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Independent Domination Number in Triangular & Quadrilateral Snake graph

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***Abstract:** Let V be the vertex set and E be the edge set of a graph G, the vertex set V has a subset S such that S contains vertices which is adjacent to atleast one vertex in V which is not in S, then S is said to be dominating set of G. If the vertex in S is not adjacent to each other, then S is said to be independent dominating set of G and so i(G) denotes the independent domination number, the minimum cardinality of an independent dominating set in G. In this paper, we obtain independent domination number for a triangular snake, alternate triangular snake, double triangular snake, alternate double triangular snake, quadrilateral snake, alternate quadrilateral snake, double quadrilateral snake and alternate double quadrilateral snake graphs.*

***Keywords:** Alternate double triangular snake, alternate double quadrilateral snake, alternate quadrilateral snake, alternate triangular snake, double triangular snake, , double quadrilateral snake, , quadrilateral snake, triangular snake.*

I. INTRODUCTION

The dominating sets were initially started with the game of chess in India over 400 years ago, in which we study sets of chess pieces which cover different squares of the chessboard. Later the eight Queens and Five Queens problems again gave the interest in dominating concepts example, In the books of Ahrens in 1901 [16]. Finally, Berge in 1958 [2] and ore in 1962 [9] published the books related to the topic domination which gave proper mathematical definition but by 1972 Cockayne and Hedetniemi [3,4] again gone through domination and began to study it, ultimately in 1975 a survey of the results were published and the independent domination number were introduced and it is denoted in the form i(G) . This rekindled the researchers to work on it. Goddard et al. [15], Kostichka [1] and Lam et al. [10] researched much about the independent dominating number in regular graph and cubic graph. Favaron [8] studied the sharp upper bounds of ***i(G)*** for general graphs and the work was extended by Haviland [6]. Cockayne et al. [5] found the product of the i(G) of a graph and its complement with its upper bound, while Shiu et al. [14] found the i(G) of triangle-free graphs and characterizing the extremal graphs with its upper bounds.

II. DEFINITIONS

Definition 1. [11] For $v \in V(G)$, the open neighbourhood of v, denoted as $N_G(v)$, is the vertices of the set adjacent with a vertex v; and the closed neighbourhood of a vertex v, denoted by $N_G[v]$, is given by $N_G(v) \cup \{v\}$. For a set $S \subseteq V(G)$, the open neighbourhood of a set S is defined by $N_G(S) = \bigcup_{v \in S} N_G(v)$ and the closed neighbourhood of S is defined as $N_G[S] = N_G(S) \cup S$. For brevity, we denote $N_G(S)$ by $N(S)$ and $N_G[S]$ by $N[S]$.

Definition 2. [11] Let G be a graph with Vertex set V and edge set E, let S be the subset of V such that every vertex in V which is not in S must contain atleast one neighbour in S. The domination number of G is denoted by $\gamma(G)$, the minimum cardinality of dominating set of G.

Definition 3. [13] If S is both an independent and dominating set of a graph G then S is said to be an independent dominating set of graph G. The independent domination number is denoted by *i(G)*, the minimum cardinality of an independent dominating set in G.

Definition 4. [12] **Triangular snake:**
The triangular snake T_n is obtained from the path P_n by replacing each edge of the path by a triangle C_3.

Definition 5. [12] **Alternate triangular snake:**
An alternate triangular snake $A(T_n)$ is obtained from a path $a_1, a_2, \dots a_n$ by joining $a_i \, and \, a_{i+1}$ (alternately) to a new vertex v_i.

Definition 6. [12] **Double Triangular snake:**
A double triangular snake $D(T_n)$ consists of two triangular snakes that have a common path.

Definition 7. [12] **Double Alternate triangular snake:**
A double alternate triangular snake $A(D(T_n))$ consists of two alternate triangular snakes that have a common path.

Definition 8. [7] **Quadrilateral snake:**
A quadrilateral snake Q_n is obtained from a path $a_1, a_2, \dots a_n$ by joining $a_i \, and \, a_{i+1}$ to new vertices $b_i \, and \, c_i$ respectively and joining the vertices $b_i \, and \, c_i$ for i=1,2,...n-1. That is every edge of a path is replaced by a cycle C_4.

Definition 9. [7] **Alternate quadrilateral snake:**
An alternate quadrilateral snake $A(Q_n)$ is obtained from a path $a_1, a_2, \dots a_n$ by joining $a_i \, and \, a_{i+1}$ to new vertices $b_i \, and \, c_i$ respectively and joining the vertices $b_i \, and \, c_i$ for i≡1(mod2) and I≤n-1 and then joining $b_i \, and \, c_i$. That is every alternate edge of a path is replaced by a cycle C_4.

Definition 10. [7] **Double Quadrilateral snake:**
A double quadrilateral snake $D(Q_n)$ is obtained from two quadrilateral snakes that have a common path.

Definition 11. [7] **Double Alternate quadrilateral snake:**
An alternate double quadrilateral snake $A(D(Q_n))$ is obtained from two alternative quadrilateral snakes that have a common path.

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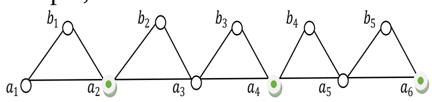
Theorem 1:
Let T_n be the triangular snake obtained from the path P_n by replacing each edge of the path by a triangle C_3 then

$$i(T_n)=\begin{cases}\left\lceil\frac{n+1}{2}\right\rceil & when\ n\ is\ odd\\ \left\lceil\frac{n}{2}\right\rceil & when\ n\ is\ even\end{cases}$$

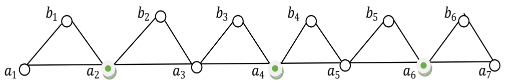
Proof:
Let $a_1, a_2, \dots a_n$ be the vertices of T_n and b_i be the new vertex joining $a_i\ and\ a_{i+1}$ of a path by a triangle C_3.
Now we select the vertices for the set S (independent domination set) from a graph T_n, in such a way that each vertex in S is adjacent to atleast one vertex in V-S and that each vertex in S is independent.
Choosing such a way we get the independent domination number for T_n as (i.e.,)

$$i(T_n)=\begin{cases}\left\lceil\frac{n+1}{2}\right\rceil & when\ n\ is\ odd\\ \left\lceil\frac{n}{2}\right\rceil & when\ n\ is\ even\end{cases}$$

for example, for n is odd



For n is even



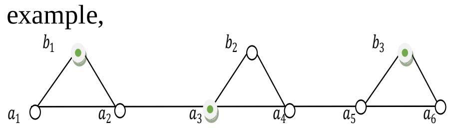
The highlighted vertices represent the independent domination set.

Theorem 2:
Let $A(T_n)$ be the alternate triangular snake obtained from the path P_n by replacing each alternate edge of the path by a triangle C_3 then $i(A(T_n))= n$

Proof:
Let $a_1, a_2, \dots a_n$ be the vertices of $A(T_n)$ and b_i be the new vertex joining alternatively $a_i\ and\ a_{i+1}$ of a path by a triangle C_3.
Now we select the vertices for the set S (independent domination set) from a graph $A(T_n)$, in such a way that each vertex in S is adjacent to atleast one vertex in V-S and that each vertex in S is independent.
Choosing such a way we get the independent domination number for $A(T_n)$ as (i.e.,)

$$i(A(T_n))= n$$

for example,



The highlighted vertices represent the independent domination set.

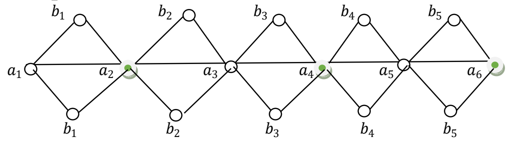
Theorem 3:
Let $D(T_n)$ be the double triangular snake obtained from two triangular snake with a common path then

$$i(T_n)=\begin{cases}\left\lceil\frac{n+1}{2}\right\rceil & when\ n\ is\ odd\\ \left\lceil\frac{n}{2}\right\rceil & when\ n\ is\ even\end{cases}$$

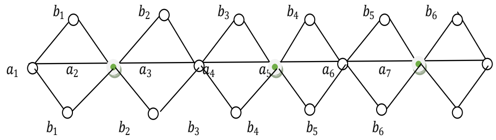
Proof:
Let $a_1, a_2, \dots a_n$ be the vertices of $D(T_n)$ and b_i be the new vertex joining $a_i\ and\ a_{i+1}$ both above and below the common path of $a_1, a_2, \dots a_n$.
Now we select the vertices for the set S (independent domination set) from a graph $D(T_n)$, in such a way that each vertex in S is adjacent to atleast one vertex in V-S and that each vertex in S is independent.
Choosing such a way we get the independent domination number for $D(T_n)$ as (i.e.,)

$$i(D(T_n))=\begin{cases}\left\lceil\frac{n+1}{2}\right\rceil & when\ n\ is\ odd\\ \left\lceil\frac{n}{2}\right\rceil & when\ n\ is\ even\end{cases}$$

for example, when n is odd,



When n is even,



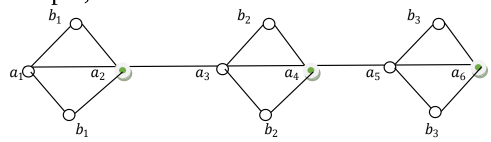
The highlighted vertices represent the independent domination set.

Theorem 4:
Let $D(A(T_n))$ be the double alternative triangular snake obtained from alternate double triangular snake then $i(D(A(T_n)))= n$

Proof:
Let $a_1, a_2, \dots a_n$ be the vertices of $D(A(T_n))$ and b_i be the new vertex joining alternatively $a_i\ and\ a_{i+1}$ both above and below the common path of $a_1, a_2, \dots a_n$.
Now we select the vertices for the set S (independent domination set) from a graph $D(A(T_n))$, in such a way that each vertex in S is adjacent to atleast one vertex in V-S and that each vertex in S is independent.
Choosing such a way we get the independent domination number for $D(A(T_n))$ as (i.e.,)

$$i(D(A(T_n)))= n$$

for example,



The highlighted vertices represent the independent domination set.

Theorem 5:
Let Q_n be the quadrilateral snake obtained from the path P_n by replacing each edge of the path by a cycle C_4 then $i(Q_n)= n+1$

Proof:
Let $a_1, a_2, \dots a_n$ be the vertices of Q_n and $b_i and\ c_i$ be the new vertex joining $a_i\ and\ a_{i+1}$ of a path by a cycle C_4.
Now we select the vertices for the set S (independent domination set) from a graph Q_n, in such a way that each vertex in S is adjacent to atleast one vertex in V-S and that each vertex in S is independent.

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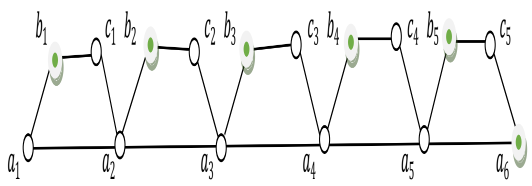
Choosing such a way we get the independent domination number for Q_n as (i.e.,)

$$i(Q_n) = n+1$$

for example,



The highlighted vertices represent the independent domination set.

Theorem 6:

Let $A(Q_n)$ be the alternate quadrilateral snake obtained from the path P_n by replacing each alternate edge of the path by a cycle C_4 then $i(AT_n) = \begin{cases} \left\lceil \frac{3n+1}{2} \right\rceil & when\ n\ is\ odd \\ \left\lceil \frac{3n}{2} \right\rceil & when\ n\ is\ even \end{cases}$

Proof:

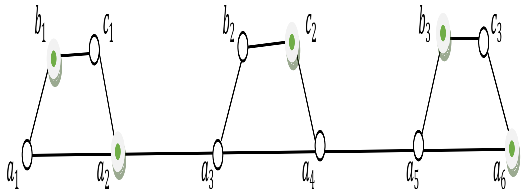
Let $a_1, a_2, \ldots a_n$ be the vertices of $A(Q_n)$ and $b_i and\ c_i$ be the new vertex joining alternatively $a_i and\ a_{i+1}$ of a path by a cycle C_4.

Now we select the vertices for the set S (independent domination set) from a graph $A(Q_n)$, in such a way that each vertex in S is adjacent to atleast one vertex in V-S and that each vertex in S is independent.

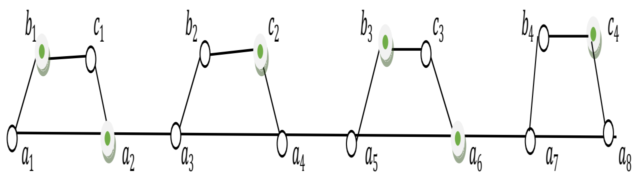
Choosing such a way we get the independent domination number for $A(Q_n)$ as (i.e.,)

$$i(AQ_n) = \begin{cases} \left\lceil \frac{3n+1}{2} \right\rceil & when\ n\ is\ odd \\ \left\lceil \frac{3n}{2} \right\rceil & when\ n\ is\ even \end{cases}$$

for example, when n is odd



When n is even,



The highlighted vertices represent the independent domination set.

Theorem 7:

Let $D(Q_n)$ be the double quadrilateral snake obtained from two quadrilateral snake with a common path then $i(Q_n)$=2n

Proof:

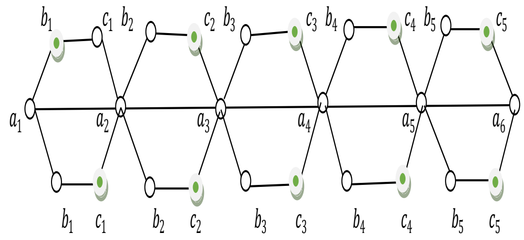
Let $a_1, a_2, \ldots a_n$ be the vertices of $D(Q_n)$ and $b_i and\ c_i$ be the new vertex joining $a_i and\ a_{i+1}$ both above and below the common path of $a_1, a_2, \ldots a_n$.

Now we select the vertices for the set S (independent domination set) from a graph $D(Q_n)$, in such a way that each vertex in S is adjacent to atleast one vertex in V-S and that each vertex in S is independent.

Choosing such a way we get the independent domination number for $D(Q_n)$ as (i.e.,)

$$i(D(Q_n)) = 2n$$

for example,



The highlighted vertices represent the independent domination set.

Theorem 8:

Let $D(A(Q_n))$ be the double alternative quadrilateral snake obtained from alternate double quadrilateral snake then $i(D(A(Q_n)))$= 2n

Proof:

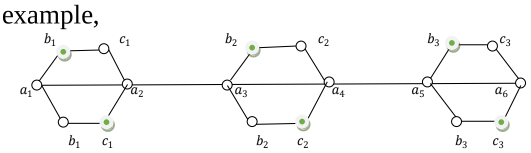
Let $a_1, a_2, \ldots a_n$ be the vertices of $D(A(Q_n))$ and $b_i and\ c_i$ be the new vertex joining alternatively $a_i and\ a_{i+1}$ both above and below the common path of $a_1, a_2, \ldots a_n$.

Now we select the vertices for the set S (independent domination set) from a graph $D(A(Q_n))$, in such a way that each vertex in S is adjacent to atleast one vertex in V-S and that each vertex in S is independent.

Choosing such a way we get the independent domination number for $D(A(Q_n))$ as (i.e.,)

$$i(D(A(Q_n))) = 2n$$

for example,



Note:

The independent domination number for $A(T_n)$ and $D(A(T_n))$ is equal (i.e.,) n. Similarly, the independent domination number for $D(Q_n)$ and $A(Q_n)$ is equal (i.e.,) 2n.

III. CONCLUSION

We hereby found the independent domination number for triangular snake, alternate triangular snake, double triangular snake, alternate double triangular snake, quadrilateral snake, alternate quadrilateral snake, double quadrilateral snake and alternate double quadrilateral snake graphs.

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