Root Square Mean Labeling (RSML) of New Crown Graphs

R. Abdul Saleem, R. Mani

***Abstract:** A graph G with a links and b nodes is called RSMG if it is possible to label the links $x\epsilon V$ with distinct labels $f(x)$ from $1,2,\dots,q+1$ in such a way that when every node $e=uv$ is $f(e=uv)=\left\lceil\sqrt{\frac{f(u)^2+f(v)^2}{2}}\right\rceil$ or $\left\lfloor\sqrt{\frac{f(u)^2+f(v)^2}{2}}\right\rfloor$, then the resulting nodes labels are separate. In this case f is called a RSML of G. In this article we prove that some new crown graphs such as*

$C_n \cup (P_k \odot K_1)$, $C_n \cup (P_k \odot K_2)$,
$C_n \cup (P_k \odot K_3)$, $(C_n \odot K_1) \cup (P_k \odot K_1)$
, $(C_n \odot K_1) \cup (P_k \odot K_2)$, $(C_n \odot K_1) \cup (P_k \odot K_3)$ *are root square mean graphs.*

***Keywords:** Graph, RSML, Path, Cycle, crown.*

I. INTRODUCTION

In this article, the graphs which are simple, countable and undirected with a links and b nodes. For a labeling graph we refer to Gallain [2]. For all other standard terminology and notations, we follow Harary [3].

The idea of RSML has been introduced by Sandhya, Somasundaram and Anusa in 2014 [8]. Some new results proved of Root Square Mean Labeling of Some Crown Graphs by R. Abdul Saleem and R. Mani [1]. In this article, we examine the RSML of New Crown graphs. Some new examples are presented and verified. We now give the definitions which are necessary for the present investigation.

Definition 1.1:

A graph G with a links and b nodes is called RSMG if it is possible to label the links $x\epsilon V$ with distinct labels $f(x)$ from $1,2,\dots,q+1$ in such a way that when every node $e=uv$ is $f(e=uv)=\left\lceil\sqrt{\frac{f(u)^2+f(v)^2}{2}}\right\rceil$ or $\left\lfloor\sqrt{\frac{f(u)^2+f(v)^2}{2}}\right\rfloor$, then the resulting nodes labels are separate. In this case f is called a RSML of G.

Definition 1.2:

A walk in which $u_1u_2\dots u_n$ are distinct is called a path. A path on n vertices is denoted by P_n.

Definition 1.3:

A closed path is called a cycle. A cycle on n vertices is denoted by C_n.

Definition 1.4:

The two graphs $G_1=(V_1,E_1)$ and $G_2=(V_2,E_2)$ is a graph $G=G_1\cup G_2$ with link set $V=V_1\cup V_2$ and the node set $=E_1\cup E_2$.

Definition 1.5:

The Corona of graphs G_1 and G_2 is the graph $G=G_1\odot G_2$ formed by taking one reproduction of G_1 and $|V(G_1)|$ reproductions of G_2 where the i^{th} link of G_1 is adjacent to each link in the i^{th} reproduction of G_2.

Definition 1.6:

Let $G_1, G_2, \dots, G_n$, $n\geq 2$ be n replicas of a immovable graph G. The graph G found by adding a node between G_i and G_{i+1} for $i=1,2,\dots,n-1$ is called a path union of G.

II. MAIN RESULTS

Theorem 2.1

$C_n \cup (P_k \odot K_1)$ is a RSMG.

Proof:

Let $u_1u_2\dots u_n$ and $v_1v_2\dots v_n$ be the vertices of two cycles C_n in G.

Let $u_1=w_1w_2\dots w_k=v_1$ be the vertices of the path P_k.

Let $w_1', w_2', \dots, w_n'$ be the pendent vertices attached at $w_1w_2\dots w_k$ respectively.

Let $G=C_n\cup(P_k\odot K_1)$ .Define a function $f:V(G)\to\{1,2,\dots,3n+k\}$ by

$f(u_i)=i+1$ for $1\leq i\leq n-1$
$f(u_n)=1$
$f(v_i)=n+k+i+4$ for $1\leq i\leq n-1$
$f(v_n)=n+k+4$
$f(w_i)=n+2i-2$ for $2\leq i\leq k-1$
$f(w_i')=n+2i+1$ for $1\leq i\leq n-1$

Then clearly the nodes labels are separate.

Hence f is a RSML of G.

Example 2.1.1:

The RSML of $C_4 \cup (P_k \odot K_1)$ is given below:

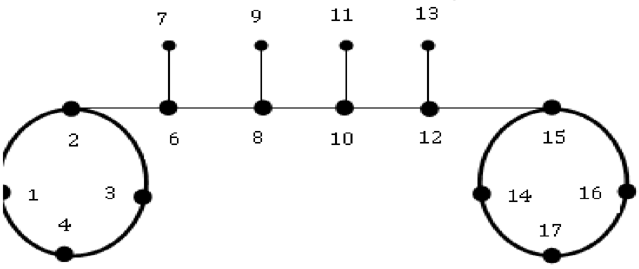

Figure 1

Theorem 2.2

$C_n \cup (P_k \odot K_2)$ is a RSMG.

Proof:

Let $u_1u_2\dots u_n$ and $v_1v_2\dots v_n$ be the links of two cycles C_n in G.

Let $u_1=w_1w_2\dots w_k=v_1$ be the vertices of the path P_k.

Let $w_1', w_2', \dots, w_n'$ and $w_1'', w_2'', \dots, w_n''$ be the pendent vertices attached at $w_1w_2\dots w_k$ respectively.

Let $G=C_n\cup(P_k\odot K_2)$. Define a function $f:V(G)\to\{1,2,\dots,4n+k\}$ by

$f(u_i)=i+1$ for $1\leq i\leq n-1$
$f(u_n)=1$

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R. Abdul Saleem, Assistant Professor, Department of Mathematics, The Quaide Milleth College for Men, Chennai – 600100, Tamil Nadu, India.
R. Mani, Research Scholar, Department of Mathematics, Government Arts College, Chidambaram– 608102, India

$f(v_i) = n + k + i + 8$ for $1 \le i \le n-1$
$f(v_n) = n + k + 8$
$f(w_i) = n + 3i - 4$ for $2 \le i \le k-1$
$f(w_i') = n + 3i$ for $1 \le i \le n-1$
$f(w_i'') = n + 3i + 1$ for $1 \le i \le n-1$

Then clearly the node labels are separate.

Hence f is a RSML of G.

Example 2.2.1:

The RSML of $C_4 \cup (P_k \odot K_2)$ is given below:

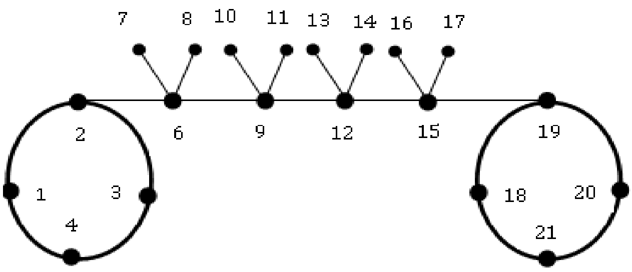

Figure 2

Theorem 2.3

$C_n \cup (P_k \odot K_3)$ is a RSMG.

Proof:

Let $u_1 u_2 \dots u_n$ and $v_1 v_2 \dots v_n$ be the vertices of two cycles C_n in G. Let $u_1 = w_1 w_2 \dots w_k = v_1$ be the vertices of the path P_k. Let $w_1', w_2', \dots, w_n'$, $w_1'', w_2'', \dots, w_n''$ and $w_1''', w_2''', \dots, w_n'''$ be the pendent vertices attached at $w_1 w_2 \dots w_k$ respectively.

Let $G = C_n \cup (P_k \odot K_3)$. be a function $f: V(G) \to \{1, 2, \dots, 5n + k\}$ by

$f(u_i) = i + 1$ for $1 \le i \le n-1$
$f(u_n) = 1$
$f(v_i) = n + k + i + 12$ for $1 \le i \le n-1$
$f(v_n) = n + k + 12$
$f(w_i) = n + 4i - 6$ for $2 \le i \le k-1$
$f(w_i') = n + 4i - 1$ for $1 \le i \le n-1$
$f(w_i'') = n + 4i$ for $1 \le i \le n-1$
$f(w_i''') = n + 4i + 1$ for $1 \le i \le n-1$

Then clearly the edge labels are distinct.

Hence f is a RSML of G.

Example 2.3.1:

The RSML of $C_4 \cup (P_k \odot K_3)$ is given below:

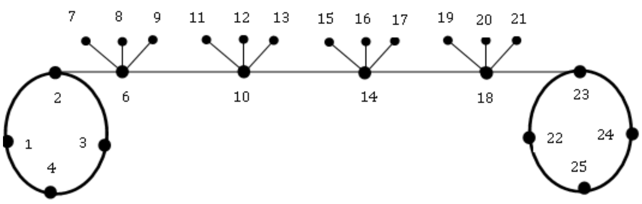

Figure 3

Theorem 2.4

$(C_n \odot K_1) \cup (P_k \odot K_1)$ is a RSMG.

Proof:

Let $u_1 u_2 \dots u_n$ and $v_1 v_2 \dots v_n$ be the vertices of two cycles C_n in G.

Let $u_1 = w_1 w_2 \dots w_k = v_1$ be the vertices of the path P_k.

Let $w_1', w_2', \dots, w_n'$ be the pendent vertices attached at $w_1 w_2 \dots w_k$ respectively.

Let $u_1', u_2', \dots, u_n'$ and $v_1', v_2', \dots, v_n'$ be the pendent vertices attached at $u_1, u_2, \dots, u_n$ and $v_1, v_2, \dots, v_n$ respectively.

Let $G = (C_n \odot K_1) \cup (P_k \odot K_1)$.

Define a function $f: V(G) \to \{1, 2, \dots, 5n + k\}$ by

$f(u_i) = 2i + 1$ for $1 \le i \le n-1$
$f(u_n) = 1$
$f(v_i) = 2n + k + 2i + 4$ for $1 \le i \le n-1$
$f(v_n) = 2n + k + 4$
$f(w_i) = 2n + 2i - 2$ for $2 \le i \le k-1$
$f(w_i') = 2n + 2i$ for $1 \le i \le n-1$
$f(u_i') = 2i + 2$ for $1 \le i \le n-1$
$f(u_n') = 2$
$f(v_i') = 2n + k + 2i + 5$ for $1 \le i \le n-1$
$f(v_n') = 2n + k + 5$

Then clearly the edge labels are distinct.

Hence f is a RSML of G.

Example 2.4.1:

The RSML of $(C_4 \odot K_1) \cup (P_k \odot K_1)$ is given below:

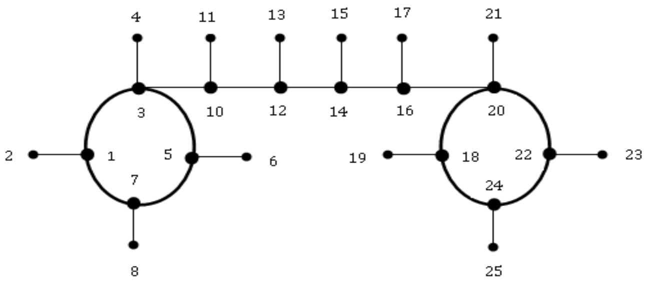

Figure 4

Theorem 2.5

$(C_n \odot K_1) \cup (P_k \odot K_2)$ is a RSMG.

Proof:

Let $u_1 u_2 \dots u_n$ and $v_1 v_2 \dots v_n$ be the vertices of two cycles C_n in G.

Let $u_1 = w_1 w_2 \dots w_k = v_1$ be the vertices of the path P_k.

Let $w_1', w_2', \dots, w_n'$ and $w_1', w_2', \dots, w_n'$ be the pendent vertices attached at $w_1 w_2 \dots w_k$ respectively.

Let $u_1', u_2', \dots, u_n'$ and $v_1', v_2', \dots, v_n'$ be the pendent vertices attached at $u_1, u_2, \dots, u_n$ and $v_1, v_2, \dots, v_n$ respectively.

Let $G = (C_n \odot K_1) \cup (P_k \odot K_2)$.

Define a function $f: V(G) \to \{1, 2, \dots, 6n + k\}$ by

$f(u_i) = 2i + 1$ for $1 \le i \le n-1$
$f(u_n) = 1$
$f(v_i) = 2n + k + 2i + 8$ for $1 \le i \le n-1$
$f(v_n) = 2n + k + 8$
$f(w_i) = 2n + 3i - 4$ for $2 \le i \le k-1$
$f(w_i') = 2n + 3i$ for $1 \le i \le n-1$
$f(w_i'') = 2n + 3i + 1$ for $1 \le i \le n-1$
$f(u_i') = 2i + 2$ for $1 \le i \le n-1$
$f(u_n') = 2$
$f(v_i') = 2n + k + 2i + 9$ for $1 \le i \le n-1$
$f(v_n') = 2n + k + 9$

Then clearly the edge labels are distinct.

Hence f is a Root Square mean labeling of G.

Example 2.5.1:

The RSML of $(C_4 \odot K_1) \cup (P_k \odot K_2)$ is given below:

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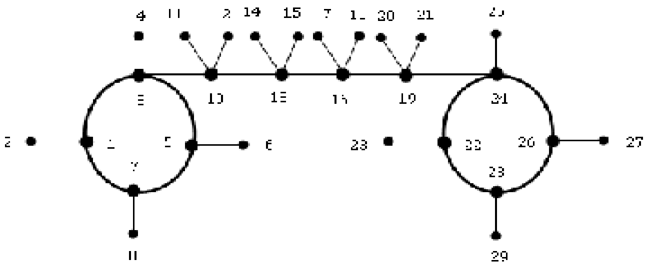

Figure 5

Theorem 2.6

$(C_n \odot K_1) \cup (P_k \odot K_3)$ is a RSMG.

Proof:

Let $u_1 u_2 \dots u_n$ and $v_1 v_2 \dots v_n$ be the vertices of two cycles C_n in G.

Let $u_1 = w_1 w_2 \dots w_k = v_1$ be the vertices of the path P_k.

Let $w_1', w_2', \dots, w_n'$, $w_1'', w_2'', \dots, w_n''$ and $w_1''', w_2''', \dots, w_n'''$ be the pendent vertices attached at $w_1 w_2 \dots w_k$ respectively.

Let $u_1', u_2', \dots, u_n'$ and $v_1', v_2', \dots, v_n'$ be the pendent vertices attached at $u_1, u_2, \dots, u_n$ and $v_1, v_2, \dots, v_n$ respectively.

Let $G = (C_n \odot K_1) \cup (P_k \odot K_3)$.

Define a function $f: V(G) \to \{1, 2, \dots, 9n + k\}$ by

$f(u_i) = 2i + 1$ for $1 \le i \le n - 1$

$f(u_n) = 1$

$f(v_i) = 2n + k + 2i + 12$ for $1 \le i \le n - 1$

$f(v_n) = 2n + k + 12$

$f(w_i) = 2n + 4i - 6$ for $2 \le i \le k - 1$

$f(w_i') = 2n + 4i - 1$ for $1 \le i \le n - 1$

$f(w_i'') = 2n + 4i$ for $1 \le i \le n - 1$

$f(w_i''') = 2n + 4i + 1$ for $1 \le i \le n - 1$

$f(u_i') = 2i + 2$ for $1 \le i \le n - 1$

$f(u_n') = 2$

$f(v_i') = 2n + k + 2i + 13$ for $1 \le i \le n - 1$

$f(v_n') = 2n + k + 13$

Then clearly the edge labels are separate.

Hence f is a RSML of G.

Example 2.6.1:

The RSML of $(C_4 \odot K_1) \cup (P_k \odot K_3)$ is given below:

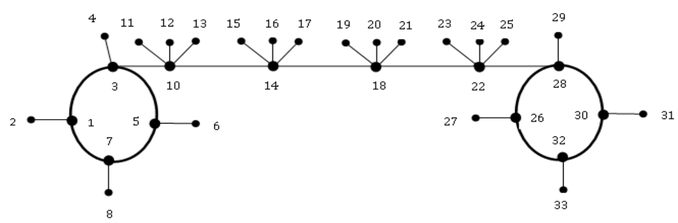

Figure 6

III. CONCLUSION

As all graphs are not RSMG's, it is very motivating to examine graphs which admits RSML. In this paper we prove that Path, Cycle, Crown are RSMG. Then, we present six new results on RSML of graphs. It is possible to investigate comparable results for numerous other graphs.

REFERENCES

1. Abdul Saleem. R and Mani. R, "Root Square Mean Labeling of Some Crown Graphs., *The IJAEMA*, V-XI, Issue X, Oct 2019, PP. 70-78.
2. Gallian. J.A, 2010, " A dynamic Survey of graph Labeling". *The EJC* 17#DS6.
3. Harary. F, 1988, " Graph Theory", *Nph Reading*, New Delhi.
4. Meena. S and Mani. R, "RSML of Some Cycle Related Graphs., *IJSART*, Vol. 5, Issue 7, Jul 2019, PP. 786-789.
5. Sandhya. S.S, Somasundaram. S, Anusa. S, "Some Results on RSMG's" "*JSR*. Vol. 5, Issue 7, Jul 2019, PP. 786-789.
6. Sandhya. S.S, Somasundaram. S, Anusa. S, "Some More Results on RSMG,s" Communicated to "*JSR*". Vol.9, 2014.
7. Sandhya. S.S, Somasundaram. S, Anusa. S, "Further Results on RSML" , "*Bulletin of PAMS"*. Vol.9, 2014, no.14, 667-676.
8. Sandhya. S.S, Somasundaram. S, Anusa. S, "RSMLG's" *IJCMS*, Vol.9, 2014.