Restrained Step Domination Number for Some Amusing Product Graph of Paths and Cycle

G.Mahadevan, M.Vimala Suganthi

Abstract: *G. Mahadevan, et, al., introduced the concept of restrained step domination number of a graph. A set $S \subseteq V$ of a graph G is said to be restrained step dominating set, if $< S >$ is the restrained dominating set and $< V - S >$ is a perfect matching. The minimum cardinality taken over all the restrained step dominating set is called the restrained step domination number of G and is denoted by γ_{rsd} (G). In this paper we explore this parameter for some product graph of path and cycle.*

***Keywords** : complementary perfect domination, Restrained domination, restrained step domination.*

I. INTRODUCTION

Paulraj Joseph et.al., [4], in the year 2006 introduced the concept of complementary perfect domination. A set is called a complementary perfect dominating set if S is a dominating set of G and the induced subgraph $< V - S >$ has a perfect matching. The minimum cardinality taken over all complementary perfect dominating sets is called the complementary perfect domination number and is denoted by γ_{cp} (G). Further results of complementary perfect domination number is discussed in [7,12]. The concept of restrained domination number was introduced by Gayla.S et.al., in the year 1999 [1]. A dominating set is said to be restrained dominating set if every vertex in $< V - S >$ is adjacent to atleast one vertex in S as well as in $V - S$. The minimum cardinality taken over all restrained dominating sets in G is restrained dominating number and denoted by γ_r(G). Further results of restrained dominating number is been discussed in [8,11]. Inspired by the above, imposing a condition on the complement of restrained dominating set, G. Mahadevan, et.al., [5] introduced the concept of restrained step domination number of a graph in the year 2018. A set $S \subseteq V$ of a graph G is said to be restrained step dominating set, if $< S >$ is the restrained dominating set and $< V - S >$ is a perfect matching. The minimum cardinality taken over all the restrained step dominating set is called the restrained step dominating number of G and is denoted by γ_{rsd} (G).

The corona $G_1 \odot G_2$ is defined as the graph G obtained by taking one copy of G_1 of order p_1 and p_1 copies of G_2 and then joining the i^{th} vertex of G_1 to every vertex in the i^{th} copy of G_2. In this graph vertices are denoted as v_i and w_j^i where v_i are the vertices of the graph G_1, w_j are the vertices of the graph G_2 and w_j^i denotes the vertices of the copies of the graph G_2 attached to the vertex of G_1. For any two simple

graphs G and H, the tensor product of G and H has vertex set $V(G \otimes H)=V(G) \times V(H)$, edge set $E(G \otimes H)=\{(a,b)(c,d)/ac \in E(G) \text{ and } bd \in E(H)\}$.

Preliminary result: We use the following preliminary result in our subsequent discussions.

Theorem 1.1 [5] For a connected graph C_p, $p \geq 3$,

$$\gamma_{rsd}(C_p) = \begin{cases} \frac{p}{3} & \text{if } p \equiv 0 \pmod 3 \\ \frac{p+2}{3} & \text{if } p \equiv 1 \pmod 3 \\ \frac{p+4}{3} & \text{if } p \equiv 2 \pmod 3. \end{cases}$$

Theorem 1.2 [13]If $n \leq p$ and $p \equiv 0,1 \pmod 3$, then

$$\gamma_{rstc}(C_p \otimes C_n) = \begin{cases} 2\left\lceil \frac{n}{3} \right\rceil \left\lceil \frac{p}{3} \right\rceil + \left\lceil \frac{n}{3} \right\rceil p & if\ n \equiv 0\ (mod\ 3) \\ 2\left\lfloor \frac{n}{3} \right\rfloor \left\lceil \frac{p}{3} \right\rceil + \left\lceil \frac{n}{3} \right\rceil p & if\ n \equiv 1\ (mod\ 3) \\ 2\left\lfloor \frac{n}{3} \right\rfloor \left\lceil \frac{p}{3} \right\rceil + (\left\lceil \frac{n}{3} \right\rceil + 1)p & if\ n \equiv 2\ (mod\ 3) \end{cases}$$

Theorem 1.3 [13] If $n \leq p$ and $p \equiv 2 \pmod 3$, then

$$\gamma_{rstc}(C_p \otimes C_n) = \begin{cases} \left\lceil \frac{n}{3} \right\rceil (2\left\lceil \frac{p}{3} \right\rceil + p) + 2 & if\ n \equiv 0\ (mod\ 3) \\ \left\lfloor \frac{n}{3} \right\rfloor (2\left\lceil \frac{p}{3} \right\rceil) + 2 + \left\lceil \frac{n}{3} \right\rceil p & if\ n \equiv 1\ (mod\ 3) \\ \left\lfloor \frac{n}{3} \right\rfloor \left(2\left\lceil \frac{p}{3} \right\rceil\right) + 2 + (\left\lceil \frac{n}{3} \right\rceil + 1)p & if\ n \equiv 2\ (mod\ 3) \end{cases}$$

II. RESTRAINED STEP DOMINATION NUMBER OF CORONA PRODUCT OF GRAPHS

Theorem 2.1 For a corona product $C_p \odot P_s$ where C_p is the cycle with p vertices and P_s is the path with s vertices,

$$\text{then } \gamma_{rsd}(C_p \odot P_s) = \begin{cases} \frac{p(s+3)}{3} & \text{if } s \equiv 0 \pmod 3 \\ \frac{p(s+5)}{3} & \text{if } s \equiv 1 \pmod 3 \\ \frac{p(s+1)}{3} & \text{if } s \equiv 2 \pmod 3. \end{cases}$$

Proof Let $C_p \odot P_s$ be the corona product graph. Let the vertices in the cycle C_p be $\{v_1, v_2, \ldots, v_p\}$ and the vertices in the path P_s be $\{w_1, w_2, \ldots, w_s\}$ as $C_p \odot P_s$ is the corona product the vertices in the graph are

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$\{v_1,v_2,\ldots,v_p,w_1^1,w_2^1,\ldots,w_s^1,\ldots,w_1^p,w_2^p,\ldots,w_s^p\}$. Every vertices of the path $w_1^i,w_2^i,\ldots,w_s^i$ is connected to the vertex v_i, where $1 \le i \le p$. In the graph v_i, $w_1^i,w_2^i,\ldots,w_s^i$, where $1 \le i \le p$ forms the cycle with s+1 vetices. Hence $\gamma_{rsd}(C_p \odot P_s) = p(\gamma_{rsd}(C_{s+1}))$, by theorem 1.1 implies

$$\gamma_{rsd}(C_p \odot P_s) = \begin{cases} \frac{p(s+3)}{3} & \text{if } s \equiv 0 \pmod 3 \\ \frac{p(s+5)}{3} & \text{if } s \equiv 1 \pmod 3 \\ \frac{p(s+1)}{3} & \text{if } s \equiv 2 \pmod 3. \end{cases}$$

Example Consider the graphs $C_3 \odot P_3$, $C_3 \odot P_4$, $C_3 \odot P_5$

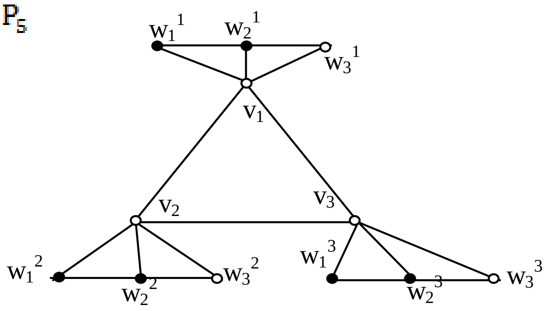

Figure 2.1

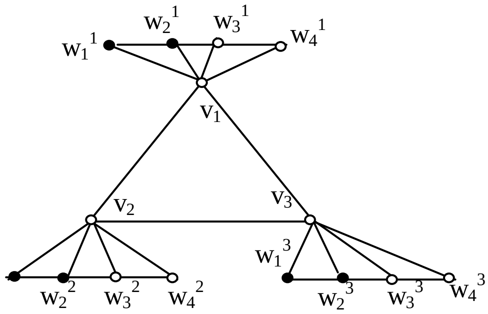

Figure 2.2

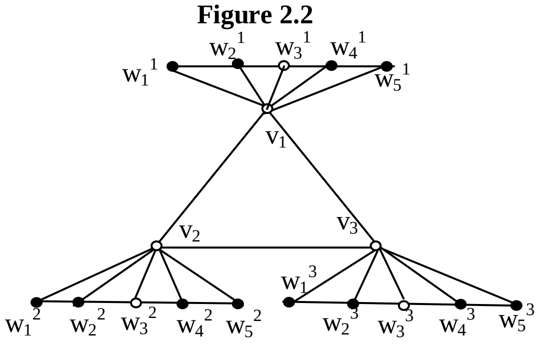

Figure 2.3

Illustration In figure 2.1, $S=\{v_1,v_2,v_3,w_3^1,w_3^2,w_3^3\}$ is the rsd-set, |S|= 6. $\gamma_{rsd}(C_p \odot P_s) = \frac{p(s+3)}{3}$, if $s \equiv 0 \pmod 3$. Hence $\gamma_{rsd}(C_3 \odot P_3) = \frac{3(3+3)}{3} = 6$. In figure 2.2, $S=\{v_1,v_2,v_3,w_3^1,w_4^1, w_3^2, w_4^2,w_3^3, w_4^3\}$ is the rsd-set, |S|= 9. $\gamma_{rsd}(C_p \odot P_s) = \frac{p(s+5)}{3}$, if $s \equiv 1 \pmod 3$. Hence $\gamma_{rsd}(C_3 \odot P_4) = \frac{3(4+5)}{3} = 9$. In figure 2.3, $S=\{v_1,v_2,v_3,w_3^1,w_3^2,w_3^3\}$ is the rsd-set, |S|= 6. $\gamma_{rsd}(C_p \odot P_s) = \frac{p(s+1)}{3}$, if $s \equiv 0 \pmod 3$. Hence $\gamma_{rsd}(C_3 \odot P_5) = \frac{3(5+1)}{3} = 6$.

Theorem 2.2 For a corona product $P_p \odot P_s$ where C_p and P_s is the paths with p and s vertices, then

$$\gamma_{rsd}(P_p \odot P_s) = \begin{cases} \frac{p(s+3)}{3} & \text{if } s \equiv 0 \pmod 3 \\ \frac{p(s+5)}{3} & \text{if } s \equiv 1 \pmod 3 \\ \frac{p(s+1)}{3} & \text{if } s \equiv 2 \pmod 3. \end{cases}$$

Proof As $\gamma_{rsd}(P_p \odot P_s) = \gamma_{rsd}(C_p \odot P_s)$ the proof is same as the theorem 2.1

Theroem 2.3 For a corona product $P_p \odot C_s$ where P_p is the path with p vertices and C_s is the cycle with s vertices, then

$$\gamma_{rsd}(P_p \odot C_s) = \begin{cases} \frac{p(s+3)}{3} & \text{if } s \equiv 0 \pmod 3 \\ \frac{p(s+5)}{3} & \text{if } s \equiv 1 \pmod 3 \\ \frac{p(s+7)}{3} & \text{if } s \equiv 2 \pmod 3. \end{cases}$$

Proof Let $P_p \odot C_s$ be the corona product graph. Let the vertices in the path P_p be $\{w_1,w_2,\ldots,w_p\}$ and let the vertices in the cycle C_s be $\{v_1,v_2,\ldots,v_s\}$. Let the vertices in the product graph be $\{w_1,w_2,\ldots,w_p, v_1^1,v_2^1,\ldots v_s^1,\ldots,v_1^p,v_2^p,\ldots,v_s^p\}$. Clearly, each vertices w_i dominates the each cycles $v_1^i,v_2^i,\ldots v_s^i$ where, $1 \le i \le p$. Hence $\{w_1,w_2,\ldots,w_p\}$ is a rsd-set whose cardinality is p. Also rsd-number for p-copies of the cycle $v_1^i,v_2^i,\ldots v_s^i$ where, $1 \le i \le p$ is $p.\gamma_{rsd}(C_s)$. Hence $\gamma_{rsd}(P_p \odot C_s) = p + p.\gamma_{rsd}(C_s)$,by theorem 1.1.Therefore,

$$\gamma_{rsd}(P_p \odot C_s) = \begin{cases} \frac{p(s+3)}{3} & \text{if } s \equiv 0 \pmod 3 \\ \frac{p(s+5)}{3} & \text{if } s \equiv 1 \pmod 3 \\ \frac{p(s+7)}{3} & \text{if } s \equiv 2 \pmod 3. \end{cases}$$

Example

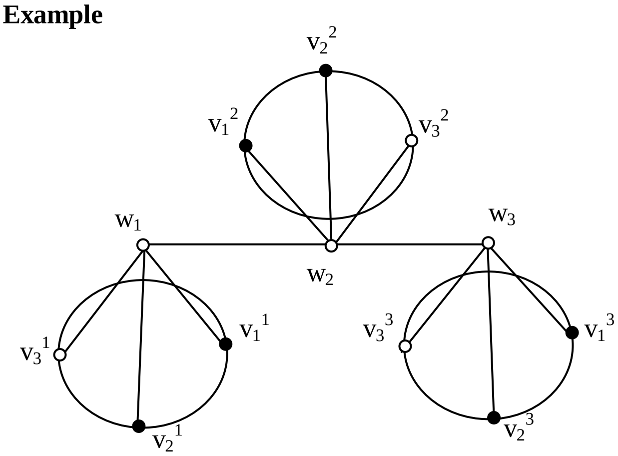

Figure 2.4

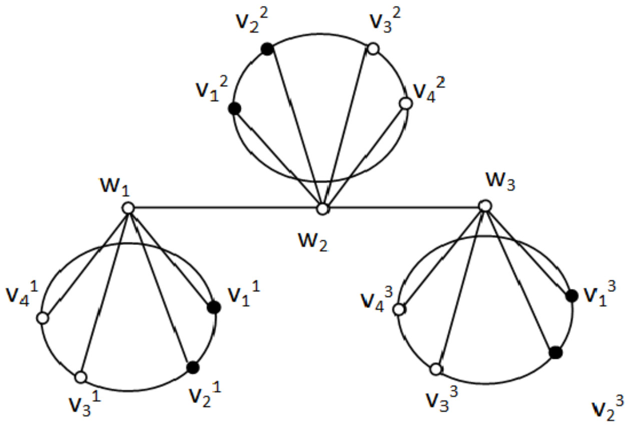

Figure 2.5

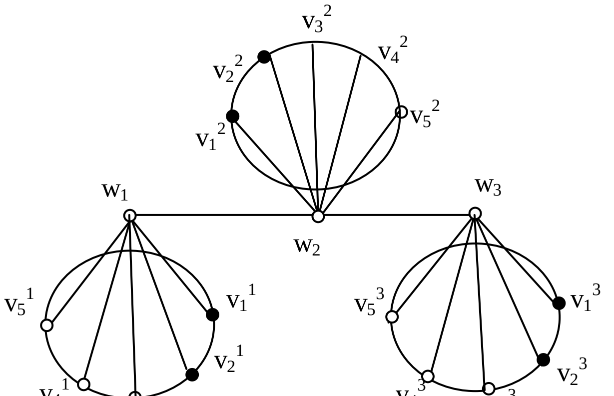

Figure 2.6

Illustration In figure 2.4, S={w_1,w_2,w_3,v_3^1,v_3^2,v_3^3} is the rsd-set, |S|= 6. $\gamma_{rsd}(P_p \odot C_s) = \frac{p(s+3)}{3}$, if $s \equiv 0 \pmod 3$. Hence $\gamma_{rsd}(P_3 \odot C_3) = \frac{3(3+3)}{3}$ =6. In figure 2.5, S={w_1,w_2,w_3,v_3^1,v_4^1, v_3^2, v_4^2,v_3^3, v_4^3} is the rsd-set, |S|= 9. $\gamma_{rsd}(P_p \odot C_s) = \frac{p(s+5)}{3}$, if $s \equiv 1 \pmod 3$. Hence $\gamma_{rsd}(P_3 \odot C_4) = \frac{3(4+5)}{3}$ =9. In figure 2.6, S={ w_1,w_2,w_3,v_3^1,v_4^1, v_5^1, v_3^2, v_4^2, v_5^2 ,v_3^3, v_4^3 , v_5^3} is the rsd-set, |S|= 12 $\gamma_{rsd}(P_p \odot C_s) = \frac{p(s+7)}{3}$, if $s \equiv 2 \pmod 3$. Hence $\gamma_{rsd}(P_3 \odot C_5) = \frac{3(5+7)}{3}$=12.

Theorem 2.4 For a corona product $C_p \odot C_s$ where C_p and C_s are the cycles with r and s vertices respectively, then

$$\gamma_{rsd}(C_p \odot C_s) = \begin{cases} \frac{p(s+3)}{3} & \text{if } s \equiv 0 \pmod 3 \\ \frac{p(s+5)}{3} & \text{if } s \equiv 1 \pmod 3 \\ \frac{p(s+7)}{3} & \text{if } s \equiv 2 \pmod 3. \end{cases}$$

Proof As $\gamma_{rsd}(C_p \odot C_s) = \gamma_{rsd}(P_p \odot C_s)$ the proof is same as the theorem 2.3.

III. RESTRAINED STEP DOMINATION NUMBER FOR TENSOR PRODUCT OF GRAPHS

Theorem 3.1 If n≤p and p ≡ 0,1,2(mod 3) and p is odd, then

γ_{rsd} ($P_n \otimes P_p$)=

$$\begin{cases} 2\left\lceil\frac{n}{3}\right\rceil\left\lceil\frac{p}{3}\right\rceil + \left\lceil\frac{p}{3}\right\rceil n & if\ n \equiv 0\ (mod\ 3) \\ 2\left\lceil\frac{n}{3}\right\rceil\left\lceil\frac{p}{3}\right\rceil + \left\lceil\frac{p}{3}\right\rceil n & if\ n \equiv 1\ (mod\ 3) \\ 2(\left\lceil\frac{n}{3}\right\rceil - 1)\left\lceil\frac{p}{3}\right\rceil + \left\lceil\frac{p}{3}\right\rceil n & if\ n \equiv 2\ (mod\ 3). \end{cases}$$

Proof: Let $P_n \otimes P_p$ be the tensor product graphs of two paths. Case 1: n is odd. Let S_1= { v_{1j}: $j \equiv 0,1\ mod\ 6\}$, S_2={v_{2j}: $j \equiv 2,4\ mod\ 6\}$, S_3={v_{ij}: $3 \le j \le p$, $1 \le i \le n-2$ i $\equiv 2,4\ mod\ 6\ and\ j \equiv 1\ mod\ 3\}$, S_4={v_{ij}: $3 \le j \le p$, $1 \le i \le n-2$ i $\equiv 0,1\ mod\ 6\ and\ j \equiv 2\ mod\ 3\}$, S_5={v_{ij}: $3 \le j \le p$, $1 \le i \le n-2\ and\ j \equiv 0\ mod\ 3\}$,S_6={$v_{n-1)j}$: $4 \le j \le p$ and $j \equiv 0\ mod\ 3\} \cup \{v_{\{n-1\}1}\}$ and S_7={v_{ip}: $1 \le i \le n$ }. If $n \equiv 0\ (mod\ 3)$, S= $S_1 \cup S_2 \cup S_3 \cup S_4 \cup S_5 \cup S_6$ is the restrained step dominating set whose cardinality is $2\left\lceil\frac{n}{3}\right\rceil\left\lceil\frac{p}{3}\right\rceil + \left\lceil\frac{p}{3}\right\rceil n$. If $n \equiv 1\ (mod\ 3)$, S= $S_1 \cup S_2 \cup S_3 \cup S_4 \cup S_5 \cup S_6$ is the restrained step dominating set whose cardinality is $2\left\lfloor\frac{n}{3}\right\rfloor\left\lceil\frac{p}{3}\right\rceil + \left\lceil\frac{p}{3}\right\rceil n$. If $n \equiv 2\ (mod\ 3)$, S= $S_1 \cup S_2 \cup S_3 \cup S_4 \cup S_5 \cup S_6 \cup S_7$ is the restrained step dominating set whose cardinality is $2(\left\lceil\frac{n}{3}\right\rceil - 1)\left\lceil\frac{p}{3}\right\rceil + \left\lceil\frac{p}{3}\right\rceil n$.

Thus, γ_{rsd} ($C_n \otimes C_p$)≤|S|.

Case 2: n is even . Let S_1= { v_{1j}: $j \equiv 0,1\ mod\ 6\}$, S_2={v_{2j}: $j \equiv 2,4\ mod\ 6\}$, S_3={v_{ij}: $3 \le j \le p$, $1 \le i \le n-2$ i $\equiv 2,4\ mod\ 6\ and\ j \equiv 1\ mod\ 3\}$, S_4={v_{ij}: $3 \le j \le p$, $1 \le i \le n$ i $\equiv 0,1\ mod\ 6\ and\ j \equiv 2\ mod\ 3\}$, S_5={v_{ij}: $3 \le j \le p$, $1 \le i \le n\ and\ j \equiv 0\ mod\ 3\}$ and S_6={v_{ip}: $1 \le i \le n$ }. If $n \equiv 0\ (mod\ 3)$, S= $S_1 \cup S_2 \cup S_3 \cup S_4 \cup S_5$ is the restrained step dominating set whose cardinality is $2\left\lceil\frac{n}{3}\right\rceil\left\lceil\frac{p}{3}\right\rceil + \left\lceil\frac{p}{3}\right\rceil n$. If $n \equiv 1\ (mod\ 3)$, S= $S_1 \cup S_2 \cup S_3 \cup S_4 \cup S_5$ is the restrained step dominating set whose cardinality is $2\left\lfloor\frac{n}{3}\right\rfloor\left\lceil\frac{p}{3}\right\rceil + \left\lceil\frac{p}{3}\right\rceil n$. If $n \equiv 2\ (mod\ 3)$, S= $S_1 \cup S_2 \cup S_3 \cup S_4 \cup S_5 \cup S_6$ is the restrained step dominating set whose cardinality is $2(\left\lceil\frac{n}{3}\right\rceil - 1)\left\lceil\frac{p}{3}\right\rceil + \left\lceil\frac{p}{3}\right\rceil n$.

Thus, γ_{rsd} ($C_n \otimes C_p$)≤|S|.

In all the above cases γ_{rsd} ≤|S|, if there exists a restrained step dominating set T ⊆S, then the set <V - T> has atleast one non-independent K_2, which contradicts the definition implies γ_{rsd} ≥|S|. Hence γ_{rsd} ($C_n \otimes C_p$)=|S|.

Theorem 3.2 If $n \leq p$ and $p \equiv 0,1,2 \pmod 3$ and p is even, then

$\gamma_{rsd}(P_n \otimes P_p) =$

$$\begin{cases} 2\left\lceil\frac{n}{3}\right\rceil\left\lceil\frac{p}{3}\right\rceil+\left\lceil\frac{p}{3}\right\rceil n & if\ n\equiv 0\ (mod\ 3) \\ 2\left\lceil\frac{n}{3}\right\rceil\left\lceil\frac{p}{3}\right\rceil+\left\lceil\frac{p}{3}\right\rceil n & if\ n\equiv 1\ (mod\ 3) \\ 2(\left\lceil\frac{n}{3}\right\rceil-1)\left\lceil\frac{p}{3}\right\rceil+\left\lceil\frac{p}{3}\right\rceil n & if\ n\equiv 2\ (mod\ 3). \end{cases}$$

Proof: Restrained step dominating sets are taken as in theorem 3.1, but differs only in cardinality i.e, Case 1: n is odd. If $n\equiv 0\ (mod\ 3)$, S= $S_1 \cup S_2 \cup S_3 \cup S_4 \cup S_5 \cup S_6$ is the restrained step dominating set whose cardinality is $2\left\lceil\frac{n}{3}\right\rceil\left\lceil\frac{p}{3}\right\rceil+\left\lceil\frac{p}{3}\right\rceil n.$ If $n\equiv 1\ (mod\ 3)$, S= $S_1 \cup S_2 \cup S_3 \cup S_4 \cup S_5 \cup S_6$ is the restrained step dominating set whose cardinality is $2\left\lceil\frac{n}{3}\right\rceil\left\lceil\frac{p}{3}\right\rceil+\left\lceil\frac{p}{3}\right\rceil n.$ If $n\equiv 2\ (mod\ 3)$, S= $S_1 \cup S_2 \cup S_3 \cup S_4 \cup S_5 \cup S_6 \cup S_7$ is the restrained step dominating set whose cardinality is $2(\left\lceil\frac{n}{3}\right\rceil-1)\left\lceil\frac{p}{3}\right\rceil+\left\lceil\frac{p}{3}\right\rceil n.$

Thus, $\gamma_{rsd}(C_n \otimes C_p) \leq |S|$.

Case 2: n is even . If $n\equiv 0\ (mod\ 3)$, S= $S_1 \cup S_2 \cup S_3 \cup S_4 \cup S_5$ is the restrained step dominating set whose cardinality is $2\left\lceil\frac{n}{3}\right\rceil\left\lceil\frac{p}{3}\right\rceil+\left\lceil\frac{p}{3}\right\rceil n.$ If $n\equiv 1\ (mod\ 3)$, S= $S_1 \cup S_2 \cup S_3 \cup S_4 \cup S_5$ is the restrained step dominating set whose cardinality is $2\left\lceil\frac{n}{3}\right\rceil\left\lceil\frac{p}{3}\right\rceil+\left\lceil\frac{p}{3}\right\rceil n.$ If $n\equiv 2\ (mod\ 3)$, S= $S_1 \cup S_2 \cup S_3 \cup S_4 \cup S_5 \cup S_6$ is the restrained step dominating set whose cardinality is $2(\left\lceil\frac{n}{3}\right\rceil-1)\left\lceil\frac{p}{3}\right\rceil+\left\lceil\frac{p}{3}\right\rceil n.$

Thus, $\gamma_{rsd}(C_n \otimes C_p) \leq |S|$.

In all the above cases $\gamma_{rsd} \leq |S|$, if there exists a restrained step dominating set T $\subseteq$S, then the set <V - T> has atleast one non-independent K_2, which contradicts the definition implies $\gamma_{rsd} \geq |S|$. Hence $\gamma_{rsd}(C_n \otimes C_p) = |S|$.

Example 3.1 consider the graph $P_6 \otimes P_6$

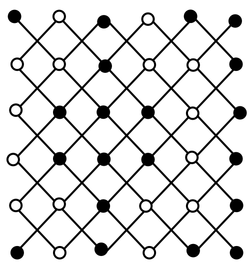

Figure 3.1

Here the darkened vertices are the restrained step dominating set . Whose cardinality is 20.

$\gamma_{rsd}(P_p \otimes P_n) = 2\left\lceil\frac{n}{3}\right\rceil\left\lceil\frac{p}{3}\right\rceil+\left\lceil\frac{n}{3}\right\rceil p$

Implies $\gamma_{rsd}(P_6 \otimes P_6) = 2\left\lceil\frac{6}{3}\right\rceil\left\lceil\frac{6}{3}\right\rceil+\left\lceil\frac{6}{3}\right\rceil 6 = 20$

Theorem 3.3 If $n \leq p$ and $p \equiv 0,1 \pmod 3$, then

$\gamma_{rsd}(C_p \otimes C_n) =$

$$\begin{cases} 2\left\lceil\frac{n}{3}\right\rceil\left\lceil\frac{p}{3}\right\rceil+\left\lceil\frac{n}{3}\right\rceil p & if\ n\equiv 0\ (mod\ 3) \\ 2\left\lceil\frac{n}{3}\right\rceil\left\lceil\frac{p}{3}\right\rceil+\left\lceil\frac{n}{3}\right\rceil p & if\ n\equiv 1\ (mod\ 3) \\ 2\left\lceil\frac{n}{3}\right\rceil\left\lceil\frac{p}{3}\right\rceil+(\left\lceil\frac{n}{3}\right\rceil+1)p & if\ n\equiv 2\ (mod\ 3) \end{cases}$$

Proof is same as theorem 1.2.

Theorem 3.4 If $n \leq p$ and $p \equiv 2 \pmod 3$, then

$\gamma_{rsd}(C_p \otimes C_n)$
=

$$\begin{cases} \left\lceil\frac{n}{3}\right\rceil(2\left\lceil\frac{p}{3}\right\rceil+p)+2 & if\ n\equiv 0\ (mod\ 3) \\ \left\lfloor\frac{n}{3}\right\rfloor(2\left\lceil\frac{p}{3}\right\rceil)+2+\left\lceil\frac{n}{3}\right\rceil p & if\ n\equiv 1\ (mod\ 3) \\ \left\lfloor\frac{n}{3}\right\rfloor\left(2\left\lceil\frac{p}{3}\right\rceil\right)+2+(\left\lceil\frac{n}{3}\right\rceil+1)p & if\ n\equiv 2\ (mod\ 3) \end{cases}$$

Proof is same as theorem 1.2.

IV. CONCLUSION

A conclusion section is not required. Although a conclusion may review the main points of the paper, do not replicate the abstract as the conclusion. A conclusion might elaborate on the importance of the work or suggest applications and extensions.

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