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Some Expansion of Fuzzy Paranormal Operators

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Abstract: *Let $\mathbb{H}$ be a Fuzzy Hilbert space over the fields of $\mathbb{R}/\mathbb{C}$ and FB($\mathbb{H}$) is the set of all fuzzy continuous linear operator on $\mathbb{H}$ .In this paper we introduce the expansion of different fuzzy paranormal operators like n- fuzzy paranormal operator, *- fuzzy paranormal operator and n^th^ -fuzzy paranormal operator, which all are developed from paranormal operators and their characteristics. The study resulted the properties of an n- fuzzy paranormal operator, * fuzzy paranormal operator and n^{th} -fuzzy paranormal operator and their relationship between them. To investigate the nature of these operators, all it needs the nature of the n- fuzzy paranormal operator.*

Keywords: Adjoint Fuzzy operator, Fuzzy Hilbert space (FH-space), Self adjoint fuzzy operator, Fuzzy paranormal operator, n - fuzzy paranormal operator, * - fuzzy paranormal operator and n^{th} -fuzzy paranormal operator.

I.INTRODUCTION

The notion of fuzzy norm on a linear space is first introduced by Katsaras[5]. In 1991, the definition of fuzzy inner product space is first introduced by Biswas[7]. Riesz theorem was given by Youngfsu[9] using fuzzy concept in 2007. In 2009 Goudarzi and Vaezpour [6] have been introduced the definition of fuzzy Hilbert space. They introduced triplet ($\mathbb{H}$, $\mathbb{F}$, *), where $\mathbb{H}$ is the Fuzzy Hilbert space, $\mathbb{F}$ is the Fuzzy set on $\mathbb{H}^2 \times \mathbb{R}$, and * is continuous t - norm. Sudad M.Rasheed [4] introduced the concept of adjoint fuzzy operator. That is $<\mathbb{U}a, b> = < a, \mathbb{U}^*b >$. Also self adjoint fuzzy operator if $\mathbb{U} = \mathbb{U}^*$ where $\mathbb{U}^*$ is adjoint fuzzy operator of $\mathbb{U}$. Radharamani et al.[1] introduced fuzzy normal operator if $\mathbb{U}\mathbb{U}^* = \mathbb{U}^*\mathbb{U}$ in 2018. And also fuzzy unitary operator and fuzzy hyponormal operator was introduced. An operator $\mathbb{U} \in FB(\mathbb{H})$ which has the property of $\|\mathbb{U}a\|^2 \leq \|\mathbb{U}^2 a\|$ for every unit vector a$\in \mathbb{H}$ is named as fuzzy paranormal operator and introduced by Radharamani et al.[3] in 2019. Then the fuzzy paranormal operator definition is expanded into positive integer n, $\|\mathbb{U}^{2n}a\| \geq \|\mathbb{U}^n a\|^2$ for some unit vector a$\in \mathbb{H}$.

In this paper, we introduce an operator $\mathbb{U}$ with the property of $\|\mathbb{U}^{2n}a\| \geq \|\mathbb{U}^n a\|^2$ is called n^{th} -fuzzy paranormal operator and we explained some characteristics of n^{th} -fuzzy paranormal operator. Before that requires to defining and explaining the theory of n- fuzzy paranormal operator. We give different properties of n -fuzzy paranormal operator and also give important result about it. An operator $\mathbb{U} \in FB(\mathbb{H})$ with the characteristics of $\|\mathbb{U}^2 a\|\|a\| \geq \|\mathbb{U}^* a\|^2$ is said to be * - fuzzy paranormal operator with $\mathbb{U}^*$ is Adjoint fuzzy operator of $\mathbb{U}$.

In this study, the consolation of the n^{th}- Fuzzy paranormal operator and * - fuzzy paranormal operator and is more emphasized on the characteristics and relationships of the n^{th} -Fuzzy paranormal operator and * - fuzzy paranormal operator on FH- space. Therefore it necessary to study more about the properties of the n^{th} -Fuzzy paranormal operator and * - fuzzy paranormal operator on FH- space.

II.PRELIMINARIES

Definition 2.1: [4] Fuzzy Hilbert space (FH-space)

Let ($\mathbb{H}$,$\mathbb{F}$,*) be a fuzzy Hilbert space with IP: <a,b>=sup{ u $\in \mathbb{R}$; $\mathbb{F}(a, b, u) < 1$} $\forall a, b \in \mathbb{H}$ .If $\mathbb{H}$ is complete in the $\|.\|$,then $\mathbb{H}$ is called Fuzzy Hilbert space(FH-space).

Definition 2.2: [4] Adjoint Fuzzy operator

Let ($\mathbb{H}$, $\mathbb{F}$,*) be a Fuzzy Hilbert space and let $\mathbb{U} \in FB(\mathbb{H})$ be $\mathcal{T}_F$ continuous linear functional.Then $\exists$ unique $\mathbb{U}^* \in FB(\mathbb{H})$ such that $\langle \mathbb{U}a, b\rangle = \langle a, \mathbb{U}^* b\rangle \forall$ a, b $\in \mathbb{H}$.

Note:

Let $FB(\mathbb{H})$ *be* the set of all fuzzy continuous linear operator on $\mathbb{H}$.

Definition2.3: [4] Self-Adjoint Fuzzy operator

Let ($\mathbb{H}$,$\mathbb{F}$,*) be a FH – space with IP: $\langle a, b\rangle = Sup\{u \in \mathbb{R}: \mathbb{F}a,b,u<1 \ \forall\ a,b \in \mathbb{H}$ and let $\mathbb{U} \in FB\mathbb{H}$Then $\mathbb{U}$ is self-adjoint Fuzzy operator, if $\mathbb{U} = \mathbb{U}^*$, where $\mathbb{U}^*$ is adjoint Fuzzy operator of $\mathbb{T}$.

Theorem 2.4: [4]

Let ($\mathbb{H}$,$\mathbb{F}$,*) be a Fuzzy Hilbert space with IP: $\langle a, b\rangle = Sup\,\{u \in \mathbb{R}: \mathbb{F}(a, b, u) < 1\} \,\forall\, a, b \in \mathbb{H}$. Let $\mathbb{U} \in FB(\mathbb{H})$ *then* $\|\mathbb{U}a\| = \|\mathbb{U}^*a\| \,\forall\, a, b \in \mathbb{H}$.

Theorem 2.5: [4]

Let ($\mathbb{H}$,$\mathbb{F}$,*) be a FH – space with IP: $\langle a, b\rangle = Sup\,\{u \in \mathbb{R}: \mathbb{F}a,b,u<1 \ \forall\ a,b \in \mathbb{H}$ and let $\mathbb{U} \in FB\mathbb{H}$ then $\mathbb{U}$ is self –adjoint Fuzzy operator.

Definition 2.6: [2] Fuzzy Unitary operator

Let ($\mathbb{H}$,$\mathbb{F}$,*) be a Fuzzy Hilbert space with IP: $\langle a, b\rangle = Sup\,\{u \in \mathbb{R}: \mathbb{F}(a, b, u) < 1\} \,\forall\, a, b \in \mathbb{H}$ and let $\mathbb{U} \in FB(\mathbb{H})$. Then $\mathbb{U}$ is a fuzzy unitary operator if it satisfies $\mathbb{U}\mathbb{U}^* = I = \mathbb{U}^*\mathbb{U}$.

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Definition 2.7: [1] Fuzzy Normal Operator

Let $(\mathbb{H},\mathbb{F},*)$ be a Fuzzy Hilbert space with IP: $\langle a,b\rangle = Sup\,\{u \in \mathbb{R}: \mathbb{F}(a,b,u) < 1\}\ \forall\, a,b \in \mathbb{H}$ and let $\mathbb{U} \in FB(\mathbb{H})$. Then $\mathbb{U}$ is a fuzzy normal operator if it commutes with its adjoint fuzzy operator of $\mathbb{U}$ .i.e $\mathbb{U}\mathbb{U}^* = \mathbb{U}^*\mathbb{U}$.

Definition 2.8: [4] Fuzzy paranormal operator

Let $(\mathbb{H},\mathbb{F},*)$ be a Fuzzy Hilbert space with IP: $\langle a,b\rangle = Sup\,\{u \in \mathbb{R}: \mathbb{F}(a,b,u) < 1\}\ \forall\, a,b \in \mathbb{H}$ and let $\mathbb{U} \in FB(\mathbb{H})$Then $\mathbb{U}$ is a Fuzzy paranormal operator if $\|\mathbb{U}^2 a\|\|a\| \geq \|\mathbb{U}a\|^2\ \forall\ a \in \mathbb{H}$.

Note:

Let $\mathbb{U} \in FB(\mathbb{H})$. Then $\mathbb{U}$ is a fuzzy paranormal operator if $\|\mathbb{U}^2 a\| \geq \|\mathbb{U}a\|^2\ \forall$ unit vector $a \in \mathbb{H}$.

Theorem (2.9):[4]

Let $(\mathbb{H},\mathbb{F},*)$ be a FH – space with IP: $\langle a,b\rangle = Sup\,\{u \in \mathbb{R}: \mathbb{F}a,b,u<1\ \ \forall\ a,b \in \mathbb{H}$ and let $\mathbb{U}^*$ be the adjoint Fuzzy operator of $\mathbb{U} \in FB(\mathbb{H})$, then

i. $(\mathbb{U}^*)^* = \mathbb{U}$
ii. $(\alpha\,\mathbb{U})^* = \alpha\mathbb{U}^*$
iii. $(\alpha\,\mathbb{U} + \beta\mathbb{V})^* = \alpha\mathbb{U}^* + \beta\mathbb{V}^*$ where α , β are scalars and $\mathbb{U} \in FB(E)$.
iv. $(\mathbb{U}\mathbb{V})^* = \mathbb{V}^*\mathbb{U}^*$

III.MAIN RESULTS

Definition: 3.1

Let $(\mathbb{H},\mathbb{F},*)$ be a fuzzy Hilbert space with IP: <a,b>=sup{u∈ $\mathbb{R}$; $\mathbb{F}(a,b,u) < 1\}\ \forall a,b \in \mathbb{H}$ and let $\mathbb{U} \in FB(\mathbb{H})$. The opertaor $\mathbb{U}$ is called n- fuzzy paranormal operator if $\|\mathbb{U}^{n+1}a\| \geq \|\mathbb{U}a\|^{n+1}$ for n∈ N *and* for some unit vector a∈ $\mathbb{H}$

Theorem: 3.2

Let $\mathbb{U} \in FB(\mathbb{H})$. If $\mathbb{U}$ is n-fuzzy paranormal operator then $\mathbb{U}$ is fuzzy paranormal.

Proof:

Given $\mathbb{U}$ is n- fuzzy paranormal operator.
By the definition, $\|\mathbb{U}^{n+1}a\| \geq \|\mathbb{U}a\|^{n+1}$
Put n=1, $\|\mathbb{U}^{1+1}a\| \geq \|\mathbb{U}a\|^{1+1}$
$\|\mathbb{U}^{2}a\| \geq \|\mathbb{U}a\|^{2}$
Therefore $\mathbb{U}$ is fuzzy paranormal.

Theorem: 3.3

Let $\mathbb{U} \in FB(\mathbb{H})$. If $\mathbb{U}$ is fuzzy paranormal then $\mathbb{U}$ is n-fuzzy paranormal operator for n ∈ N.

Proof:

Take a∈ $\mathbb{H}$,
By using the mathematical induction, if n=1
$\|\mathbb{U}^{1+1}a\| = \|\mathbb{U}^{2}a\| \geq \|\mathbb{U}a\|^{2}$
$\geq \|\mathbb{U}a\|^{1+1}$
i.e $\|\mathbb{U}^{1+1}a\| \geq \|\mathbb{U}a\|^{1+1}$
if n=2,
$\|\mathbb{U}^{2+1}a\| = \|\mathbb{U}^{3}a\| \geq \|\mathbb{U}a\|^{3}$
$\geq \|\mathbb{U}a\|^{2+1}$
i.e $\|\mathbb{U}^{2+1}a\| \geq \|\mathbb{U}a\|^{2+1}$etc.,
if n=k is true
$\|\mathbb{U}^{k+1}a\| \geq \|\mathbb{U}a\|^{k+1}$
Now we have to prove that it is true for n=k+1,
Let $\|\mathbb{U}a\|^{k+1+1} = \|\mathbb{U}a\|^{k+1}\|\mathbb{U}a\|$
$\leq \|\mathbb{U}^{k+1}a\|\|\mathbb{U}a\|$
$\|\mathbb{U}a\|^{(k+1)+1} \leq \|\mathbb{U}^{(k+1)+1}a\|$
Hence $\mathbb{U}$ is n-fuzzy paranormal operator.

Definition: 3.4

Let $(\mathbb{H},\mathbb{F},*)$ be a fuzzy Hilbert space with IP: <a,b>=sup{ u ∈ $\mathbb{R}$; $\mathbb{F}(a,b,u) < 1\}\ \ \forall a,b \in \mathbb{H}$ and let $\mathbb{U} \in FB(\mathbb{H})$. The operator $\mathbb{U}$ is called n[th]- fuzzy paranormal operator if $\|\mathbb{U}^{2n}a\|\|a\| \geq \|\mathbb{U}^{n}a\|^2$ for n∈ N.

Note:

Let $\mathbb{U} \in FB(\mathbb{H})$ *be* a n[th]- fuzzy paranormal operator if $\|\mathbb{U}^{2n}a\| \geq \|\mathbb{U}^{n}a\|^2$, for n∈ N and ∀ unit vector $a \in \mathbb{H}$.

Theorem: 3.5

Let $\mathbb{U},\mathbb{V} \in FB(\mathbb{H})$. If $\mathbb{U},\mathbb{V}$ are Self adjoint fuzzy operators then $\mathbb{U}+\mathbb{V}$ is n[th] - fuzzy paranormal operator for n ∈ N.

Proof:

Take n∈ N for a ∈ $\mathbb{H}$ with $\|a\|$=1.
Let $\|(\mathbb{U}+\mathbb{V})^n a\|^2 = \langle(\mathbb{U}+\mathbb{V})^n a, (\mathbb{U}+\mathbb{V})^n a\rangle$
$= Sup\,\{u \in \mathbb{R}: \mathbb{F}((\mathbb{U}+\mathbb{V})^n a, (\mathbb{U}+\mathbb{V})^n a, u) < 1\}$
$= Sup\,\{u \in \mathbb{R}: \mathbb{F}(((\mathbb{U}+\mathbb{V})^n)^*\,(\mathbb{U}+\mathbb{V})^n a, a, u) < 1\}$
$= Sup\{u \in \mathbb{R}: \mathbb{F}((\mathbb{U}^* + \mathbb{V}^*)^n(\mathbb{U}+\mathbb{V})^n a, a, u) < 1\}$
$= Sup\,\{u \in \mathbb{R}: \mathbb{F}((\mathbb{U}+\mathbb{V})^n(\mathbb{U}+\mathbb{V})^n a, a, u) < 1\}$
$= \langle(\mathbb{U}+\mathbb{V})^n(\mathbb{U}+\mathbb{V})^n a, a\rangle$
$=\langle(\mathbb{U}+\mathbb{V})^{2n} a, a\rangle$
$\|(\mathbb{U}+\mathbb{V})^n a\|^2 \leq \|(\mathbb{U}+\mathbb{V})^{2n} a\|\|a\|$
Implies that $\|(\mathbb{U}+\mathbb{V})^n a\|^2 \leq \|((\mathbb{U}+\mathbb{V})^{2n} a\|$
Therefore $\mathbb{U}+\mathbb{V}$ is n[th] -fuzzy paranormal operator.

Theorem: 3.6

Let $\mathbb{U},\mathbb{V} \in FB(\mathbb{H})$. If $\mathbb{U},\mathbb{V}$ are Self adjoint fuzzy operators then $\mathbb{U}\mathbb{V}$ is n[th]- fuzzy paranormal operator for n ∈ N.

Proof:

Take n∈ N for a ∈ $\mathbb{H}$ with $\|a\|$=1.
Since $\mathbb{U}$ *and* $\mathbb{V}$ are self adjoint fuzzy operators.
We know that $\mathbb{U} = \mathbb{U}^*, \mathbb{V} = \mathbb{V}^*$.
To prove that $\mathbb{U}\mathbb{V}$is an n[th] - fuzzy paranormal operator.
Let $\|(\mathbb{U}\mathbb{V})^n\, a\|^2 = \langle(\mathbb{U}\mathbb{V})^n\, a, (\mathbb{U}\mathbb{V})^n\, a\rangle$
$= Sup\,\{u \in \mathbb{R}: \mathbb{F}((\mathbb{U}\mathbb{V})^n\, a, (\mathbb{U}\mathbb{V})^n\, a, u) < 1\}$
$= Sup\,\{u \in \mathbb{R}: \mathbb{F}(((\mathbb{U}\mathbb{V})^n\,)^*\,(\mathbb{U}\mathbb{V})^n\, a, a, u) < 1\}$
$= Sup\,\{u \in \mathbb{R}: \mathbb{F}(((\mathbb{U}\mathbb{V})^*\,)^n\,(\mathbb{U}\mathbb{V})^n\, a, a, u) < 1\}$
$= Sup\,\{u \in \mathbb{R}: \mathbb{F}((\mathbb{V}^*\,\mathbb{U}^*\,)^n\,(\mathbb{U}\mathbb{V})^n\, a, a, u) < 1\}$
$= Sup\,\{u \in \mathbb{R}: \mathbb{F}((\mathbb{V}\mathbb{U})^n\,(\mathbb{U}\mathbb{V})^n\, a, a, u) < 1\}$
$= \langle(\mathbb{U}\mathbb{V})^n\,(\mathbb{U}\mathbb{V})^n\, a, a\rangle$
$\leq \|(\mathbb{U}\mathbb{V})^{2n}\, a\|\|a\|$
$\|(\mathbb{U}\mathbb{V})^n\, a\|^2 \leq \|(\mathbb{U}\mathbb{V})^{2n}\, a\|\|a\|$
Implies that $\|(\mathbb{U}\mathbb{V})^n\, a\|^2 \leq \|(\mathbb{U}\mathbb{V})^{2n}\, a\|$
Therefore $\mathbb{U}\mathbb{V}$ is n[th]- fuzzy paranormal operator.

Theorem: 3.7

Every fuzzy paranormal operator is 1[th]-fuzzy paranormal operator.

Proof:

Take n∈ N for a ∈ $\mathbb{H}$ with $\|a\|$=1.
Since $\mathbb{U} \in FB(\mathbb{H})$ is fuzzy paranormal operator
$\|\mathbb{U}^2\, a\| \geq \|\mathbb{U}\, a\|^2$
$\|\mathbb{U}^{2.1}a\| \geq \|\mathbb{U}^1 a\|^2$
Hence $\mathbb{U}$ is 1[th] fuzzy paranormal operator.

Theorem: 3.8

Let $\mathbb{U} \in FB(\mathbb{H})$ is a Fuzzy Hilbert space and Self-adjoint fuzzy operator. If $\mathbb{U}$ is a n[th]-fuzzy paranormal operator then $\mathbb{U}^n$ is n[th]-fuzzy paranormal operator for n∈ N.

Proof:

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Take n$\in N$ for a $\in$ $\mathbb{H}$ with $\|a\|$=1.

We know that $\mathbb{U}$ is n[th]-fuzzy paranormal operator.

i.e $\|\mathbb{U}^n a\|^2 \le \|\mathbb{U}^{2n} a\|$

To show that $\|(\mathbb{U}^n)^n a\|^2 \le \|(\mathbb{U}^n)^{2n} a\|$

Let $\|(\mathbb{U}^n)^n a\|^2 = \langle(\mathbb{U}^n)^n a, (\mathbb{U}^n)^n a\rangle$

$$= Sup\,\{u \in \mathbb{R}: \mathbb{F}((\mathbb{U}^n)^n a, (\mathbb{U}^n)^n a, u) < 1\}$$
$$= Sup\,\{u \in \mathbb{R}: \mathbb{F}(((\mathbb{U}^n)^n)^*(\mathbb{U}^n)^n a, a, u) < 1\}$$
$$= Sup\,\{u \in \mathbb{R}: \mathbb{F}(((\mathbb{U}^*)^n)^n(\mathbb{U}^n)^n a, a, u) < 1\}$$
$$= Sup\,\{u \in \mathbb{R}: \mathbb{F}((\mathbb{U}^n)^n(\mathbb{U}^n)^n a, a, u) < 1\}$$
$$= Sup\{u \in \mathbb{R}: \mathbb{F}((\mathbb{U}^n)^{2n} a, a, u) < 1\}$$
$$= \langle(\mathbb{U}^n)^{2n} a, a\rangle$$
$$\|(\mathbb{U}^n)^n a\|^2 \le \|(\mathbb{U}^n)^{2n} a\|\|a\|$$

Implies that $\|(\mathbb{U}^n)^n a\|^2 \le \|(\mathbb{U}^n)^{2n} a\|$

Therefore $\mathbb{U}^n$ is n[th]- fuzzy paranormal operator.

Theorem: 3.9

An operator $\mathbb{U} \in FB(\mathbb{H})$ is n[th]-fuzzy paranormal operator if and only if $\mathbb{U}^{*^{2n}}\mathbb{U}^{2n} - 2\mathbb{U}^{*^n}\mathbb{U}^n + \lambda^2 \ge 0 \text{ for all } \lambda \ge 0, n \in N$.

Proof:

Take n$\in N$ For every a in $\mathbb{H}$ with $\|a\| = 1$

$$\mathbb{U}^{*^{2n}}\mathbb{U}^{2n} - 2\lambda\mathbb{U}^{*^n}\mathbb{U}^n + \lambda^2 \ge 0 \Leftrightarrow$$
$$\langle(\mathbb{U}^{*^{2n}}\mathbb{U}^{2n} - 2\lambda\mathbb{U}^{*^n}\mathbb{U}^n + \lambda^2)a, a\rangle \ge 0$$
$$\mathbb{U}^{*^{2n}}\mathbb{U}^{2n} - 2\lambda\mathbb{U}^{*^n}\mathbb{U}^n + \lambda^2 \ge 0 \Leftrightarrow$$
$$Sup\{u \in \mathbb{R}: \mathbb{F}\big((\mathbb{U}^{*^{2n}}\mathbb{U}^{2n} - 2\lambda\mathbb{U}^{*^n}\mathbb{U}^n + \lambda^2)a, a, u\big) < 1\} \ge 0$$
$$\mathbb{U}^{*^{2n}}\mathbb{U}^{2n} - 2\lambda\mathbb{U}^{*^n}\mathbb{U}^n + \lambda^2 \ge 0 \Leftrightarrow$$
$$Sup\,\left\{u \in \mathbb{R}: \mathbb{F}\left(\left(\mathbb{U}^{*^{2n}}\mathbb{U}^{2n}\right)a, a, u\right) < 1\right\}$$
$$-2\lambda Sup\,\left\{u \in \mathbb{R}: \mathbb{F}\left(\left(\mathbb{U}^{*^n}\mathbb{U}^n\right)a, a, u\right) < 1\right\}$$
$$+\lambda^2\, Sup\,\{u \in R: \mathbb{F}(a, a, u) < 1\} \ge 0$$
$$\mathbb{U}^{*^{2n}}\mathbb{U}^{2n} - 2\lambda\mathbb{U}^{*^n}\mathbb{U}^n + \lambda^2 \ge 0 \Leftrightarrow$$
$$Sup\,\{u \in \mathbb{R}: \mathbb{F}(\mathbb{U}^{2n} a, \mathbb{U}^{2n} a, u) < 1\}$$
$$-2\lambda Sup\,\{u \in \mathbb{R}: \mathbb{F}(\mathbb{U}^n a, \mathbb{U}^n a, u) < 1\}$$
$$+\lambda^2\, Sup\,\{u \in \mathbb{R}: \mathbb{F}(a, a, u) < 1\} \ge 0$$
$$\Leftrightarrow \langle\mathbb{U}^{2n} a, \mathbb{U}^{2n} a\rangle - 2\lambda\langle\mathbb{U}^n a, \mathbb{U}^n a\rangle + \lambda^2\langle a, a\rangle \ge 0$$
$$\Leftrightarrow \|\mathbb{U}^{2n} a\|^2 - 2\lambda\|\mathbb{U}^n a\|^2 + \lambda^2\|a\|^2 \ge 0$$

Since if a > 0, b and c are real numbers then at^2+bt+c $\ge$ 0 for every real t if and only if b^2 - 4ac $\le$ 0 in an analogous manner, using elementary property of real quadratic forms.

$$\Leftrightarrow 4\|\mathbb{U}^n a\|^4 - 4\|a\|^2\|\mathbb{U}^{2n} a\|^2 \le 0$$
$$\Leftrightarrow \|\mathbb{U}^n a\|^4 \le \|a\|^2\|\mathbb{U}^{2n} a\|^2$$
$$\Leftrightarrow \|\mathbb{U}^n a\|^2 \le \|\mathbb{U}^{2n} a\|\|a\|$$
$$\Leftrightarrow \|\mathbb{U}^{2n} a\| \ge \|\mathbb{U}^n a\|^2$$

Hence $\mathbb{U}$ is n[th]-fuzzy paranormal operator.

Theorem: 3.10

Let $\mathbb{U} \in FB(\mathbb{H})$. *If* $\mathbb{U}$ is n[th]- fuzzy paranormal operator and self-adjoint fuzzy operator then $\mathbb{U}^*$ is n[th]- fuzzy paranormal operator for n $\in N$.

Proof:

Take n$\in N$ for a $\in$ $\mathbb{H}$ with $\|a\|$=1.

Since $\mathbb{U}$ is n[th]- fuzzy paranormal operator.

i.e $\|\mathbb{U}^{2n} a\| \ge \|\mathbb{U}^n a\|^2$

Let $\|(\mathbb{U}^*)^n a\|^2 = \langle(\mathbb{U}^*)^n a, (\mathbb{U}^*)^n a\rangle$

$$= Sup\,\{u \in \mathbb{R}: \mathcal{F}((\mathbb{U}^*)^n a, (\mathbb{U}^*)^n a, u) < 1\}$$
$$= Sup\,\{u \in \mathbb{R}: \mathcal{F}(((\mathbb{U}^*)^n)^*(\mathbb{U}^*)^n a, a, u) < 1\}$$
$$= Sup\,\{u \in \mathbb{R}: \mathcal{F}(((\mathbb{U}^*)^*)^n(\mathbb{U}^*)^n a, a, u) < 1\}$$
$$= Sup\,\{u \in \mathbb{R}: \mathcal{F}((\mathbb{U}^*)^n(\mathbb{U}^*)^n a, a, u) < 1\}$$
$$= Sup\,\{u \in \mathbb{R}: \mathcal{F}((\mathbb{U}^*)^{2n} a, a, u) < 1\}$$
$$= \langle(\mathbb{U}^*)^{2n} a, a\rangle$$
$$\|(\mathbb{U}^*)^n a\|^2 \le \|(\mathbb{U}^*)^{2n} a\|\|a\|$$

Implies that $\|(\mathbb{U}^*)^{2n} a\| \ge \|(\mathbb{U}^*)^n a\|^2$

Therefore $\mathbb{U}^*$ is n[th] - fuzzy paranormal.

Theorem: 3.11

If $\mathbb{U} \in FB(\mathbb{H})$ is n[th]-fuzzy paranormal operator and self-adjoint fuzzy operator. Then $\mathbb{U}^n$ is n[th]-fuzzy paranormal operator for n$\in N$.

Proof:

Take n$\in N$ for a $\in$ $\mathbb{H}$ with $\|a\|$=1.

Since $\mathbb{U}$ is n[th]- fuzzy paranormal operator.

i.e $\|\mathbb{U}^{2n} a\| \ge \|\mathbb{U}^n a\|^2$

Let $\|(\mathbb{U}^n)^n a\|^2 = \langle(\mathbb{U}^n)^n a, (\mathbb{U}^n)^n a\rangle$

$$= Sup\,\{u \in \mathbb{R}: \mathcal{F}((\mathbb{U}^n)^n a, (\mathbb{U}^n)^n a, u) < 1\}$$
$$= Sup\,\{u \in \mathbb{R}: \mathcal{F}(((\mathbb{U}^n)^n)^*(\mathbb{U}^n)^n a, a, u) < 1\}$$
$$= Sup\,\{u \in \mathbb{R}: \mathcal{F}(((\mathbb{U}^*)^n)^n(\mathbb{U}^n)^n a, a, u) < 1\}$$
$$= Sup\,\{u \in \mathbb{R}: \mathcal{F}((\mathbb{U}^n)^n(\mathbb{U}^n)^n a, a, u) < 1\}$$
$$= Sup\,\{u \in \mathbb{R}: \mathcal{F}((\mathbb{U}^n)^{2n} a, a, u) < 1\}$$
$$= \langle(\mathbb{U}^n)^{2n} a, a\rangle$$
$$\|(\mathbb{U}^n)^n a\|^2 \le \|(\mathbb{U}^n)^{2n} a\|\|a\|$$

Implies that $\|(\mathbb{U}^n)^{2n} a\| \ge \|(\mathbb{U}^n)^n a\|^2$

Therefore $\mathbb{U}^n$ -is n[th]- fuzzy paranormal.

Theorem: 3.12

Let $\mathbb{U} \in FB(\mathbb{H})$ is fuzzy paranormal operator. Then $\mathbb{U}$ is n[th]-fuzzy paranormal operator for n$\in N$.

Proof:

Take n$\in N$ for a $\in$ $\mathbb{H}$ with $\|a\|$=1.

Since $\mathbb{U} \in FB(\mathbb{H})$ is fuzzy paranormal operator.

$\|\mathbb{U}a\|^2 \le \|\mathbb{U}^2 a\|$.

By using the mathematical induction,

If n=1, then it is correct because $\mathbb{U}$ is fuzzy paranormal operator.

Let $\|\mathbb{U}^{2.1} a\| = \|\mathbb{U}^{2.1} a\|$

$$= \|\mathbb{U}^2 a\| \ge \|\mathbb{U}a\|^2$$
$$\ge \|\mathbb{U}^1 a\|^2$$

i.e $\|\mathbb{U}^{2.1} a\| \ge \|\mathbb{U}^1 a\|^2$

If n=2, then

i.e $\|\mathbb{U}^{2.2} a\| \ge \|\mathbb{U}^2 a\|^2$etc.,

If n=k is true, then

$$\left\|\mathbb{U}^{2k} a\right\| \ge \left\|\mathbb{U}^k a\right\|^2$$

Now we have to prove that it is true for n=k+1

Let $\left\|\mathbb{U}^{k+1} a\right\|^2 = \langle\mathbb{U}^{k+1} a, \mathbb{U}^{k+1} a\rangle$

$$= Sup\,\{u \in \mathbb{R}: \mathbb{F}(\mathbb{U}^{k+1} a, \mathbb{U}^{k+1} a, u) < 1\}$$
$$= Sup\,\{u \in \mathbb{R}: \mathbb{F}(\mathbb{U}\mathbb{U}^k a, \mathbb{U}\mathbb{U}^k a, u) < 1\}$$
$$= \langle\mathbb{U}\mathbb{U}^k a, \mathbb{U}\mathbb{U}^k a\rangle$$
$$= \left\|\mathbb{U}\mathbb{U}^k a\right\|^2$$
$$\le \|\mathbb{U}\|^2\left\|\mathbb{U}^k a\right\|^2$$
$$\le \|\mathbb{U}\|^2\left\|\mathbb{U}^{2k} a\right\|\|a\|$$
$$\le \|\mathbb{U}^2\|\left\|\mathbb{U}^{2k} a\right\|\|a\|$$
$$\le \left\|\mathbb{U}^2\,\mathbb{U}^{2k} a\right\|\|a\|$$
$$\le \left\|\mathbb{U}^{2k}\,\mathbb{U}^2 a\right\|\|a\|$$
$$\le \left\|\mathbb{U}^{2k+2} a\right\|\|a\|$$
$$\left\|\mathbb{U}^{k+1} a\right\|^2 \le \left\|\mathbb{U}^{2(k+1)} a\right\|\|a\|$$

Implies that $\left\|\mathbb{U}^{k+1} a\right\|^2 \le \left\|\mathbb{U}^{2(k+1)} a\right\|$

$\therefore$ $\mathbb{U}$ is n[th]- fuzzy paranormal operator for n$\in N$.

Theorem: 3.13

Let $\mathbb{U} \in FB(\mathbb{H})$. If $\mathbb{U}$ is n- fuzzy paranormal operator Then $\mathbb{U}$ is n[th]- fuzzy paranormal operator for n$\in N$.

Proof:

By the definition of n- fuzzy paranormal operator

$\|\mathbb{U}^{n+1} a\| \ge \|\mathbb{U}a\|^{n+1}$ for n$\in N$.

It will be shown that $\|\mathbb{U}^{2n} a\| \ge \|\mathbb{U}^n a\|^2$ for n$\in N$

Take n∈ N for a ∈ ℍ with $\|a\|$=1.

We will prove, By using the mathematical induction,

Since 𝕌 is n - fuzzy paranormal operator

If n=1 is true then

$\|\mathbb{U}^1 a\|^2=\langle \mathbb{U}^1 a, \mathbb{U}^1 a\rangle$

$= Sup\,\{u \in \mathbb{R}: \mathbb{F}(\mathbb{U}^1 a, \mathbb{U}^1 a, u) < 1\}$

$= Sup\,\{u \in \mathbb{R}: \mathbb{F}(\mathbb{U}^* \mathbb{U}a, a, u) < 1\}$

$= \langle \mathbb{U}^* \mathbb{U}a, a\rangle$

$\leq \|\mathbb{U}^* \mathbb{U}a\|\|a\|$

$\leq \|\mathbb{U}^* \mathbb{U}a\|\|a\|$

$\leq \|\mathbb{U}^* \mathbb{U}a\|\|a\|$

$\leq \|\mathbb{U}^2 a\|$

If n=k,then

$\|\mathbb{U}^k a\|^2=\langle \mathbb{U}^k a, \mathbb{U}^k a\rangle$

$= Sup\,\{u \in \mathbb{R}: \mathbb{F}(\mathbb{U}^k a, \mathbb{U}^k a, u) < 1\}$

$= Sup\,\{u \in \mathbb{R}: \mathbb{F}((\mathbb{U}^k)^*\mathbb{U}^k a, a, u) < 1\}$

$= Sup\,\{u \in \mathbb{R}: \mathbb{F}((\mathbb{U}^*)^k\mathbb{U}^k a, a, u) < 1\}$

$= Sup\,\{u \in \mathbb{R}: \mathbb{F}(\mathbb{U}^k \mathbb{U}^k a, a, u) < 1\}$

$= Sup\,\{u \in \mathbb{R}: \mathbb{F}(\mathbb{U}^{2k} a, a, u) < 1\}$

$= \langle \mathbb{U}^{2k} a, a\rangle$

$\|\mathbb{U}^k a\|^2 \leq \|\mathbb{U}^{2k} a\|\|a\|$

$\|\mathbb{U}^k a\|^2 \leq \|\mathbb{U}^{2k} a\|$

It will be shown that it is true for n=k+1,

Let $\|\mathbb{U}^{k+1} a\|^2=\langle \mathbb{U}^{k+1} a, \mathbb{U}^{k+1} a\rangle$

$= Sup\,\{u \in \mathbb{R}: \mathbb{F}(\mathbb{U}^{k+1} a, \mathbb{U}^{k+1} a, u) < 1\}$

$= Sup\,\{u \in \mathbb{R}: \mathbb{F}((\mathbb{U}^{k+1})^*\mathbb{U}^{k+1} a, a, u) < 1\}$

$= Sup\,\{u \in \mathbb{R}: \mathbb{F}((\mathbb{U}^*)^{k+1}\mathbb{U}^{k+1} a, a, u) < 1\}$

$= Sup\,\{u \in \mathbb{R}: \mathbb{F}(\mathbb{U}^{k+1} \mathbb{U}^{k+1} a, a, u) < 1\}$

$= Sup\,\{u \in \mathbb{R}: \mathbb{F}(\mathbb{U}^{2(k+1)} a, a, u) < 1\}$

$= \langle \mathbb{U}^{2(k+1)} a, a\rangle$

$\|\mathbb{U}^{k+1} a\|^2 \leq \|\mathbb{U}^{2(k+1)} a\|\|a\|$

$\|\mathbb{U}^{k+1} a\|^2 \leq \|\mathbb{U}^{2(k+1)} a\|$

Hence 𝕌 is nth- fuzzy paranormal.

Theorem: 3.14

If $\mathbb{U} \in FB(\mathbb{H})$ is nth- fuzzy paranormal operator and $\mathbb{U}^{-1}$ exist then $\mathbb{U}^{-1}$ is nth- fuzzy paranormal operator for n∈ N.

Proof:

Since 𝕌 is nth- fuzzy paranormal operator.

i.e $\|\mathbb{U}^{2n} a\|\|a\| \geq \|\mathbb{U}^n a\|^2$ for n∈ N

Now we replace 'a' by $(\mathbb{U}^{-1})^{2n}$ a, then

$\|\mathbb{U}^n (\mathbb{U}^{-1})^{2n} a\|^2 \leq \|\mathbb{U}^{2n} (\mathbb{U}^{-1})^{2n} a\|\|(\mathbb{U}^{-1})^{2n} a\|$

$\|\mathbb{U}^n (\mathbb{U}^n)^{-2} a\|^2 \leq \|a\|\|(\mathbb{U}^{-1})^{2n} a\|$

$\|(\mathbb{U}^{-1})^n a\|^2 \leq \|(\mathbb{U}^{-1})^{2n} a\|\|a\|$

$\|(\mathbb{U}^{-1})^{2n} a\|\|a\| \geq \|(\mathbb{U}^{-1})^n a\|^2$

Therefore $\mathbb{U}^{-1}$ is nth- fuzzy paranormal operator for n∈ N.

Theorem: 3.15

Let $\mathbb{V} \in FB(\mathbb{H})$ be a fuzzy paranormal operator and 𝕌 is unitarily equivalent to 𝕍. Then 𝕌 is nth- fuzzy paranormal operator for n∈ N.

Proof:

For 𝕌 is unitarily equivalent to 𝕍, we have $\mathbb{U} = \mathbb{T}\mathbb{V}\mathbb{T}^*$

Then,

$\mathbb{U}^{2n} = \mathbb{T}\mathbb{V}^{2n}\mathbb{T}^*$

$\Rightarrow \|\mathbb{U}^{2n} a\| = \|\mathbb{T}\mathbb{V}^{2n}\mathbb{T}^* a\|$

Let $\|\mathbb{U}^n a\|^2 = \|(\mathbb{T}\mathbb{V}\mathbb{T}^*)^n a\|^2$

$\langle \mathbb{U}^n a, \mathbb{U}^n a\rangle = \langle (\mathbb{T}\mathbb{V}\mathbb{T}^*)^n a, (\mathbb{T}\mathbb{V}\mathbb{T}^*)^n a\rangle$

$= Sup\,\{u \in \mathbb{R}: \mathbb{F}((\mathbb{T}\mathbb{V}\mathbb{T}^*)^n a, (\mathbb{T}\mathbb{V}\mathbb{T}^*)^n a, u) < 1\}$

$= Sup\,\{u \in \mathbb{R}: \mathbb{F}((\mathbb{T}\mathbb{V}^n\mathbb{T}^*)a, (\mathbb{T}\mathbb{V}^n\mathbb{T}^*)a, u) < 1\}$

$= Sup\,\{u \in \mathbb{R}: \mathbb{F}((\mathbb{T}\mathbb{V}^n\mathbb{T}^*)^*(\mathbb{T}\mathbb{V}^n\mathbb{T}^*)a, a, u) < 1\}$

[∵ 𝕋 is fuzzy isometry]

$= Sup\,\{u \in \mathbb{R}: \mathbb{F}((\mathbb{T}\mathbb{V}^n\mathbb{T}^*)(\mathbb{T}\mathbb{V}^n\mathbb{T}^*)a, a, u) < 1\}$

$= Sup\,\{u \in \mathbb{R}: \mathbb{F}((\mathbb{T}\mathbb{V}^{2n}\mathbb{T}^*)a, a, u) < 1\}$

$\leq \|\mathbb{T}\mathbb{V}^{2n}\mathbb{T}^* a\|\|a\|$

$\|\mathbb{U}^n a\|^2 \leq \|\mathbb{T}\mathbb{V}^{2n}\mathbb{T}^* a\|\|a\|$

Implies that

$\|\mathbb{U}^n a\|^2 \leq \|\mathbb{U}^{2n} a\|\|a\|$

Hence 𝕌 is nth- fuzzy paranormal operator.

Definition: 3.16

Let (ℍ,𝔽,*) be a fuzzy Hilbert space with IP: <a,b>=sup{ u∈ ℝ; $\mathbb{F}(a, b, u) < 1\}$ $\forall a, b \in \mathbb{H}$ and self adjoint fuzzy operator .An operator $\mathbb{U} \in FB(\mathbb{H})$ is called *- fuzzy paranormal operator if $\|\mathbb{U}^2 a\|\|a\| \geq \|\mathbb{U}^* a\|^2$.

Theorem: 3.17

Let (ℍ,𝔽,*) be a fuzzy Hilbert space with IP: <a,b>=sup{ u∈ ℝ; $\mathbb{F}(a, b, u) < 1\}$ $\forall a, b \in \mathbb{H}$ and self adjoint fuzzy operator . Let $\mathbb{U}, \mathbb{V} \in FB(\mathbb{H})$ be a Self adjoint fuzzy operators. Then 𝕌𝕍 is *- fuzzy paranormal operator for n∈ N.

Proof:

Take n∈ N for a ∈ ℍ with $\|a\|$=1.

Since 𝕌 *and* 𝕍 are self adjoint fuzzy operators.

we know that $\mathbb{U} = \mathbb{U}^*$, $\mathbb{V} = \mathbb{V}^*$.

To prove that 𝕌𝕍 is a *- fuzzy paranormal operator.

Let $\|(\mathbb{U}\mathbb{V})^* a\|^2=\langle (\mathbb{U}\mathbb{V})^* a, (\mathbb{U}\mathbb{V})^* a\rangle$

$= Sup\,\{u \in \mathbb{R}: \mathbb{F}((\mathbb{U}\mathbb{V})^* a, (\mathbb{U}\mathbb{V})^* a, u) < 1\}$

$= Sup\,\{u \in \mathbb{R}: \mathbb{F}(((\mathbb{U}\mathbb{V})^*)^* (\mathbb{U}\mathbb{V})^* a, a, u) < 1\}$

$= Sup\,\{u \in \mathbb{R}: \mathbb{F}((\mathbb{V}^* \mathbb{U}^*)^* (\mathbb{U}\mathbb{V})^* a, a, u) < 1\}$

$= Sup\,\{u \in \mathbb{R}: \mathbb{F}((\mathbb{V}\mathbb{U})^* (\mathbb{V}^* \mathbb{U}^*)a, a, u) < 1\}$

$= Sup\,\{u \in \mathbb{R}: \mathbb{F}((\mathbb{U}^* \mathbb{V}^*)(\mathbb{V}^* \mathbb{U}^*)a, a, u) < 1\}$

$= Sup\,\{u \in \mathbb{R}: \mathbb{F}((\mathbb{U}\mathbb{V})(\mathbb{V}\mathbb{U})a, a, u) < 1\}$

$= Sup\,\{u \in \mathbb{R}: \mathbb{F}((\mathbb{U}\mathbb{V})^2 a, a, u) < 1\}$

$= \langle (\mathbb{U}\mathbb{V})^2 a, a\rangle$

$\leq \|(\mathbb{U}\mathbb{V})^2 a\|\|a\|$

$\|(\mathbb{U}\mathbb{V})^* a\|^2 \leq \|(\mathbb{U}\mathbb{V})^2 a\|\|a\|$

Implies that $\|(\mathbb{U}\mathbb{V})^* a\|^2 \leq \|(\mathbb{U}\mathbb{V})^2 a\|$

Therefore 𝕌𝕍 is *- fuzzy paranormal operator.

Theorem: 3.18

Let (ℍ,𝔽,*) be a fuzzy Hilbert space with IP: <a,b>=sup{ u∈ ℝ; $\mathbb{F}(a, b, u) < 1\}$ $\forall a, b \in \mathbb{H}$ and self adjoint fuzzy operator. Let $\mathbb{U}, \mathbb{V} \in FB(\mathbb{H})$. If 𝕌, 𝕍 are self adjoint fuzzy operators then $\mathbb{U} + \mathbb{V}$ is *- fuzzy paranormal operator for n∈ N.

Proof:

Take n∈ N for a ∈ ℍ with $\|a\|$=1.

Let $\|(\mathbb{U} + \mathbb{V})^* a\|^2 = \langle (\mathbb{U} + \mathbb{V})^* a, (\mathbb{U} + \mathbb{V})^* a\rangle$

$= Sup\,\{u \in \mathbb{R}: \mathbb{F}((\mathbb{U} + \mathbb{V})^* a, (\mathbb{U} + \mathbb{V})^* a, u) < 1\}$

$= Sup\,\{u \in \mathbb{R}: \mathbb{F}(((\mathbb{U} + \mathbb{V})^*)^* (\mathbb{U} + \mathbb{V})^* a, a, u) < 1\}$

$= Sup\{u \in \mathbb{R}: \mathbb{F}((\mathbb{U} + \mathbb{V})(\mathbb{U}^* + \mathbb{V}^*)a, a, u) < 1\}$

$= Sup\,\{u \in \mathbb{R}: \mathbb{F}((\mathbb{U} + \mathbb{V})(\mathbb{U} + \mathbb{V})a, a, u) < 1\}$

$= \langle (\mathbb{U} + \mathbb{V})^2 a, a\rangle$

$= \langle (\mathbb{U} + \mathbb{V})^2 a, a\rangle$

$\leq \|(\mathbb{U} + \mathbb{V})^2 a\|\|a\|$

Implies that $\|(\mathbb{U} + \mathbb{V})^* a\|^2 \leq \|((\mathbb{U} + \mathbb{V})^2 a\|$

Therefore $\mathbb{U} + \mathbb{V}$ is *- fuzzy paranormal operator.

Theorem: 3.19

If $\mathbb{U} \in FB(\mathbb{H})$ is fuzzy paranormal operator and self adjoint fuzzy operator then 𝕌 is *- fuzzy paranormal operator for.

Proof:

For a ∈ ℍ with $\|a\|$=1.

We know that 𝕌 is fuzzy paranormal operator i.e $\|\mathbb{U}^2 a\| \geq \|\mathbb{U}a\|^2$ and $\mathbb{U} = \mathbb{U}^*$.

Let $\|\mathbb{U}^* a\|^2 = \langle \mathbb{U}^* a, \mathbb{U}^* a \rangle$

$= Sup\,\{u \in \mathbb{R}: \mathbb{F}(\mathbb{U}^* a, \mathbb{U}^* a, u) < 1\}$

$= Sup\,\{u \in \mathbb{R}: \mathbb{F}(\mathbb{U}\mathbb{U}^* a, a, u) < 1\}$

$= Sup\{u \in \mathbb{R}: \mathbb{F}(\mathbb{U}^2\, a, a, u) < 1\}$

$= \langle \mathbb{U}^2 a, a \rangle$

$\|\mathbb{U}^* a\|^2 \;\leq \|\mathbb{U}^2 a\|\|a\|$

Implies that $\|\mathbb{U}^* a\|^2 \leq \|\mathbb{U}^2 a\|$

Therefore $\mathbb{U}$ is *- fuzzy paranormal operator.

Theorem: 3.20

If $\mathbb{U} \in FB(\mathbb{H})$ is *- fuzzy paranormal operator and self adjoint fuzzy operator then $\mathbb{U}^n$ is *- fuzzy paranormal operator for n$\in N$.

Proof:

Since $\mathbb{U}$ is *- fuzzy paranormal operator

i.e $\|\mathbb{U}^* a\|^2 \leq \|\mathbb{U}^2 a\|\|a\|$ and $\mathbb{U} = \mathbb{U}^*$.

Let $\|(\mathbb{U}^n)^* a\|^2 = \langle (\mathbb{U}^n)^* a, (\mathbb{U}^n)^* a \rangle$

$= Sup\,\{u \in \mathbb{R}: \mathbb{F}((\mathbb{U}^n)^* a, (\mathbb{U}^n)^* a, u) < 1\}$

$= Sup\,\{u \in \mathbb{R}: \mathbb{F}(\mathbb{U}^n (\mathbb{U}^n)^* a, a, u) < 1\}$

$= Sup\,\{u \in \mathbb{R}: \mathbb{F}(\mathbb{U}^n (\mathbb{U}^*)^n a, a, u) < 1\}$

$= Sup\,\{u \in \mathbb{R}: \mathbb{F}(\mathbb{U}^n \mathbb{U}^n a, a, u) < 1\}$

$= Sup\{u \in \mathbb{R}: \mathbb{F}(\mathbb{U}^{2n}\, a, a, t) < 1\}$

$= \langle \mathbb{U}^{2n} a, a \rangle$

$\|(\mathbb{U}^n)^* a\|^2 \;\leq \|(\mathbb{U}^n)^2 a\|\|a\|$

Implies that $\|(\mathbb{U}^n)^* a\|^2 \;\leq \|(\mathbb{U}^n)^2 a\|\|a\|$

Therefore $\mathbb{U}^n$ is *- fuzzy paranormal operator for n$\in N$.

III. CONCLUSION

The conclusion that can be taken from a new idea of fuzzy paranormal operator in Fuzzy Hilbert space, example and characteristics of n- Fuzzy paranormal operator, *-Fuzzy paranormal operator and n^{th} -Fuzzy paranormal operator including addition and multiplication operators and its connection with self-adjoint fuzzy operator. In addition other characteristics are found and its connection between each of the operators of such other definition.

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