Soret and Dufour Effects on the Flow of Spilled Oil in the Soil

S. V. Hemalatha, Nirmala P. Ratchagar

***Abstract:** Study about the movement of oil flow describing the transport of hydrocarbons dissolved in the water on soil are considered including the effects of viscous and Darcy dissipation, energy flux caused by both temperature gradient and concentration gradient on the unsteady convective heat and mass transfer in the subsurface. The resulting fluid flow, heat and mass transfer processes occurring in the subsurface described by coupled non-linear system of equations are simplified and solved using perturbation technique. Numerical results obtained are graphically portrayed and the importance of the parameters are given by its impact on the result.*

***Keywords :** Chemical reaction, Concentration of hydrocarbons, Perturbation technique, Retardation.*

I. INTRODUCTION

Thermal and mass diffusion have wide applications in geophysics. It arises due to the interaction of the gravitational force and density differences and hence plays a vital role in a number of engineering and technological systems.

The relation between the fluxes when heat and mass transfer occur simultaneously in a moving fluid is of great importance. Here, we observe that an energy flux is generated by both temperature and concentration gradients which are termed as Soret and Dufour effect, respectively. In general the study on these effects are considered as second order phenomena and hence neglected. But they may become significant in areas such as hydrology, petrology, geosciences, etc.

Many investigators[1,2,3,4,5] have studied and reported results on Soret and Dufour effects due to its importance for the fluids with very light/medium molecular weight. Lakshmi Narayana and Murthy [6] studied these effects on free convection heat and mass transfer from a horizontal flat plate in a Darcy porous medium. Sharma et al. [7] studied the same effect on flow past a vertical porous plate in a porous medium with heat source and chemical reaction and radiation effect. Recently, these effects are discussed by Moorthy et al. [8] on flow past a semi-infinite horizontal plate under the influence of variable viscosity. Gbadeyan et al. [9] studied the problem of coupled heat and mass transfer by free convection of a chemically reacting viscous incompressible and electrically conducting fluid confined in a vertical channel bounded by wavy and flat wall in the presence of Soret and Dufour effects and internal heat source or sink.

Motivated by these studies, a mathematical model has been developed to study about the movement of oil flow describing the transport of hydrocarbons dissolved in the water on soil bounded by porous layers. Thermal and concentration buoyancy effects are included. The viscous and Darcy dissipations are considered in the energy equation. The hydrocarbon concentration including homogeneous first order chemical reaction is discussed in the species equation along with the energy flux caused by both temperature gradient and concentration gradient. The influence of the density variation with temperature and concentration in the body force term are considered to be variables, while all other physical properties are assumed to be constants.

The initial and boundary conditions are formulated in an appropriate manner with the slip phenomena at the boundaries and the Beavers and Joseph [10] interface condition defined by

$$\frac{du}{dy} = \frac{\alpha_p}{\sqrt{k_p}}(u_B - Q)$$

where, u is the fluid velocity, u_B is the slip velocity, α_p is the velocity slip parameter, k_p is the permeability of the porous medium and Q is the Darcy velocity.

Sahraoui and Kaviany [11] interface condition for temperature, analogous to that of Beavers and Joseph [10] given by

$$\frac{dT}{dy} = \frac{\alpha_T}{\sqrt{k_p}}(T - T_B)$$

where, T is the temperature of the porous medium, α_T is the thermal slip parameter, $T_{\mathbf{B}}$ is the ambient temperature is applied. Perturbation technique is applied to simplify the resulting differential equations and its boundary conditions.

II. MATHEMATICAL FORMULATION

The nonlinear system of equations defined by this model includes the mass, momentum, energy and species equation. The buoyancy effects including Darcian term in the momentum equation, dissipation effects in the energy equation and homogeneous first order chemical reaction as well as Soret and Dufour in species equation are included for study. These are defined by the equations (1) to (5):

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$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \quad (1)$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\upsilon\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)+g\beta_T(T-T_1)+g\beta_C(C-C_1)-\frac{\upsilon}{k_p}u \quad (2)$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=\upsilon\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)-\frac{\upsilon}{k_p}v \quad (3)$$

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k_T}{\rho C_p}\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)+\frac{D_m k_T}{C_p C_s}\left(\frac{\partial^2 C}{\partial x^2}+\frac{\partial^2 C}{\partial y^2}\right)+\frac{\upsilon}{C_p}\left(2\left(\frac{\partial u}{\partial x}\right)^2+2\left(\frac{\partial v}{\partial y}\right)^2+\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)^2\right)+\frac{\upsilon}{C_p k_p}(u^2+v^2) \quad (4)$$

$$\rho_b\frac{\partial S}{\partial t}+\beta_w\frac{\partial C}{\partial t}+\beta_w u\frac{\partial C}{\partial x}+\beta_w v\frac{\partial C}{\partial y}=\beta_w D_m\left(\frac{\partial^2 C}{\partial x^2}+\frac{\partial^2 C}{\partial y^2}\right)+\frac{D_m k_T}{T_m}\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)-\beta_w k_1(C-C_1) \quad (5)$$

where, u and v are the components of velocities along x and y directions, respectively, T is the temperature distribution in the region, C is the concentration of hydrocarbons in water, S is the concentration of adsorbed oil in soil, υ is the kinematic viscosity of oil, g is the gravitational acceleration, β_t and β_c are the thermal and concentration expansion coefficients, respectively, T_1 and C_1 are the temperature and concentration at the upper surface, respectively, ρ is the density of oil, C_p is the specific heat at constant pressure, k_T is the thermal conductivity, ρ_b is the soil bulk density, β_w is the volumetric water content of soil, D_m is the mass diffusivity, C_s is the concentration susceptibility, T_m is the mean temperature and k_1 is the chemical reaction rate parameter.

The retardation factor $R=1+\frac{\rho_b k_d}{\beta_w}$, due to linear sorption process, reduces equation (5) to

$$R\frac{\partial C}{\partial t}+u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_m\left(\frac{\partial^2 C}{\partial x^2}+\frac{\partial^2 C}{\partial y^2}\right)+\frac{D_m k_T}{T_m\beta_w}\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)-k_1(C-C_1) \quad (6)$$

where, $S=k_d C$, k_d is the adsorption coefficient.

The appropriate boundary conditions for the velocity, temperature and concentration fields are:

$$\left.\begin{array}{l} u=0,\ v=0,\ T=T_0,\ C=C_0\ \forall\ y,\ t\le 0 \\ \frac{\partial u}{\partial y}=-\frac{\alpha_p}{\sqrt{k_p}}(u-Q),\ v=0,\ \frac{\partial T}{\partial y}=-\frac{\alpha_T}{\sqrt{k_p}}(T-T_0),\ C=C_0\ at\ y=0,\ t>0 \\ \frac{\partial u}{\partial y}=\frac{\alpha_p}{\sqrt{k_p}}(u-Q),\ v=-\varepsilon e^{nt+\lambda x}u_0,\ \frac{\partial T}{\partial y}=\frac{\alpha_T}{\sqrt{k_p}}(T-T_1),\ C=C_1 at\ y=h,\ t>0 \end{array}\right\} \quad (7)$$

where, T_0 and C_0 are the temperature and concentration at the lower surface, respectively, u_0, n and λ are constants and ε is the perturbation parameter(<1). Here the fluid velocity is considered to be the slip velocity.

With the following non-dimensional quantities:

$$u^*=\frac{u}{u_0},\ v^*=\frac{v}{u_0},\ t^*=\frac{tu_0{}^2}{\upsilon},\ x^*=\frac{xu_0}{\upsilon},\ y^*=\frac{yu_0}{\upsilon},$$

$$\theta=\frac{T-T_1}{T_0-T_1},\ \phi=\frac{C-C_1}{C_0-C_1},\ Q^*=\frac{Q}{u_0}$$

where, θ and ϕ are dimensionless temperature and concentration functions respectively, the set of equations (1) to (4) and (6) reduces to the following non-dimensional equations by neglecting the '*' symbol.

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \quad (8)$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+Gr\theta+Gc\phi-\sigma^2 u \quad (9)$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}-\sigma^2 v \quad (10)$$

$$\frac{\partial \theta}{\partial t}+u\frac{\partial \theta}{\partial x}+v\frac{\partial \theta}{\partial y}=\frac{1}{\Pr}\left(\frac{\partial^2 \theta}{\partial x^2}+\frac{\partial^2 \theta}{\partial y^2}\right)+Du\left(\frac{\partial^2 \phi}{\partial x^2}+\frac{\partial^2 \phi}{\partial y^2}\right)+Ec\left(2\left(\frac{\partial u}{\partial x}\right)^2+2\left(\frac{\partial v}{\partial y}\right)^2+\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)^2\right)+Ec\sigma^2(u^2+v^2) \quad (11)$$

$$R\frac{\partial \phi}{\partial t}+u\frac{\partial \phi}{\partial x}+v\frac{\partial \phi}{\partial y}=\frac{1}{Sc}\left(\frac{\partial^2 \phi}{\partial x^2}+\frac{\partial^2 \phi}{\partial y^2}\right)+So\left(\frac{\partial^2 \theta}{\partial x^2}+\frac{\partial^2 \theta}{\partial y^2}\right)-K\phi \quad (12)$$

where,

$Gr=\frac{\upsilon g\beta_T(T_0-T_1)}{u_0{}^3}$ is the thermal Grashof number,

$Gc=\frac{\upsilon g\beta_C(C_0-C_1)}{u_0{}^3}$ is the species Grashof number,

$\sigma=\frac{\upsilon}{u_0\sqrt{k_p}}$ is the porous parameter,

$Ec=\frac{u_0{}^2}{(T_0-T_1)C_p}$ is the Eckert number,

$\Pr=\frac{\rho C_p\upsilon}{k_T}$ is the Prandtl number,

$Du=\frac{D_m k_T}{C_p C_s\upsilon}\left(\frac{C_0-C_1}{T_0-T_1}\right)$ is the Dufour number,

$So=\frac{D_m k_T}{T_m\beta_w\upsilon}\left(\frac{T_0-T_1}{C_0-C_1}\right)$ is the Soret number,

$Sc=\frac{\upsilon}{D_m}$ is the Schmidt number and

$K = \frac{k_1 \upsilon}{u_0^{\,2}}$ is the dimensionless chemical reaction rate parameter.

The boundary conditions (7) in non-dimensional form are:

$$\left.\begin{array}{l} u=0,\, v=0,\, \theta=1\ ,\, \phi=1\ \forall\, y,\, t\le 0 \\ \frac{\partial u}{\partial y}=-\alpha_p\sigma(u-Q),\, v=0,\, \frac{\partial\theta}{\partial y}=-\alpha_T\sigma(\theta-1),\, \phi=1\ at\ y=0,\, t>0 \\ \frac{\partial u}{\partial y}=\alpha_p\sigma(u-Q),\, v=-\varepsilon\, e^{nt+\lambda x}\ ,\, \frac{\partial\theta}{\partial y}=\alpha_T\sigma\theta,\, \phi=0\ \ at\ y=h,\, t>0 \end{array}\right\} \quad (13)$$

III. METHOD OF SOLUTION

The system of nonlinear partial differential equations (9) to (12) defined for oil flow, energy and concentration are reduced to ordinary differential equations using:

$$\left.\begin{array}{l} u(x,y,t)=u_0(y)+\varepsilon\, e^{nt+\lambda x}\, u_1(y)+O(\varepsilon^2) \\ v(x,y,t)=\varepsilon\, e^{nt+\lambda x}\, v_1(y)+O(\varepsilon^2) \\ \theta(x,y,t)=\theta_0(y)+\varepsilon\, e^{nt+\lambda x}\, \theta_1(y)+O(\varepsilon^2) \\ \phi(x,y,t)=\phi_0(y)+\varepsilon\, e^{nt+\lambda x}\, \phi_1(y)+O(\varepsilon^2) \end{array}\right\} \quad (14)$$

Substituting equations (14) into equations (9) to (12), neglecting the higher order of (ε^2) and equating the like terms we obtain the following set of nonlinear ordinary differential equations for u_0, θ_0, ϕ_0 and $u_1, v_1, \theta_1, \phi_1$

Zeroth order equations:

$$u_{0yy}-\sigma^2 u_0+Gr\,\theta_0+Gc\,\phi_0=0 \quad (15)$$

$$\theta_{0yy}+\Pr Du\phi_{0yy}+\Pr Ec(u_{oy})^2+\Pr Ec\sigma^2 u_0^{\,2}=0 \quad (16)$$

$$\phi_{0yy}-Sc\,K\phi_0+Sc\,So\,\theta_{0yy}=0 \quad (17)$$

First order equations:

$$u_{1yy}+(\lambda^2-\sigma^2-\lambda u_0-n)u_1-v_1u_{0y}+Gr\,\theta_1+Gc\,\phi_1=0 \quad (18)$$

$$v_{1yy}+(\lambda^2-\sigma^2-\lambda u_0-n)v_1=0 \quad (19)$$

$$\theta_{1yy}+(\lambda^2-\Pr(\lambda u_0+n))\theta_1+2Ec\Pr(u_{0y}u_{1y}+\lambda u_{oy}v_1 +\sigma^2 u_0u_1)+Du\Pr(\phi_{1yy}+\lambda^2\phi_1)-\Pr v_1\theta_{0y}=0 \quad (20)$$

$$\phi_{1yy}+(\lambda^2-Sc(\lambda u_0+nR+K))\phi_1-Sc\,v_1\phi_{0y}) +ScSo(\theta_{1yy}+\lambda^2\theta_1)=0 \quad (21)$$

subject to the boundary conditions,

$$\left.\begin{array}{l} u_{0y}=-\alpha_p\sigma(u_0-Q),\ \ \theta_{0y}=-\alpha_T\sigma(\theta_0-1),\, \phi_0=1\ at\ \ y=0 \\ u_{0y}=\alpha_p\sigma(u_0-Q),\ \ \theta_{0y}=\alpha_T\sigma\theta_0,\, \phi_0=0\ \ at\ \ y=1 \end{array}\right\} \quad (22)$$

$$\left.\begin{array}{l} u_{1y}=-\alpha_p\sigma u_1, v_1=0,\ \ \theta_{1y}=-\alpha_T\,\sigma\,\theta_1,\, \phi_1=0\ at\ \ y=0 \\ u_{1y}=\alpha_p\sigma u_1,\ v_1=-1,\ \theta_{1y}=\alpha_T\,\sigma\,\theta_1,\, \phi_1=0\ at\ \ y=1 \end{array}\right\} \quad (23)$$

All the characteristics of oil flow are obtained using the coupled equations (15)-(21) and the boundary conditions (22) and (23).

IV. RESULTS AND DISCUSSION

Numerical solution obtained for oil flow, thermal and concentration effects are discussed for variations in the pertinent parameters.

Figs. 1 and 2 portrays the axial velocity(u) profile for different Grashof numbers and porous parameter, respectively. They indicate that increasing Grashof number enhances velocity profile while an increase in porous parameter retards the velocity profile. These opposing nature of parameters are due to the advancement in the buoyancy ratio that tends to accelerate the fluid flow and the frictional drag resistance against the flow that decelerates the fluid flow in the porous region, respectively. But in the case for the transverse directional flow(Fig. 3), the porous parameter enhances the vertical velocity(v).

Figs. 4 to 7 show the influence of Prandtl number, Dufour number, Eckert number and porous parameter on the dimensionless temperature profile at time t = 0.5. The influence of Prandtl number and Dufour number on the temperature profile are observed through Figs. 4 and 5, respectively. They depict that boosting these parameters reduce the temperature. But this gradual reduction in temperature happens only at the end y=0 and finds no difference at the other end. Figs. 6 and 7 display the effect of Eckert number and porous parameter on the temperature distribution, respectively. From these figures, temperatures are seen to fall with rise in parameter values at the lower surface but rises at the upper surface. The predicted results clearly indicate that the presence of pertinent parameters damps the heat transfer process significantly.

The effects of chemical reaction rate parameter, Schmidt number, Soret number, porous parameter and retardation factor for hydrocarbon concentration distribution on both spatial and temporal variations are analyzed through the Figs. 8 to 12. The Figs. 8, 9 and 10 display the effect of the chemical reaction rate parameter, Schmidt number and Soret number on the concentration profile, respectively which reveals that the hydrocarbon concentration reduces due to a rise in all these parameters, causing the concentration buoyancy to decrease yielding a reduction in the fluid velocity. From Fig. 11 we observe that porous parameter improves the hydrocarbon concentration. Fig. 12 shows that the hydrocarbon concentration increases in water along with time for different retardation factor level with retarding effect.

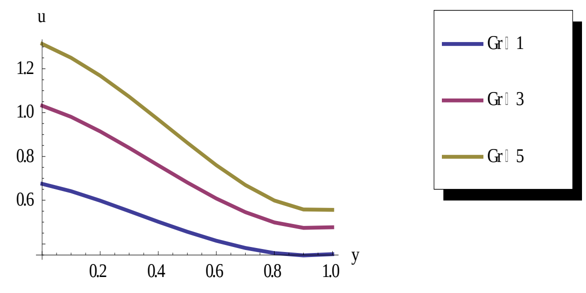

Fig. 1. Effect of Grashof number on u velocity

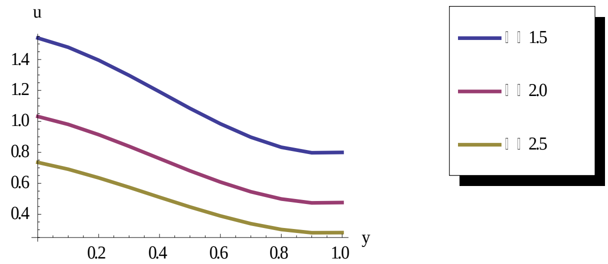

Fig. 2. Effect of porous parameter on u velocity

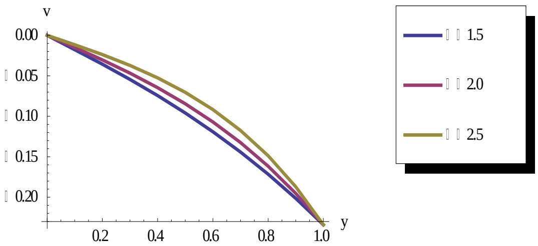

Fig. 3. Effect of porous parameter on v velocity

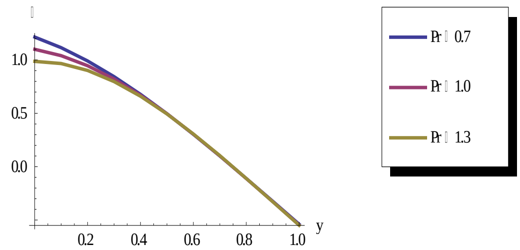

Fig. 4. Effect of Prandtl number on temperature distribution

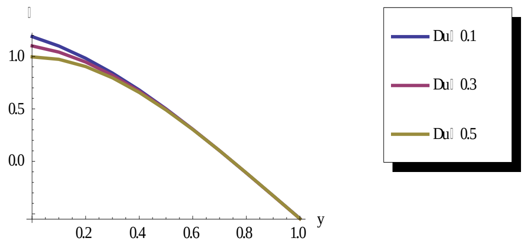

Fig. 5. Effect of Dufour number on temperature distribution

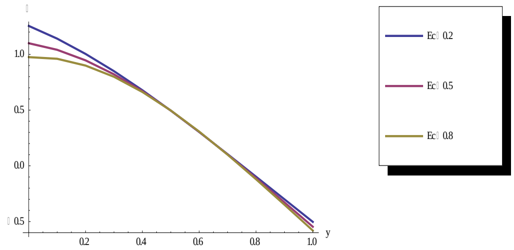

Fig. 6. Effect of Eckert number on temperature distribution

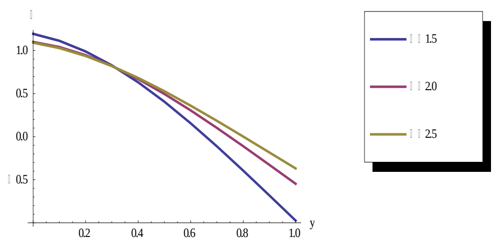

Fig. 7. Effect of porous parameter on temperature distribution

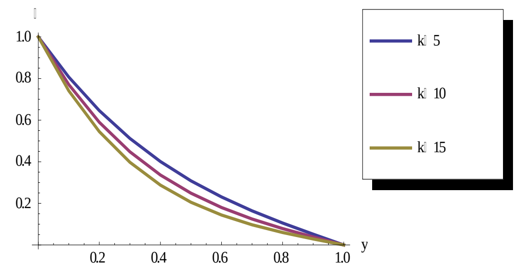

Fig. 8. Effect of chemical reaction parameter on concentration distribution

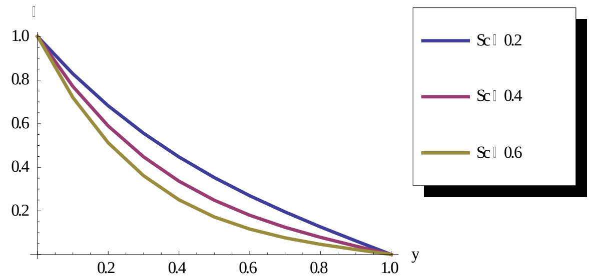

Fig. 9. Effect of Schmidt number on concentration distribution

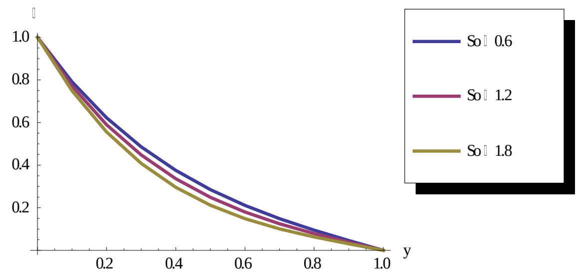

Fig. 10. Effect of Soret number on concentration distribution

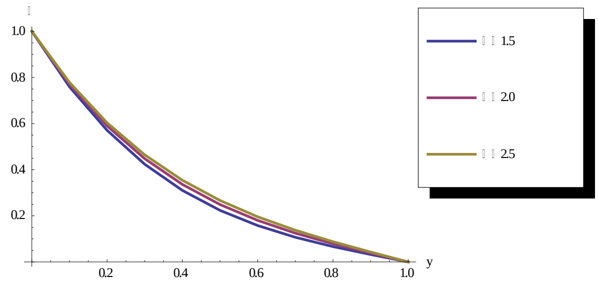

Fig. 11. Effect of porous parameter on concentration distribution

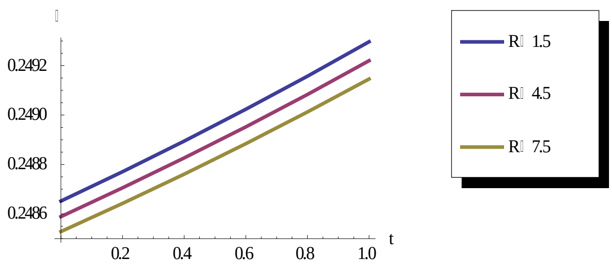

Fig. 12. Time evolution of concentration distribution for different retardation factor

V. CONCLUSION

This paper is focused on the study of transport of petroleum hydrocarbons on the soil subsurface including the effects of viscous and Darcy dissipation, energy flux caused by both temperature gradient and concentration gradient on the unsteady convective heat and mass transfer. The governing equations of the model are highlighted and non-dimensionalized. The mathematical equations thus obtained are simplified using perturbation technique and then solved. Series of computation was carried out to study the effects of all the pertinent parameters. The discussion was made using graphical representation related to the characteristics such as velocity, temperature and concentration.

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