Complex Method on Octagonal Number

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Abstract: ***In Number theory Study of polygonal numbers is rich in varity. In this paper we establish a Complex Octagonal Number using Continued Fraction algorithm.***

Keywords: ***Algorithm, Continuedfraction, complex Number. Octagonal Number.***

I. INTRODUCTION

A Simple continued fraction [1] is an expression of the form

$$a_0 + \cfrac{b_0}{a_1 + \cfrac{b_1}{a_2 + \cfrac{b_2}{\ddots}}}$$

Where the a_i are a possibly infinite sequence of integers such that a_1 is non-negative and the rest of the sequence is positive. We write $\langle a_1; a_2, a_3 \dots \dots \rangle$. The above fraction also calls them Regular continued fractions.

II. CONTINUED FRACTION ALGORITHM

Suppose we wish to find continued fraction expansion[2] of $x \in R$.
Let $x_0 \in x$ and set $a_0 = [x_0]$,
Define $x_1 = \frac{1}{x_0 - [x_0]}$ and set $a_1 = [x_1]$ and $x_2 = \frac{1}{x_1 - [x_1]} \Rightarrow$ $a_2 = [x_2] \dots . x_k = \frac{1}{x_{k-1} - [x_{k-1}]} \Rightarrow a_k = [x_k] \dots$
This process is continued infinitely or to some finite stage till an $x_i \in N$ exists such that $a_i = [x_i]$.

III. OCTAGONAL NUMBER

A. Definition: Centered Octagonal Number[3]
The Number 1,9,25,49,81,121,…….. are called centered octagonal numbers. The number that represents associate in nursing polygonal shape with a dot within the center and every one dots different dots encompassing the middle dot in associate in nursing polygonal shape lattice .

The n^{th} centered octagonal number is given by the formula
$O_n = 4n(n-1) + 1$

B. Theorem:

For $n \geq 3$,

$$\frac{O_n}{O_{n+1}} + i\frac{O_{n+2}}{O_{n+3}} = \begin{cases} \langle 0; 1, \left[\frac{n}{2}\right], 8n \rangle + i \langle 0; 1, \left[\frac{n+1}{2}\right], 8(n+2) \rangle & when\ n\ is\ odd \\ \langle 0; 1, \frac{n}{2} - 1,1,1,2n-1,2 \rangle + i \langle 0; 1, \frac{(n+2)}{2} - 1,1,1,2(n+2)-1,2 \rangle & when\ n\ is\ even \end{cases}$$

Proof:

Case (i):- n is odd

Let n = 2k-1, Where 3$\leq k \leq n$

Then

$$\frac{O_{2k-1}}{O_{2k}} + i\frac{O_{2k+1}}{O_{2k+2}} = \langle 0; 1, \left[\frac{2k-1}{2}\right], 8(2k-1) \rangle + i \langle 0; 1, [k], 8(2k+1) \rangle$$

Next we have to prove that n = 2k+1

To find the continued fraction of

$$\frac{O_{2k+1}}{O_{2k+2}} + i\frac{O_{2k+3}}{O_{2k+4}}$$

A. Real Part:-[3]

$$\frac{O_{2k+1}}{O_{2k+2}} = \frac{4(2k+1)(2k+1-1)+1}{4(2k+2)(2k+2-1)+1}$$
$$= \frac{16k^2+8k+1}{16k^2+2k+9}$$

$$x_0 = \frac{16k^2+8k+1}{16k^2+2k+9}, a_0 = 0$$
$$x_1 \Rightarrow 1 + \frac{16k+8}{16k^2+8k+1} \Rightarrow a_1 = 1$$
$$x_2 \Rightarrow k + \frac{1}{16k+8} \Rightarrow a_2 = k$$
$$x_3 \Rightarrow 16k+8 \Rightarrow a_3 = 16k+8$$
=8(2k+1)

$$\therefore \frac{O_{2k+1}}{O_{2k+2}} = \langle 0; 1, k, 8(2k+1) \rangle$$

B. Imaginary part:-

$$\frac{O_{2k+3}}{O_{2k+4}} \Rightarrow \frac{4(2k+3)(2k+3-1)+1}{4(2k+4)(2k+4-1)+1}$$
$$= \frac{4[4k^2+6k-2k+6k+9-3]+1}{4[4k^2+8k-2k+8k+16-4]+1}$$
$$= \frac{4[4k^2+4k+6k+6]+1}{4[4k^2+14k+12]+1}$$
$$= \frac{4[4k^2+10k+6]+1}{4[4k^2+14k+12]+1}$$
$$= \frac{16k^2+40k+24+1}{16k^2+56k+48+1}$$
$$\frac{O_{2k+3}}{O_{2k+4}} \Rightarrow \frac{16k^2+40k+25}{16k^2+56k+49}$$
$$x_0 = \frac{16k^2+40k+25}{16k^2+56k+49}; \ a_0 = 0$$

Then

$$x_1 \Rightarrow \frac{16k^2+56k+49}{16k^2+40k+25} = 1 + \frac{16k+24}{16k^2+40k+25} \Rightarrow a_1 = 1$$

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$$x_2 \Rightarrow \frac{16k^2+40k+25}{16k+24} = (k+1) + \frac{1}{16k+24} \Rightarrow a_2 = (k+1)$$

$$x_3 \Rightarrow \frac{16k+24}{1} \Rightarrow 16k+24 \Rightarrow 8(2k+3)$$

$$\therefore \frac{O_{2k+3}}{O_{2k+4}} = \langle 0; 1, (k+1), 8(2k+3)\rangle$$

$$\therefore \frac{O_{2k+1}}{O_{2k+2}} + i\frac{O_{2k+3}}{O_{2k+4}} = \langle 0; 1, k, 8(2k+1)\rangle + i\,\langle 0; 1, (k+1), 8(2k+3)\rangle$$

By the results is true for all values of n when n is odd.

Case (ii):- n is even

Let n = 2k – 2

Then

$$\frac{O_{2k-2}}{O_{2k-1}} + i\frac{O_{2k}}{O_{2k+2}} = \langle 0; 1, \frac{2k-2}{2} - 1,1,1,2(2k-2) - 1,2\rangle + i\,\langle 0; 1, \frac{2k-2+2}{2} - 1,1,1,2(2k-2) - 1,2\rangle$$

$$= \langle 0; 1, k-2,1,1,2(2k-2) -$$

1,2+i0;1,k−1,1,1,22k−2−1,2

Next we have to prove that n = 2k

To find the continued fraction of

$$\frac{O_{2k}}{O_{2k+1}} + i\frac{O_{2k+2}}{O_{2k+3}}$$

C. Real Part:-[3]

$$\frac{O_{2k}}{O_{2k+1}} = \langle 0; 1, k-1,1,1,4k-1,2\rangle$$

D. Imaginary part:-

$$\frac{O_{2k+2}}{O_{2k+3}} = \frac{4(2k+2)(2k+2-1)+1}{4(2k+3)(2k+3-1)+1}$$

$$= \frac{4[4k^2+2k+4k+2]+1}{4[4k^2+4k+6k+6]+1}$$

$$= \frac{4[4k^2+6k+2]+1}{4[4k^2+10k+6]+1}$$

$$= \frac{16k^2+24k+8+1}{16k^2+40k+24+1}$$

$$= \frac{16k^2+24k+9}{16k^2+40k+25}$$

$$\frac{O_{2k+2}}{O_{2k+3}} \Rightarrow \frac{16k^2+24k+9}{16k^2+40k+25}$$

$$x_0 = \frac{16k^2+24k+9}{16k^2+40k+25}\ ;\ a_0 = 0$$

Then

$$x_1 \Rightarrow \frac{16k^2+40k+25}{16k^2+24k+9} = 1 + \frac{16k+16}{16k^2+24k+9} \Rightarrow a_1 = 1$$

$$x_2 \Rightarrow \frac{16k^2+24k+9}{16k+16} = k + \frac{8k+9}{16k+16} \Rightarrow a_2 = k$$

$$x_3 \Rightarrow \frac{16k+16}{8k+9} \Rightarrow 1 + \frac{8k+7}{8k+9} \Rightarrow a_3 = 1$$

$$x_4 \Rightarrow \frac{8k+9}{8k+7} \Rightarrow 1 + \frac{2}{8k+7} \Rightarrow a_4 = 1$$

$$x_5 \Rightarrow \frac{8k+7}{2} \Rightarrow (4k+3) + \frac{1}{2} \Rightarrow a_5 = 4k+3$$

$$x_6 \Rightarrow \frac{2}{1} \Rightarrow a_6 = 2$$

$$\therefore \frac{O_{2k+2}}{O_{2k+3}} = \langle 0; 1, k, 1,1,4k+3,2)\rangle$$

Since

$$\frac{O_{2k}}{O_{2k+1}} + i\frac{O_{2k+2}}{O_{2k+3}} = \langle 0; 1, k-1,1,1,4k-1,2\rangle + i\langle 0; 1, k, 1,1,4k+3,2)\rangle$$

Hence by the result is true for all value of n where n is even.

Since case (i) & case (ii) for each $n \geq 3$, the continued fraction expansion of

$$\frac{O_n}{O_{n+1}} + i\frac{O_{n+2}}{O_{n+3}} = \begin{cases} \langle 0; 1, [\frac{n}{2}], 8n\rangle + i\,\langle 0; 1, [\frac{n+1}{2}], 8(n+2)\rangle \text{ when } n \text{ is odd} \\ \langle 0; 1, \frac{n}{2} - 1,1,1,2n-1,2\,\rangle + i\,\langle 0; 1, \frac{(n+2)}{2} - 1,1,1,2(n+2) - 1,2\,\rangle \text{ when } n \text{ is even} \end{cases}$$

IV. ILLUSTRATION

Let $n = 3$,

$$\frac{O_3}{O_4} + i\frac{O_5}{O_6} = \frac{25}{49} + i\frac{81}{121}$$

A. Real Part:-[3]

$$\frac{O_3}{O_4} = \frac{25}{49}\ , so\ a_0 = 0$$

$$\therefore\ x_0 = \frac{25}{49}$$

$$Then\ x_1 = \frac{1}{x_0 - [x_0]} = \frac{49}{25} = 1 + \frac{24}{25} \Rightarrow a_1 = 1$$

$$x_2 = \frac{1}{x_1 - [x_1]} = \frac{25}{24} = 1 + \frac{1}{24} \Rightarrow a_2 = 1$$

$$x_3 = \frac{1}{x_2 - [x_2]} = \frac{24}{1} = 24 \Rightarrow a_3 = 24$$

$$\therefore \frac{25}{49} = \langle 0; 1,1,24\rangle$$

B. Imaginary Part:-

$$\frac{O_5}{O_6} = \frac{81}{121}\ , a_0 = 0$$

$$x_0 = \frac{81}{121}\ , a_0 = 0$$

Then

$$x_1 = \frac{1}{x_0 - [x_0]} = \frac{121}{81} = 1 + \frac{40}{81} \Rightarrow a_1 = 1$$

$$x_2 = \frac{1}{x_1 - [x_1]} = \frac{81}{40} = 2 + \frac{1}{40} \Rightarrow a_2 = 2$$

$$x_3 = \frac{1}{x_2 - [x_2]} = \frac{40}{1} = 40 \Rightarrow a_3 = 40$$

$$\frac{81}{121} = \langle 0; 1,2,40\rangle$$

Hence

$$\frac{25}{49} + i\frac{81}{121} = \langle 0; 1,1,24\rangle + i\langle 0; 1,2,40\rangle$$

V. ILLUSTRATION

Put n = 4

$$\frac{O_4}{O_5} + i\frac{O_6}{O_7} = \frac{49}{81} + i\frac{121}{169}$$

A. Real Part:-[3]

$$\frac{O_4}{O_5} = \frac{49}{81} \Rightarrow \langle 0; 1,1,1,1,7,2\rangle$$

B. Imaginary part:-

$$\frac{O_6}{O_7} = \frac{121}{169}$$

$$Let\ x_0 = \frac{121}{169}\ , a_0 = 0$$

Then

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$$x_1 = \frac{1}{x_0-[x_0]} = \frac{169}{121} = 1 + \frac{48}{121} \Rightarrow a_1 = 1$$

$$x_2 = \frac{1}{x_1-[x_1]} = \frac{121}{48} = 2 + \frac{25}{48} \Rightarrow a_2 = 2$$

$$x_3 = \frac{1}{x_2-[x_2]} = \frac{48}{25} = 1 + \frac{23}{25} \Rightarrow a_3 = 1$$

$$x_4 = \frac{1}{x_3-[x_3]} = \frac{25}{23} = 1 + \frac{2}{23} \Rightarrow a_4 = 1$$

$$x_5 = \frac{1}{x_4-[x_4]} = \frac{23}{2} = 11 + \frac{1}{2} \Rightarrow a_5 = 11$$

$$x_6 = \frac{1}{x_5-[x_5]} = 2 \Rightarrow a_6 = 2$$

$$\frac{121}{169} = \langle 0; 1,2,1,1,11,2 \rangle$$

Since

$$\frac{49}{81} + i\frac{121}{169} = \langle 0; 1,1,1,1,7,2 \rangle + i\langle 0; 1,2,1,1,11,2 \rangle$$

VI. CONCLUSION

In this paper we have identified complex octagonal number using continued fractions.

REFERENCES

1. https://crypto.stanford.edu
2. International Journal of Engineering Science, Advanced Computing and Bio –Technology "Ratios of Polygonal Numbers as Continued Fractions" – A. Gnanam and S. Krithika.
3. The International Journal of analytical and experimental modal analysis " Algebraic Approach on Octagonal Numbers"- A.Gnanam and S. Sudha

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