

Minimum Total dominating Energy of Some Special Classes of Graphs

K. Malathy, S. Meenakshi

Abstract: Let $G = (V, E)$ be a simple, finite, connected and undirected graph with vertex set $V(G)$ and edge set $E(G)$. Let $S \subseteq V(G)$. A set S of vertices of G is a dominating set if every vertex in $V(G) - S$ is adjacent to at least one vertex in S . A set S of vertices in a graph $G(V, E)$ is called a total dominating set if every vertex $v \in V$ is adjacent to an element of S . The minimum cardinality of a total dominating set of G is called the total domination number of G which is denoted by $\gamma_t(G)$. The energy of the graph is defined as the sum of the absolute values of the eigen values of the adjacency matrix. In this paper, we computed minimum total dominating energy of some special graphs such as Paley graph, Shrikhande graph, Clebsch graph, Chvatal graph, Moser graph and Octahedron graph.

Keywords: dominating set, minimum total dominating set, minimum total dominating matrix, minimum total dominating eigen values, minimum total dominating energy of a graph.

I. INTRODUCTION

In 1960, the mathematical study of Domination Theory in graphs was started. I. Gutman introduced the concept of energy of a graph in the year 1978. The roots go back to 1862 when C.F. De Jaenisch studied the problem of determining the minimum number of queens necessary to cover an $n \times n$ chess board in such way that every square is attacked by one of the queens.

The graph invariant is closely connected to a chemical quantity known as the total π electron energy of conjugated hydrocarbon molecules. The study of Graph Energy first arouses in the field of chemistry. Chemists used Huckel's method to approximate energies associated with π - electron orbitals in a special class of molecules called conjugated hydrocarbons. Gutman first introduced the concept of 'energy of graph' for a simple graph. At first very few mathematicians seemed to be interested in this concept. However, over the years graph energy has become an interesting area of research for mathematicians and several variations have been introduced.

Let G be a graph with x -vertices and y -edges. Let $A = (a_{ij})$ be the adjacency matrix of a graph. Let $\lambda_1, \lambda_2, \dots, \lambda_n$ be the eigenvalues of adjacency matrix of a graph G . The values are in non-decreasing order, that is $\lambda_1 \geq \lambda_2 \geq \dots \geq \lambda_n$. Since $A(G)$ is real and symmetric, its eigenvalues are real number. The energy $E(G)$ of graph is defined as sum of the absolute values of its eigenvalues of graph G , ie, $E(G) = \sum_{i=1}^n |\lambda_i|$

In this paper, we computed minimum total dominating energy of some special graphs such as Paley graph, Shrikhande graph, Clebsch graph, Chvatal graph, Moser graph and Octahedron graph.

II. PRELIMINARIES

Definition 2.1 Dominating set

Let G be a simple graph with vertex set $V(G)$. Let $S \subseteq V(G)$. A set S of vertices of G is a dominating set if every vertex in $V(G) - S$ is adjacent to at least one vertex in S . The minimum cardinality of a dominating set of G is called the domination number of G which is denoted by $\gamma(G)$. [2][5]

Definition 2.2 Total Dominating set

A set S of vertices in a graph $G(V, E)$ is called a total dominating set if every vertex $v \in V$ is adjacent to an element of S . The minimum cardinality of a total dominating set of G is called the total domination number of G which is denoted by $\gamma_t(G)$. [1]

Definition 2.3 Energy

Energy of a simple graph $G = (V, E)$ with adjacency matrix A is defined as the sum of absolute values of eigen values of A denoted by $E(G)$. ie, $E(G) = \sum_{i=1}^n |\lambda_i|$ where λ_i is an eigen values of A , $i = 1, 2, \dots, n$. [3][4]

Definition 2.4 Minimum Total Dominating Energy

Let G be a simple graph of order n with vertex set $V = \{v_1, v_2, \dots, v_n\}$ and the edge set E . A vertex set S in G is a total dominating set if every vertex $v \in V$ is adjacent to an element of S

The total dominating number is the minimum cardinality taken over all the minimal total dominating sets of G . Let TDS be the minimum total dominating set of graph G . The minimum total dominating matrix of G is $n \times n$ matrix $A_{TD}(G) = (a_{ij})$ where,

$$A_{TD}(G) = \begin{cases} 1 & \text{if } v_i \text{ and } v_j \text{ are adjacent} \\ 1 & \text{if } i = j, v_i \in TDS \\ 0 & \text{otherwise} \end{cases}$$

The characteristic polynomial of $A_{TD}(G)$ is denoted by $P(G, \lambda) = \det(\lambda I - A_{TD}(G))$. The minimum total dominating eigen values of the graph G are the eigen values of $A_{TD}(G)$.

Since $A_{TD}(G)$ is real and symmetric, the eigen values are $\lambda_1, \lambda_2, \dots, \lambda_n$ in decreasing order $\lambda_1 \geq \lambda_2 \geq \dots \geq \lambda_n$. The total dominating energy of G is defined as, $E_{TD}(G) = \sum_{i=1}^n |\lambda_i|$. [6][7]

Example:

Let G be a graph with 6-vertices $V(G) = \{v_1, v_2, v_3, v_4, v_5, v_6\}$

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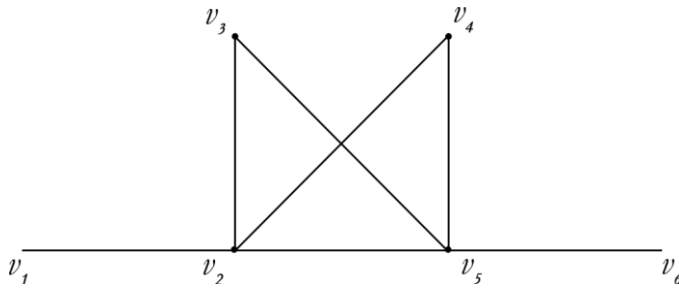


Figure (i)

The minimum total dominating set, $TDS = \{v_2, v_5\}$.

The minimum cardinality of a total dominating set $\gamma_t(G) = 2$.

The minimum total dominating matrix, $A_{TD}(G) =$

$$\begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

The Characteristic polynomial is

$$x^6 - 2x^5 - 6x^4 + 2x^3 + 5x^2 + 0x - 0$$

The eigen values are $\lambda_1 = -1.4495, \lambda_2 = -1.0000, \lambda_3 = -0.0000, \lambda_4 = 0.0000, \lambda_5 = 1.0000, \lambda_6 = 3.4495$

Therefore, the minimum total dominating energy, $E_{TD}(G) = 6.899$

III. MINIMUM TOTAL DOMINATING ENERGY OF SOME SPECIAL GRAPH

Definition 3.1 Paley graph

The ‘‘Paley graph’’ is a 6-regular graph on 13 vertices and 39 edges. It is also known as the Raymond Paley graph.[8]

Definition 3.2 Shrikhande graph

The ‘‘Shrikhande graph’’ is a named graph discovered by S.Shrikhande in 1959. It is strongly regular graph with 16 vertices and 48 edges, with each vertex having degree 6.[8]

Definition 3.3 Clebsch graph

The ‘‘Clebsch graph’’ is a named graph discovered by Alfred Clebsch in 1868. It is strongly quintic graph on 16 vertices and 40 edges. The Clebsch graph is also known as Greenwood-Gleason graph. It is a 5-regular graph.[8]

Definition 3.4 Chvatal graph

$$A_{TD}(G) = \begin{pmatrix} 1 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 \end{pmatrix}$$

The Characteristic polynomial is

The ‘‘Chvatal graph’’ is an undirected graph with 12 vertices and 24 edges, discovered by Vaclav Chvatal in 1970. It is a 4-regular graph.[8]

Definition 3.5 Moser graph

The ‘‘Moser graph’’ is an undirected graph with 7 vertices and 11 edges, named after the Mathematician Leo Moser and his brother William.[8]

Definition 3.6 Octahedron graph

The ‘‘Octahedron graph’’ is a polyhedron graph with 8 faces, 12 edges and 6 vertices. It is a 4-regular graph.[8]

3.1 Minimum Total Dominating Energy of Paley graph

Let $G = (V, E)$ be a Paley graph with 13 vertices and 39 edges. Let $V = \{v_1, v_2, v_3, \dots, v_{13}\}$ be a vertex set of the graph G .

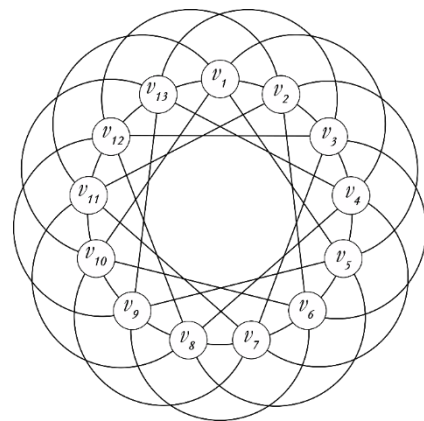


Figure (i)
PaleyGraph

The minimum total dominating set,

$$TDS = \{v_1, v_2, v_7, v_8\}$$

The minimum cardinality of a total dominating set $\gamma_t(G) = 4$.

The minimum total dominating matrix,

$$(0.0010)x^{13} - (0.0040)x^{12} - (0.0320)x^{11} + (0.0720)x^{10} + (0.3600)x^9 - (0.5570)x^8 - (1.9700)x^7 + (2.4110)x^6 + (5.7280)x^5 + (6.1660)x^4 - (8.5500)x^3 + (8.7330)x^2 + (5.1750)x - 5.2650$$

The eigen values are $\lambda_1 = 6.3312$ $\lambda_2 = -1.604$ $\lambda_3 = -1.7321$ $\lambda_4 = -1.8148$
 $\lambda_5 = -2.2361$ $\lambda_6 = -2.2032$ $\lambda_7 = 2.2361$ $\lambda_8 = 1.7321$
 $\lambda_9 = 1.5359$ $\lambda_{10} = 1.5359$ $\lambda_{11} = 1.2190$ $\lambda_{12} = -2.3028$
 $\lambda_{13} = 1.3028$

Therefore, the minimum total dominating energy of Paley graph, $E_{TD}(G) = 27.786$

3.2 Minimum Total Dominating Energy of Shrikhande graph

Let $G = (V, E)$ be a Shrikhande graph with 16 vertices and 48 edges.

Let $V = \{v_1, v_2, v_3 \dots \dots v_{16}\}$ be a vertex set of the graph G .

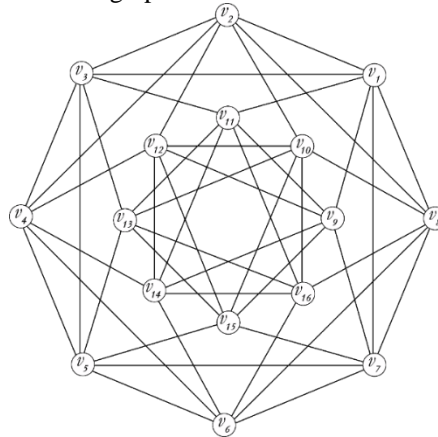


Figure (ii)
Shrikhande Graph

The minimum total dominating set, $TDS = \{v_{12}, v_{13}, v_{16}, v_6\}$.

The minimum cardinality of a total dominating set $\gamma_t(G) = 4$.

The minimum total dominating matrix

$$A_{TD}(G) = \begin{pmatrix} 0 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 0 \end{pmatrix}$$

The Characteristic polynomial is

$$(0.0000)x^{16} - (0.0000)x^{15} - (-0.0004)x^{14} + (0.0005)x^{13} + (0.0062)x^{12} - (0.0010)x^{11} - (0.0494)x^{10} - (0.0245)x^9 + (0.2166)x^8 + (0.2152)x^7 - (0.5058)x^6 - (0.7588)x^5 + (0.4908)x^4 + (1.2466)x^3 + (0.1165)x^2 - (0.7722)x - 0.3696$$

The eigen values are $\lambda_1 = 6.2067$ $\lambda_2 = 1.2804$ $\lambda_3 = 2.2167$ $\lambda_4 = 2.2167$
 $\lambda_5 = 2.3631$ $\lambda_6 = 1.8906$ $\lambda_7 = -1.0893$ $\lambda_8 = -1.3392$
 $\lambda_9 = -1.3392$ $\lambda_{10} = -1.8198$ $\lambda_{11} = -1.8198$ $\lambda_{12} = -1.7669$
 $\lambda_{13} = -2.0000$ $\lambda_{14} = -2.0000$ $\lambda_{15} = 2.0000$ $\lambda_{16} = -2.0000$

Therefore, the minimum total dominating energy of Shrikhande graph, $E_{TD}(G) = 33.348$

3.3 Minimum Total Dominating Energy of Clebsch graph

Let $G = (V, E)$ be a Clebsch graph with 16 vertices and 40 edges.

Let $V = \{v_1, v_2, v_3 \dots \dots v_{16}\}$ be a vertex set of the graph G .



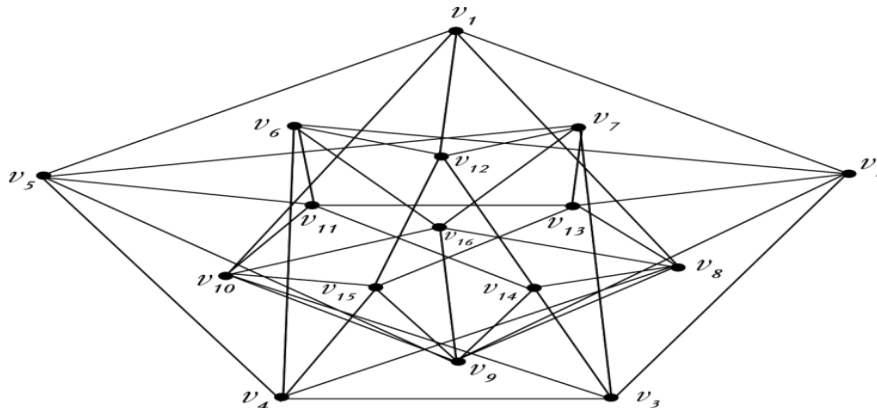


Figure (iii)
ClebschGraph

The minimum total dominating set, $TDS = \{v_7, v_8, v_9, v_{10}, v_{16}\}$.

The minimum cardinality of a total dominating set $\gamma_t(G) = 5$.

The minimum total dominating matrix

$$A_{TD}(G) = \begin{pmatrix} 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

The Characteristic polynomial is

$$(0.0001)x^{16} - (0.0005)x^{15} - (0.0025)x^{14} + (0.0117)x^{13} + (0.0192)x^{12} - (0.1102)x^{11} - (0.0268)x^{10} + (0.5033)x^9 - (0.3055)x^8 - (1.0018)x^7 + (1.3688)x^6 + (0.1920)x^5 - (1.4314)x^4 + (1.1414)x^3 - (0.4546)x^2 + (0.1120)x - 0.0152$$

The eigen values are $\lambda_1 = 5.7202$ $\lambda_2 = -2.7308$ $\lambda_3 = -2.7308$ $\lambda_4 = -2.2119$

$$\lambda_5 = -2.2119 \quad \lambda_6 = -1.5661 \quad \lambda_7 = 2.3628 \quad \lambda_8 = 1.5367$$

$$\lambda_9 = 1.5367 \quad \lambda_{10} = 0.0987 \quad \lambda_{11} = 0.0987 \quad \lambda_{12} = 1.7311$$

$$\lambda_{13} = 1.3194 \quad \lambda_{14} = 1.0000 \quad \lambda_{15} = 0.5235 \quad \lambda_{16} = 0.5235$$

Therefore, the minimum total dominating energy of Clebschgraph, $E_{TD}(G) = 27.902$

3.4 Minimum Total Dominating Energy of Chvatal graph

Let $G = (V, E)$ be a Chvatalgraph with 12 vertices and 24 edges.

Let $V = \{v_1, v_2, v_3 \dots \dots v_{12}\}$ be a vertex set of the graph G.

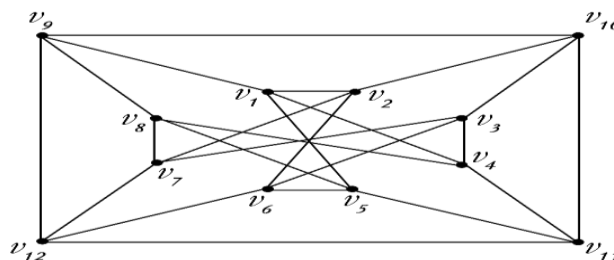


Figure (iv)

ChvatalGraph

The minimum total dominating set, $TDS = \{v_1, v_2, v_7, v_5\}$.

The minimum cardinality of a total dominating set $\gamma_t(G) = 4$.

The minimum total dominating matrix

$$A_{TD}(G) = \begin{pmatrix} 1 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 0 \end{pmatrix}$$

The Characteristic polynomial is

$$(0.0010)x^{12} - (0.0030)x^{11} - (0.0210)x^{10} + (0.0590)x^9 + (0.1200)x^8 - (0.4160)x^7 - (0.0550)x^6 + (1.0190)x^5 - (0.8530)x^4 - (0.0790)x^3 + (0.3160)x^2 - (0.0960)x + 0.0080$$

The eigen values are $\lambda_1 = -2.8427$ $\lambda_2 = -2.6798$ $\lambda_3 = -2.4998$ $\lambda_4 = -0.6219$
 $\lambda_5 = 0.1561$ $\lambda_6 = 0.2379$ $\lambda_7 = 1$ $\lambda_8 = 1$
 $\lambda_9 = 1.2655$ $\lambda_{10} = 1.6412$ $\lambda_{11} = 2.0302$ $\lambda_{12} = 4.3133$

Therefore, the minimum total dominating energy of Chvatalgraph, $E_{TD}(G) = 20.288$

3.5 Minimum Total Dominating Energy of Moser graph

Let $G = (V, E)$ be a Moser graph with 7 vertices and 11 edges.

Let $V = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7\}$ be a vertex set of the graph G.

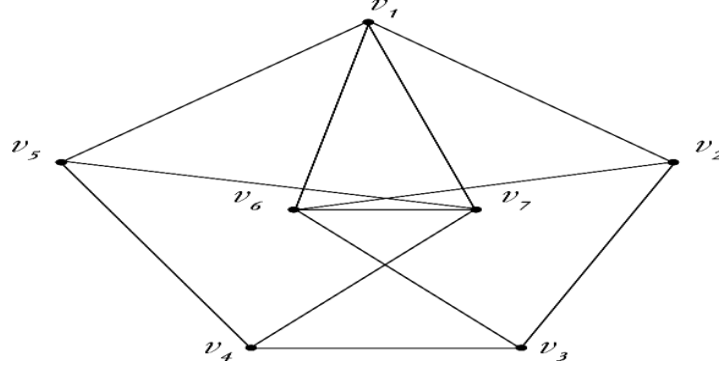


Figure (v)
MoserGraph

The minimum total dominating set, $TDS = \{v_1, v_2, v_5\}$.

The minimum cardinality of a total dominating set $\gamma_t(G) = 3$.

The minimum total dominating matrix

$$A_{TD}(G) = \begin{pmatrix} 1 & 1 & 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 1 & 1 & 0 \end{pmatrix}$$

The Characteristic polynomial is

$$x^7 - 3x^6 - 9x^5 + 15x^4 + 24x^3 - 12x^2 - 16x - 0$$

The eigen values are $\lambda_1 = -2$ $\lambda_2 = -1$ $\lambda_3 = -1$ $\lambda_4 = -0$
 $\lambda_5 = 1$ $\lambda_6 = 2$ $\lambda_7 = 4$

Therefore, the minimum total dominating energy of Moser graph, $E_{TD}(G) = 11$

3.6 Minimum Total Dominating Energy of Octahedron graph

Let $G = (V, E)$ be an Octahedron graph with 6 vertices and 12 edges.

Let $V = \{v_1, v_2, v_3, v_4, v_5, v_6\}$ be a vertex set of the graph G.

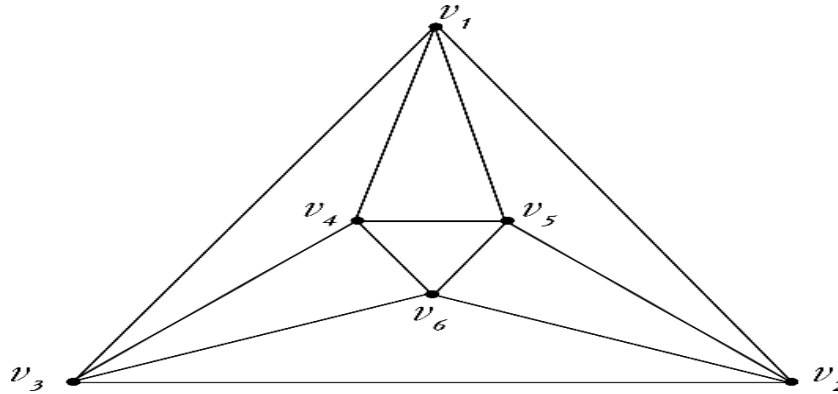


Figure (vi)
Octahedron Graph

The minimum total dominating set, $TDS = \{v_1, v_4\}$.

The minimum cardinality of a total dominating set $\gamma_t(G) = 2$.

The minimum total dominating matrix

$$A_{TD}(G) = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 \end{pmatrix}$$

The Characteristic polynomial is

$$x^6 - 2x^5 - 11x^4 + 0x^3 + 11x^2 - 4x - 0$$

The eigen values are $\lambda_1 = -1.8741$ $\lambda_2 = -1.6180$ $\lambda_3 = 0$ $\lambda_4 = 0.4865$

$$\lambda_5 = 0.6180$$

$$\lambda_6 = 4.3876$$

herefore, the minimum total dominating energy of Octahedron graph, $E_{TD}(G) = 8.984$

IV. CONCLUSION

In this paper, the energy namely Minimum total dominating energy has been found for some special graphs such as Paley graph, Shrikhande graph, Clebsch graph, Chvatal graph, Moser graph and Octahedron graph. For future research, the minimum inverse total dominating energy can be computed for some special classes of graphs.

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