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Total Mean Labeling Graphs

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Abstract*: Let G=(V, E) be a finite, undirected simple graph with p vertices and q edges. A total mean labeling of G is a bijection f from $V(G)\cup E(G)$ to {1,2,…,p+q} such that for each edge $uv\in E(G), f^*(uv)=\left\lceil \frac{f(u)+f(v)+f(uv)}{3}\right\rceil$ is distinct. A graph which admits a total mean labeling is called a total mean labeling graph. In this paper, we prove that $P_n, P_n^+, K_{1,n}, K_{2,n}$, $C_n, B_{m,n}$, Triangular snakes and Alternate Triangular snakes are total mean labeling graphs.

***Keywords* : labeling, mean labeling, total mean labeling.**

I. INTRODUCTION

By a graph $G=(V,E)$, we mean finite, undirected simple graph with p vertices and q edges. We refer to Chartrand, Lesniak and Zhang [1] for basic terminology in graphs.

An assignment of integers to the vertices or edges or both subject to certain conditions is called a graph labeling. Labeled graphs are used in many areas like, astronomy, coding theory, radar, X-ray crystallography etc.

Somasundaram and Ponraj [2] have introduced the concept of mean labeling of graphs. A graph *G* with *p* vertices and *q* edges is called a mean graph if there is an injective function f from the vertices of *G* to *{0,1,2,…,q}*, such that for each edge *uv* $f^*(uv)=\left\lceil \frac{f(u)+f(v)}{2}\right\rceil$ is distinct.

In [2] – [5], they prove the following graphs are mean graphs: $P_n, C_n, K_{2,n}, K_2+mK_1, \overline{K_n}+2K_2, C_m\cup P_n, P_m\times P_n$, triangular snakes and quadrilateral snakes.

Gayathri and Gopi [7], prove the following are mean graphs: double triangular snakes, double quadrilateral snakes, generalized antiprisms, graphs obtained by joining the 2 vertices of degree n of $K_{2,n}$ with an edge, and graphs obtained from C_n with consecutive vertices $v_1, v_2, \cdots, v_n$ by adding the chords joining v_i and v_{n-i+2} for $2\le i\le \lfloor n/2\rfloor$. In [6], they give various necessary conditions for mean labelings.

A latest survey on various graph labeling can be found in Gallian [8].

In this paper we introduce the concept of total mean labeling graphs.

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II. TOTAL MEAN LABELING GRAPH

Let $G=(V,E)$ be a finite, undirected simple graph with *p* vertices and *q* edges.

Definition 2.1: A bijection $f:V(G)\cup E(G)\to\{1,2,\cdots,p+q\}$ is said to be a total mean labeling if for each edge $uv\in E(G), f^*(uv)=\left\lceil \frac{f(u)+f(v)+f(uv)}{3}\right\rceil$ is distinct. A graph G is said to be a total mean labeling graph if it admits a total mean labeling.

Example 2.2: Let $G=2P_3, V(G)=\{u_1,u_2,u_3,v_1,v_2,v_3\}$ and $E(G)=\{u_1u_2,u_2u_3,v_1v_2,v_2v_3\}$. Define $f:V\cup E\to\{1,2,\cdots,10\}$ by $f(u_i)=2i-1, f(v_i)=2i$ for $i=1,2,3,$ $f(u_1u_2)=7, f(u_2u_3)=9,$ $f(v_1v_2)=8$ and $f(v_2v_3)=10$. Clearly *f* is a bijection and also $f^*(uv)=\left\{\left\lceil \frac{f(u)+f(v)+f(uv)}{3}\right\rceil : uv\in E(G)\right\}=\{4,5,6,7\}$. Hence *G* is a total mean labeling graph.

Theorem 2.3: For $n\ge 2$, the path P_n is a total mean labeling graph.

Proof: Let $P_n=(v_1,v_2,\cdots,v_n)$. Define $f:V(P_n)\cup E(P_n)\to$ $\{1,2,\cdots,2n-1\}$ by $f(v_i)=2i-1$ for $i=1,2,\cdots,n$ and $f(v_iv_{i+1})=2i$ for $i=1,2,\cdots,n-1$. Clearly, f is a bijection. Also, $f^*(v_iv_{i+1})=\left\lceil \frac{f(v_i)+f(v_{i+1})+f(v_iv_{i+1})}{3}\right\rceil=$ $\left\lceil \frac{2i-1+2(i+1)-1+2i}{3}\right\rceil=2i$ for $i=1,2,\cdots,n-1,$ are distinct. Hence P_n is a total mean labeling graph.

Theorem 2.4: For $n\ge 2,$ the tree $P_n^+=(P_n\circ K_1)$ is a total mean labeling graph.

Proof : The tree P_n^+ is a graph obtained by attaching a pendent edge to each vertex of the path P_n. Let $V(P_n^+)=\{u_1,u_2,\cdots,u_n,v_1,v_2,\cdots,v_n\}$ and $E(P_n^+)=$ $\{u_iu_{i+1}:1\le i\le n-1\}\cup\{u_iv_i:1\le i\le n\}$. Define $f:V\cup E\to$ $\{1,2,\cdots,4n-1\}$ by $f(u_i)=2i-1, f(v_i)=3n-1+i, f(u_iv_i)=$ $2n-1+i$ for $i=1,2,\cdots,n$ and $f(u_iu_{i+1})=2i$ for $i=1,2,\cdots,$ $n-1$. Clearly, f is a bijection. Also, $f^*(u_iu_{i+1})=$ $\left\lceil \frac{f(u_i)+f(u_{i+1})+f(u_iu_{i+1})}{3}\right\rceil=\left\lceil \frac{2i-1+2(i+1)-1+2i}{3}\right\rceil=2i$ for $i=1,2,\cdots,n-1$ and $f^*(u_iv_i)=\left\lceil \frac{f(u_i)+f(v_i)+f(u_iv_i)}{3}\right\rceil=$ $\left\lceil \frac{2i-1+3n-1+i+2n-1+i}{3}\right\rceil=\left\lceil \frac{3(n+i-1)+2n+i}{3}\right\rceil=(n+i-1)+$ $\left\lceil \frac{2n+i}{3}\right\rceil$ for $i=1,2,\cdots,n,$ are distinct. Hence P_n^+ is a total mean labeling graph.

Theorem 2.5: The star $K_{1,n}$ is a total mean labeling graph.

Proof: Let $V(K_{1,n})=\{u,v_1,v_2,\cdots,v_n\}$ and $E(K_{1,n})=\{uv_i:1\le i\le n\}$. Define $f:V\cup E\to\{1,2,\cdots,2n+1\}$ by $f(u)=1$ and $f(v_i)=2i+1, f(uv_i)=2i$ for $i=1,2,\cdots,n.$ Clearly f is a bijection. Also, $f^*(uv_i)=\left\lceil\frac{f(u)+f(v_i)+f(uv_i)}{3}\right\rceil=\left\lceil\frac{1+2i+1+2i}{3}\right\rceil=\left\lceil\frac{3i+i+2}{3}\right\rceil=i+\left\lceil\frac{i+2}{3}\right\rceil$ for $i=1,2,\cdots,n,$ are distinct. Hence $K_{1,n}$ is a total mean labeling graph.

Theorem 2.6: The complete bipartite graph $K_{2,n}$ is a total mean labeling graph.

Proof: Let $V(K_{2,n})=\{u_1,u_2\}\cup\{v_i:1\le i\le n\}$ and $E(K_{2,n})=\{u_iv_j:i=1,2;1\le j\le n\}$. Define $f:V\cup E\to\{1,2,\cdots,3n+2\}$ by $f(u_1)=1, f(u_2)=2$ and $f(v_i)=3i, f(u_1v_i)=3i+1, f(u_2v_i)=3i+2$ for $i=1,2,\cdots,n.$ Clearly f is a bijection. Also, $f^*(u_1v_i)=\left\lceil\frac{f(u_1)+f(v_i)+f(u_1v_i)}{3}\right\rceil=\left\lceil\frac{1+3i+(3i+1)}{3}\right\rceil=2i+1$ and $f^*(u_2v_i)=\left\lceil\frac{f(u_2)+f(v_i)+f(u_2v_i)}{3}\right\rceil=\left\lceil\frac{2+3i+(3i+2)}{3}\right\rceil=2i+2$ for $i=1,2,\cdots,n,$ are distinct. Hence $K_{2,n}$ is a total mean labeling graph.

Theorem 2.7: The cycle C_n is a total mean labeling graph.

Proof: Case (i): When n is odd.

Let $n=2m+1$ and let $C_n=(x,u_1,u_2,\cdots,u_m,v_m,v_{m-1},\cdots,v_1,x)$. Define $f:V\cup E\to\{1,2,\cdots,4m+2\}$ by $f(x)=1, f(u_i)=4i-1,$ $f(v_i)=4i+1$ for $i=1,2,\cdots,m,$ $f(xu_1)=2, f(v_1x)=4, f(u_mv_m)=4m+2$ and $f(u_iu_{i+1})=4i+2, f(v_{i+1}v_i)=4i+4$ for $i=1,2,\cdots,m-1.$ Clearly, f is a bijection. Now, $f^*(xu_1)=\left\lceil\frac{f(x)+f(u_1)+f(xu_1)}{3}\right\rceil=\left\lceil\frac{1+3+2}{3}\right\rceil=2,$ $f^*(v_1x)=\left\lceil\frac{f(v_1)+f(x)+f(v_1x)}{3}\right\rceil=\left\lceil\frac{5+1+4}{3}\right\rceil=4,$ $f^*(u_mv_m)=\left\lceil\frac{f(u_m)+f(v_m)+f(u_mv_m)}{3}\right\rceil=\left\lceil\frac{(4m-1)+(4m+1)+(4m+2)}{3}\right\rceil=\left\lceil\frac{12m+2}{3}\right\rceil=4m+1,$ $f^*(u_iu_{i+1})=\left\lceil\frac{f(u_i)+f(u_{i+1})+f(u_iu_{i+1})}{3}\right\rceil=\left\lceil\frac{4i-1+4(i+1)-1+4i+2}{3}\right\rceil=\left\lceil\frac{12i+4}{3}\right\rceil=4i+2$ and $f^*(v_{i+1}v_i)=\left\lceil\frac{f(v_{i+1})+f(v_i)+f(v_{i+1}v_i)}{3}\right\rceil=\left\lceil\frac{4(i+1)+1+4i+1+4i+4}{3}\right\rceil=\left\lceil\frac{12i+10}{3}\right\rceil=4i+4$ for $i=1,2,\cdots,m-1,$ are distinct.

Case (ii): When n is even

Let $n=2m$ and let $C_n=(x,u_1,u_2,\cdots,u_m,v_{m-1},v_{m-2},\cdots,v_1,x)$. Define $f:V\cup E\to\{1,2,\cdots,4m\}$ by $f(x)=1, f(u_i)=4i-1$ for $i=1,2,\cdots,m,$ $f(v_i)=4i+1$ for $i=1,2,\cdots,m-1,$ $f(xu_1)=2,$ $f(v_1x)=4, f(u_mv_{m-1})=4m,$ $f(u_iu_{i+1})=4i+2$ for $i=1,2,\cdots,m-1$ and $f(v_{i+1}v_i)=4i+4$ for $i=1,2,\cdots,m-2.$ Clearly f is a bijection. Now, $f^*(xu_1)=\left\lceil\frac{f(x)+f(u_1)+f(xu_1)}{3}\right\rceil=\left\lceil\frac{1+3+2}{3}\right\rceil=2,$ $f^*(v_1x)=\left\lceil\frac{f(v_1)+f(x)+f(v_1x)}{3}\right\rceil=\left\lceil\frac{5+1+4}{3}\right\rceil=4,$ $f^*(u_mv_{m-1})=\left\lceil\frac{f(u_m)+f(v_{m-1})+f(u_mv_{m-1})}{3}\right\rceil=\left\lceil\frac{(4m-1)+4(m-1)+1+4m}{3}\right\rceil=\left\lceil\frac{12m-4}{3}\right\rceil=4m-1,$ $f^*(u_iu_{i+1})=\left\lceil\frac{f(u_i)+f(u_{i+1})+f(u_iu_{i+1})}{3}\right\rceil=\left\lceil\frac{4i-1+4(i+1)-1+4i+2}{3}\right\rceil=\left\lceil\frac{12i+4}{3}\right\rceil=4i+2$ for $i=1,2,\cdots,m-1,$ $f^*(v_{i+1}v_i)=\left\lceil\frac{f(v_{i+1})+f(v_i)+f(v_{i+1}v_i)}{3}\right\rceil=\left\lceil\frac{4(i+1)+1+4i+1+4i+4}{3}\right\rceil=\left\lceil\frac{12i+10}{3}\right\rceil=4i+4$ for $i=1,2,\cdots,m-2,$ are distinct.

Hence C_n is a total mean labeling graph.

Theorem 2.8: The Bistar $B_{m,n}$ is a total mean labeling graph.

Proof: Let $V(B_{m,n})=\{u,v\}\cup\{u_i:1\le i\le m\}\cup\{v_i:1\le i\le n\}$ and $E(B_{m,n})=\{uv\}\cup\{uu_i:1\le i\le m\}\cup\{vv_i:1\le i\le n\}$. Define $f:V\cup E\to\{1,2,\cdots,2(m+n)+3\}$ by $f(u)=2(m+n+1),$ $f(v)=2m+2n+3,$ $f(u_i)=2i$ for $i=1,2,\cdots,m,$ $f(v_i)=2m+2i$ for $i=1,2,\cdots,n,$ $f(uv)=2m+2n+1,$ $f(uu_i)=2i-1$ for $i=1,2,\cdots,m$ and $f(vv_i)=2m+2i-1$ for $i=1,2,\cdots,n.$ Clearly f is a bijection. Now, $f^*(uv)=\left\lceil\frac{f(u)+f(v)+f(uv)}{3}\right\rceil=\left\lceil\frac{2m+2n+2+2m+2n+3+2m+2n+11}{3}\right\rceil=\left\lceil\frac{6m+6n+6}{3}\right\rceil=2(m+n+1),$ $f^*(uu_i)=\left\lceil\frac{f(u)+f(u_i)+f(uu_i)}{3}\right\rceil=\left\lceil\frac{2m+2n+2+2i+2i-1}{3}\right\rceil=\left\lceil\frac{2m+2n+4i+1}{3}\right\rceil$ for $i=1,2,\cdots,m$ and $f^*(vv_i)=\left\lceil\frac{f(v)+f(v_i)+f(vv_i)}{3}\right\rceil=\left\lceil\frac{2m+2n+3+2m+2i+2m+2i-1}{3}\right\rceil=\left\lceil\frac{6m+2n+4i+2}{3}\right\rceil$ for $i=1,2,\cdots,n,$ are distinct. Hence $B_{m,n}$ is a total mean labeling graph.

Theorem 2.9: The triangular snakes T_n is a total mean labeling graph.

Proof: Let $V(T_n)=\{u_i:1\le i\le n-1\}\cup\{v_i:1\le i\le n\}$ and $E(T_n)=\{u_iv_i:1\le i\le n-1\}\cup\{u_iv_{i+1}:1\le i\le n-1\}\cup\{v_iv_{i+1}:1\le i\le n-1\}$. Define $f:V\cup E\to\{1,2,\cdots,5n-4\}$ by $f(v_i)=4n+i$ for $i=1,2,\cdots,n$ and $f(u_i)=2i, f(u_iv_i)=4i-3,$ $f(u_iv_{i+1})=4i-1, f(v_iv_{i+1})=2n+2i$ for $i=1,2,\cdots,n-1.$ Clearly, f is a bijection. Now, $f^*(u_iv_i)=\left\lceil\frac{f(u_i)+f(v_i)+f(u_iv_i)}{3}\right\rceil=$

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$\left\lceil \frac{2i+4n+i+4i-3}{3} \right\rceil = \left\lceil \frac{3(n+2i-1)+n+i}{3} \right\rceil = n+2i-1+\left\lceil \frac{n+i}{3} \right\rceil$, $f^*(u_iv_{i+1}) = \left\lceil \frac{f(u_i)+f(v_{i+1})+f(u_iv_{i+1})}{3} \right\rceil =$ $\left\lceil \frac{2i+4n+i+1+4i-1}{3} \right\rceil = \left\lceil \frac{3(n+2i)+n+i}{3} \right\rceil = n+2i+\left\lceil \frac{n+i}{3} \right\rceil$ and $f^*(v_iv_{i+1}) = \left\lceil \frac{f(v_i)+f(v_{i+1})+f(v_iv_{i+1})}{3} \right\rceil =$ $\left\lceil \frac{4n+i+4n+i+1+2n+2i}{3} \right\rceil = \left\lceil \frac{3(3n+i)+n+i+1}{3} \right\rceil =$ $3n+i+\left\lceil \frac{n+i+1}{3} \right\rceil$ for $i=1,2,\cdots,n-1$, are distinct. Hence T_n is a total mean labeling graph.

Theorem 2.10: The alternate triangular snake $A(T_{2n})$ (Fig. 1) is a total mean labeling graph.

Proof: Let $V(A(T_{2n})) = \{u_i : 1 \le i \le n\} \cup \{v_i : 1 \le i \le n\} \cup \{w_i : 1 \le i \le n\}$ and $E(A(T_{2n})) = \{u_iv_i : 1 \le i \le n\} \cup \{u_iw_i : 1 \le i \le n\} \cup \{v_iw_i : 1 \le i \le n\} \cup \{w_iv_{i+1} : 1 \le i \le n-1\}$. Define $f : V \cup E \to \{1,2,\cdots,7n-1\}$ by $f(u_i) = 6i-4$, $f(v_i) = 6i-2$, $f(w_i) = 6i$ for $i=1,2,\cdots,n$, $f(u_iv_i) = 8i-7$, $f(u_iw_i) = 8i-5$, $f(v_iw_i) = 8i-3$, $f(w_iv_{i+1}) = 8i-1$ for $i=1,2,\cdots,n-1$, $f(u_nv_n) = 6n$, $f(u_nw_n) = 6n+1$, $f(v_nw_n) = 6n+2$. Clearly, f is a bijection. Now, $f^*(u_iv_i) = \left\lceil \frac{f(u_i)+f(v_i)+f(u_iv_i)}{3} \right\rceil =$ $\left\lceil \frac{6i-4+6i-2+8i-7}{3} \right\rceil = \left\lceil \frac{20i-13}{3} \right\rceil$, $f^*(u_iw_i) =$ $\left\lceil \frac{f(u_i)+f(w_i)+f(u_iw_i)}{3} \right\rceil = \left\lceil \frac{6i-4+6i+8i-5}{3} \right\rceil = \left\lceil \frac{20i-9}{3} \right\rceil$, $f^*(v_iw_i) = \left\lceil \frac{f(v_i)+f(w_i)+f(v_iw_i)}{3} \right\rceil = \left\lceil \frac{6i-2+6i+8i-3}{3} \right\rceil =$ $\left\lceil \frac{20i-5}{3} \right\rceil$, $f^*(w_iv_{i+1}) = \left\lceil \frac{f(w_i)+f(v_{i+1})+f(w_iv_{i+1})}{3} \right\rceil =$ $\left\lceil \frac{6i+6(i+1)-2+8i-1}{3} \right\rceil = \left\lceil \frac{20i+3}{3} \right\rceil$ for $i=1,2,\cdots,n-1$, $f^*(u_nv_n) = \left\lceil \frac{f(u_n)+f(v_n)+f(u_nv_n)}{3} \right\rceil = \left\lceil \frac{6n-4+6n-2+6n}{3} \right\rceil =$ $\left\lceil \frac{18n-6}{5} \right\rceil = 6n-2$, $f^*(u_nw_n) = \left\lceil \frac{f(u_n)+f(w_n)+f(u_nw_n)}{3} \right\rceil =$ $\left\lceil \frac{6n-4+6n+6n+1}{3} \right\rceil = \left\lceil \frac{18n-3}{3} \right\rceil = 6n-1$ and $f^*(v_nw_n) =$ $\left\lceil \frac{f(v_n)+f(w_n)+f(v_nw_n)}{3} \right\rceil = \left\lceil \frac{6n-2+6n+6n+2}{3} \right\rceil = \left\lceil \frac{18n}{3} \right\rceil = 6n$ are distinct. Hence $A(T_{2n})$ is a total mean labeling graph.

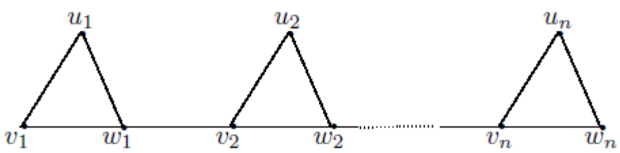

Fig. 1. Alternate Triangular Snake Graph $A(T_{2n})$

III. CONCLUSION

In this paper, we have introduce the notion of total mean labeling graph and determined several families of total mean labeling graphs. Characterizing the class of graphs which are not total mean labeling is under study.

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