequence for a graph was the elementary prerequisite which paved way for the study of degree sequence of graphs. Erdos and Gallai gave an existential characterization for the degree sequence of graphs. Then the constructive characterization was found independently by Havel and later by Hakimi that is now referred to as the Havel and Hakimi algorithm. Next sequence to be developed was the Eccentric Sequence of graphs and this was first in the class of distance related sequences to be studied for undirected graphs. Some basic results in this concept were due to Lesniak-Foster, Ostrand , Behzad, and Simpson. Next the minimal eccentric sequences were mainly studied by Nandakumar . The eccentric sequence of digraphs were developed quite later. Gimbert and Lopez worked on the concept of eccentric sequence of digraphs .

Next distance based sequences to be studied were the path degree sequences and distance degree sequences. Randic studied these sequences to differentiate chemical isomers by their graph structure. Path degree sequence of a graph finds its application in describing atomic environments and in various classification schemes for molecules.

In this paper we have discussed the eccentric sequence of an Integral graph, the eccentric sequence of line graph of complete graph and the eccentric sequence of the Mycielski of Graph(Grotzsch graph).

II. PRELIMINARIES

Let G (V, E) to be a graph where V is the list of vertices and E is the list of edges. Here G is a finite undirected graph without multiple edges and self loops

Consider a, b as two vertices in Graph G. The length of shortest a-b path from the vertex a to the vertex b is called the distance d(a,b) from a to b.

The list of number of incident edges of all vertices of the graph is called the Degree Sequence of a graph.

The distance of a farthest vertex from g is called the eccentricity e(g) of vertex g.

Diameter of the graph G is the highest of the eccentricities of G. Radius of graph G is the least of the eccentricities of G.

If all the vertices of the graph G have same eccentricity, then the graph is said to
be self- centered.

A vertex x is an eccentric vertex of another vertex y if
d(x, y) = e(x), $x \neq y$

The Eccentric Sequence of a graph G is a list of the eccentricities of its vertices.

Example:

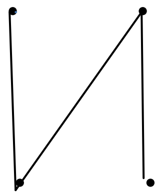

Fig (i) Graph G

The Eccentricity Sequence of the above graph G is 2,2,3,3.

A graph G whose spectrum of its adjacency matrix consists only of integers is defined as an Integral Graph. In other words, a graph is an integral graph if all of the eigen values of the characteristic polynomial of its adjacency matrix are integers. The line graph of *G* is another graph L(G) such that each vertex of L(G) represents an edge of *G*; and two vertices of L(G) are adjacent if and only if their corresponding edges are incident in *G*.

Revised Manuscript Received on December 5, 2019.

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The Mycielski graph $\mu(G)$ of the graph G has G itself as its subgraph, along with n+1 additional vertices.There is a vertex u_i which corresponds to each vertex v_i of G and an extra vertex x. Each vertex u_i is adjacent to x such that these vertices form a subgraph in the form of a star. Also for every edge of G , the Mycielski graph includes two edges. So if G has n vertices and m edges then $\mu(G)$ has 2n+1vertices and 3m+n edge.

III. MAIN RESULTS

A Theorem

Statement:
The Eccentric Sequence of the integral graph of the complete graph G = K_n where 'n ' is the number of vertices of G is {1,1,1,…1,1}. The cardinality of the Eccentric Sequence depends on 'n'.
Proof:
Let G = K_n be the complete graph with 'n' number of vertices. The Integral Graph of the complete graph K_n is the complete graph itself for all 'n'. The Eccentric sequence of K_1 is {1}. The Eccentric Sequence of K_2 is {1,1}. The Eccentric Sequence of K_3 is {1,1,1} and so on. The eccentricity of every vertex of a complete graph is obviously 1 as every pair of distinct vertices is connected by an unique edge. Or in other words every vertex of G is adjacent with every other vertex of G. Therefore the Eccentric Sequence of the complete graph K_n is {1,1,1,…1,1}. As mentioned earlier the cardinality of the Eccentric Sequence of the complete graph depends on the number of vertices of G = K_n

Hence the theorem.

B Theorem

Statement:
The Eccentric Sequence of the line graph of the complete graph K_n is {2,2,2…2,2}.The cardinality of the Eccentric Sequence of the line graph of K_n depends on the number of edges of the complete graph.
Proof:
Let G = K_n be the complete graph. Let n be the number of vertices and m be the number of edges of G. Line graph of K_n is denoted as L(K_n) . For any graph G with n vertices and m edges, the number of vertices of the line graph L(G) is m and the number of edges of L(G) is the difference between half the sum of the squares of the degrees of the vertices in G and the number of edges of G (ie m).
For the complete graph K_n its line graph L(K_n) has m vertices ie each vertex of L(K_n) represents an edge of K_n .In general , two vertices of any line graph L(G) are adjacent if and only if their corresponding edges are incident to a vertex in G. From L(K_n) we infer that every pair of v_i and v_{i+k} vertices are adjacent that is there is an edge between v_i and v_{i+k} because v_i and v_{i+k} in L(K_n) are the corresponding edges e_i and e_{i+k} that are incident in K_n. So the distance between every pair of vertex v_i and v_{i+k} is 1. Similarly if two edges $e_i$and e_j are not incident in K_n then the corresponding vertices v_i and v_j in L(K_n) will not be adjacent and the distance between such vertices in L(K_n) will be 2 which is maximum. So the eccentricity of every vertex v_i , i=1,2,…n is 2. Hence the Eccentricity Sequence of the line graph of the complete graph. L(K_n) is {2,2,2, ... 2,2} . Since the number of vertices of L(K_n) is the number of edges of K_n , the cardinality of L(K_n) depends on the number of edges of K_n.

Hence the theorem.

C Theorem

Statement:
The eccentric matrix of G(T,s) the s subdivided tree is given by
E(G(T,s)) =

$$\begin{bmatrix} [P]_{aXa} & 0 \\ 0 & [Q]_{s(a-1)Xs(a-1)} \end{bmatrix}_{(a+s(a-1))X(a+s(a-1))}$$

Proof:
Here we give the eccentricity of each vertex of the tree and subdivided vertices. On subdivision of edges s times, we see that the distance between the tree vertices gets multiplied by s+1. We note that the degree of u is same in G(T,s)and the neighbour vertices of u in T are at a distance s+1 in G(T,r) with the distance between u and s new vertices being 1,2,3,…s. So the number of vertices at each distance can be up to (s+1) eccT(u).For the new vertices introduced at every edge we write the eccentricity of vertices . In other words we write the eccentricity matrix. E(G(T,s)) will have a+s(a-1) vertices of which the first a entries will be vertices of T. So in E(G(T,s))each entry of the submatrix of order aXa is just the multiple of (s+1) of the entries of E(T) as they correspond to the tree vertices. We denote this block be P. The next block matrix of order s(a-1)Xs(a-1) has all new vertices introduced as subdivision vertices. We denote this by Q. If we consider the (i , j) th entry in E(T) , if it is one, then in E(G(T,s)) , the (i , j) th entry with the subdivision vertices will be (s+1) times. The eccentricity of any vertex of G(T,s) will be at most (s+1)eccT(x) for any vertex x in the tree T. Also the pendant vertices of T and G(T,s) are the same.

Hence the theorem

D Result

The eccentric sequence of a Mycielski graph M_k of order k with chromatic number k having the smallest possible number of vertices.
Solution
A Mycielski graph M_k of order k is a triangle free graph with chromatic number k with smallest possible number of vertices. For example triangle free graphs with chromatic number 4 include the Grotzsch graph(11 vertices), Chvatal graph(12 vertices), 13- cyclotomic graph(13 vertices), Clebsch graph(16 vertices), Brinkmann graph(21 vertices), Foster graph(30 vertices), Wong graph(30 vertices) and so on.
Grotzsch graph is the smallest of these graphs and therefore the Mycielski graph of order 4. We now compute the eccentric sequence of the Mycielski graph of order 4(ie the Grotzsch graph).
The Grotzsch graph is a symmetric graph. The eccentricity of every vertex is 2.

There are 11 vertices in G. So the eccentric sequence of the Grotzsch graph G is {2,2,2,2,2,2,2,2,2,2,2}

IV. CONCLUSION

In this paper we have computed the eccentric sequence of an integral graph, the eccentric sequence of the line graph of complete graph and the eccentric sequence of the Mycielski graph. We observe that the eccentricity of all the vertices of the above discussed graphs are the same. Then they are all self-centered graphs. So the eccentric sequence of self-centered class of graphs can be studied and discussed later. Also various properties can be found. The study of eccentric sequence of graphs has given rise to many open problems which can be discussed later.

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