

A New Efficient Technique Based on Majority Logic Decoding to Decode Linear Block Codes



Issam Abderrahman Joundan, Moulay Seddiq El Kasmi Alaoui, Karim Rkizat, Said Nouh, Abdelwahed Namir

Abstract: One-Step Majority logic decoding (OSMLD) codes present a very powerful error correcting schemes due to the simplicity of their decoder. However, families of these codes are limited to a few numbers. In this paper, we present a new adaptation which generalizes the majority-logic decoding for non OSMLD codes. This technique uses MIM method to select codewords from the dual of the code with last digit equal to one and then estimates the best threshold from which we obtain the best performance in term of the bit error rate (BER). The comparison between performances of the proposed technique and other coder/decoder schemes on AWGN (Additive White Gaussian Noise) channel proves its capacity in correcting more errors.

Keywords: Error correcting codes, HSDec, Hash table, MIM, syndrome decoding, QR code, BCH Code, OSMLD codes, Majority logic decoding;

I. INTRODUCTION

The majority-logic decoding is an efficient decoder for decoding certain classes of cyclic codes due to its good performances with a very low complexity when used for transmission over AWGN channels. The first majority-logic decoding algorithm was devised in 1954 by Reed [1] for a class of multiple-error-correcting codes discovered by Muller [2]. Reed's algorithm was later extended and generalized by many coding investigators. The first unified formulation of majority-logic decoding algorithms was due to Massey [3]. Consider an (n,k) cyclic code C with parity-check matrix H . The row space of H is an $(n, n-k)$ cyclic code, denoted by C^\perp , which is the dual code of C , or the null space of C . For any vector v in C and any vector w in C^\perp , the inner product of v

and w is zero, that is, $w \bullet v = w_0 v_0 + w_1 v_1 + \dots + w_{n-1} v_{n-1} = 0$ (1.1). In fact, an n -tuple v is a code vector in C if and only if, for any vector w in C^\perp , $w \bullet v = 0$. The equality of (1.1) is called a parity-check equation. Clearly, there are 2^{n-k} such parity-check equations.

Now suppose that a code vector v in C is transmitted. Let $e = (e_0, e_1, \dots, e_{n-1})$ and $r = (r_0, r_1, \dots, r_{n-1})$ be the error vector and the received vector respectively. Then $r = v + e$ (1.2). For any vector w in the dual code C^\perp , we can form the following linear sum of the received digits called a parity-check sum or simply check sum: $A = w \bullet r = w_0 r_0 + w_1 r_1 + \dots + w_{n-1} r_{n-1}$ (1.3). Combining (1.2) and (1.3) and using the fact that $w \bullet v = 0$, we obtain the following relationship between the check sum A and error digits in e : $A = w_0 e_0 + w_1 e_1 + \dots + w_{n-1} e_{n-1}$ (1.4). An error digit e_i is said to be checked by the check sum A if the coefficient $w_i = 1$.

Suppose that there exist J vectors in the dual code C^\perp : $w_1 = (w_{1,0}, w_{1,1}, \dots, w_{1,n-1})$, $w_2 = (w_{2,0}, w_{2,1}, \dots, w_{2,n-1})$, ..., $w_J = (w_{J,0}, w_{J,1}, \dots, w_{J,n-1})$. Which have the following properties:

The $(n-1)^{th}$ component of each vector is a "1", $w_{1,n-1} = w_{2,n-1} = \dots = w_{J,n-1} = 1$.

For $i \neq n-1$, there is at most one vector whose i^{th} component is a "1"; for example, if $w_{1,i} = 1$ then $w_{2,i} = w_{3,i} = \dots = w_{J,i} = 0$.

Since $w_{1,n-1} = w_{2,n-1} = \dots = w_{J,n-1} = 1$, these J check sums are related to the error digits in the following manner:

$$\begin{aligned} A_1 &= w_{1,0} e_0 + w_{1,1} e_1 + \dots + w_{1,n-2} e_{n-2} + e_{n-1} \\ A_2 &= w_{2,0} e_0 + w_{2,1} e_1 + \dots + w_{2,n-2} e_{n-2} + e_{n-1} \\ &\vdots \\ A_J &= w_{J,0} e_0 + w_{J,1} e_1 + \dots + w_{J,n-2} e_{n-2} + e_{n-1} \end{aligned}$$

These J check sums are said to be orthogonal on the error digit e_{n-1} . Based on the facts above, an algorithm for decoding e_{n-1} can be formulated as follows:

The error digit e_{n-1} is decoded as 1 if a clear majority of the parity-check sums orthogonal on e_{n-1} is 1; otherwise, e_{n-1} is decoded as 0.

Correct decoding of e_{n-1} is guaranteed, if there is $\lfloor J/2 \rfloor$ or fewer errors in the error vector e . If it is possible to form J parity-check sums orthogonal on e_{n-1} , it is possible to form J parity-check sums orthogonal on any error digit because of the cyclic symmetry of the code.

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The decoding of other error digits is identical to the decoding of e_{n-1} . The decoding algorithm described above is called one-step majority-logic decoding.

Let d_{\min} be the minimum distance of the code. Clearly, the one-step majority-logic decoding is effective for this code only if $\lfloor J/2 \rfloor$ is equal to or close to the error-correcting capability of the code; in other words, J should be equal to or close to $d_{\min}-1$, we say that the code is completely orthogonalisable. However, families of cyclic codes having this property are limited to a few numbers [4] and some construction of OSMLD codes which are Quasi-Cyclic are presented in [5].

The remainder of this paper is structured as follows. In section 2 we present some decoding algorithms as related works. In section 3 we present the proposed technique, in the section 4, we present the simulation of results of the proposed decoders and we make a comparison with other decoders. Finally, a conclusion and a possible future direction of this research are outlined in section 5.

II. RELATED WORKS

The quadratic residue (QR) codes were first defined in 1964 by Andrew Gleason who demonstrated many of their important properties in a brief letter. Quadratic Residue codes are a powerful cyclic codes, contain the famous Hamming and Golay codes, they offer also the best known minimum distance codes for lengths 191, 199 and 223. Unfortunately, there is no known specific decoder for this family of codes. The Bose, Chaudhuri, and Hocquenghem (BCH) codes form a large class of powerful random error-correcting cyclic codes. This class of codes is a remarkable generalization of the Hamming codes for multiple-error correction. Binary BCH codes were discovered by Hocquenghem in 1959 and independently by Bose and Chaudhuri in 1960. The cyclic structure of these codes was proved by Peterson in 1960. The well known specific decoder for this family of codes is the Berlekamp-Massey (BM) decoder.

In [6], an approach based on the link between syndromes and correctable errors pattern is developed by using hash techniques. In [7], the authors have presented a method called NESWDA to decode up to five errors in a binary systematic quadratic residue QR(47,24,11) code, this method is based on the weight of syndrome difference and properties of cyclic codes. The disadvantage of these two methods is that it's just applicable for Quadratic Residue code.

In [8], a deep learning method to improve belief propagation algorithm was proposed, by attribution of weights to the edges of the Tanner graph the authors generalized the standard belief propagation algorithm.

In [9], the authors have developed a Cyclic Weight (CW) algorithm decoding to decode the binary systematic QR(47,24,11) code. In addition to the properties of the cyclic codes, they based on the weights of syndromes; the same authors and in a previous paper [10] they have presented an algebraic decoding algorithm to correct all patterns of four or fewer errors in the binary QR (41, 21, 9) code. In order to decode up to five possible errors in a binary systematic QR(47,24,11) code, the authors of [11] have presented a table lookup decoding algorithm.

By using the Lagrange interpolation formula, the authors

of [12] have calculated the needed primary unknown syndrome for the binary QR code and proposed hardware architecture to implement it, also by using the developed Berlekamp-Massey (BM) algorithm and Chien search they decoded the binary QR code. In [13] the authors have proposed a decoding of quadratic residue codes by using hashing search to determine error patterns.

In [14, 15] several hard decoder based on genetic algorithms (GA) are developed, the first one is the HDGA (Hard decision Decoder based on Genetic Algorithms) [14], it used information sets to decode linear block codes. In [15], an efficient decoder called ARDecGA (Artificial reliabilities based decoding algorithm) is presented. It uses a generalized parity check matrix to compute a vector of artificial reliabilities of the binary received word h and it exploited a genetic algorithm to find the maximum likelihood binary word to this vector. The algebraic hard decision decoder [16, 17] of Berlekamp-Massey is based on compute of syndromes and it has an efficient mechanism to localize all corrigible errors, it is applicable on BCH codes. Another version of this last decoder is adapted for Quadratic residue codes [18].

Recently, several decoders based on hash techniques have been developed [19, 20, 21]; for example in [19], the authors have developed two hard decoders HSDec and HWDDec applicable to linear block codes and based on syndrome and hash techniques.

III. THE PROPOSED DECODER

The proposed technique GOSMLD works as follows:

Inputs: - A generator matrix G of the linear code $C(n, k)$.
Step 1: by using the MIM Method, Find a list of vectors $w_i = (w_{i,0}, w_{i,1}, \dots, w_{i,n-1})$ from the dual code C^\perp with $w_{i,n-1} = 1$.
Step 2: Estimate the best threshold from which, we obtain the best performance in term of the bit error rate (BER).

IV. SIMULATION RESULTS AND COMPARISON

To show the efficiency of the proposed technique, we give in this section its error correcting performances for some non OSMLD cyclic codes. A comparison with other decoding algorithms over the Additive White Gaussian Noise (AWGN) channel with a BPSK (Binary Phase Shift Keying) modulation is done. All simulations are obtained by using the parameters given in Table 1. The error correcting performances will be represented in terms of Bit Error Rate (BER) in each Signal to Noise Ratio (SNR= E_b/N_0).

TABLE I. DEFAULT SIMULATION PARAMETERS.

Simulation parameters	Value
Channel	AWGN
Modulation	BPSK
Minimum number of residual bit in errors	200
Minimum number of transmitted blocks	1000

In order to show the efficiency of the proposed decoders we made several simulations for plotting their error correcting performances for many QR codes and BCH codes. The figure 1 represents the performances of the GOSMLD for some QR codes of length up to 73. Knowing that if the data are transmitted without coding in the sending step and without correction in the receiving step over AWGN channel then the BER reaches the value 10^{-5} at the SNR=9.6 therefore the proposed scheme permits to obtain a gain of coding about 2.2 dB for QR(23,12,7), 2 dB for QR(31,16,7), 3dB for QR(47,24,11), 2.8dB for QR(71,36,11) and about 2.9dB for QR(73,37,13). The figures 2 and 3 represent the performances of the GOSMLD for some BCH codes of lengths 31 and 63 respectively. These figures show that our scheme permits to obtain a gain of coding go to 2.1dB for length 31 and go to 2.8dB for length 63.

In order to study the impact of the threshold at the Bit Error Rate (BER), a number of simulations are done. The figure 4 summarizes the obtained results for the same number M of codewords of the dual code find by the MIM Method [22]. The obtained results show that the value of the threshold has a great impact on Bit Error Rate. The best value of the threshold for QR(31,16,7) is 0.57. That is, if the sum of parity-check sum exceeds 68 then flips the last digit. For the other digit, view the cyclic structure of the code, we use the same way on the shift vector.

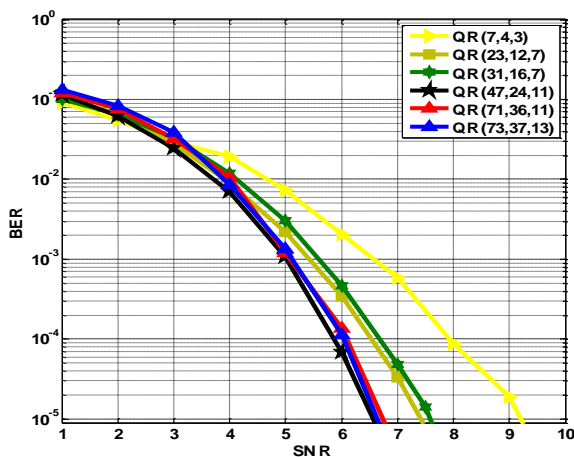


Fig. 1. Performances of GOSMLD for some QR codes of length up to 73.

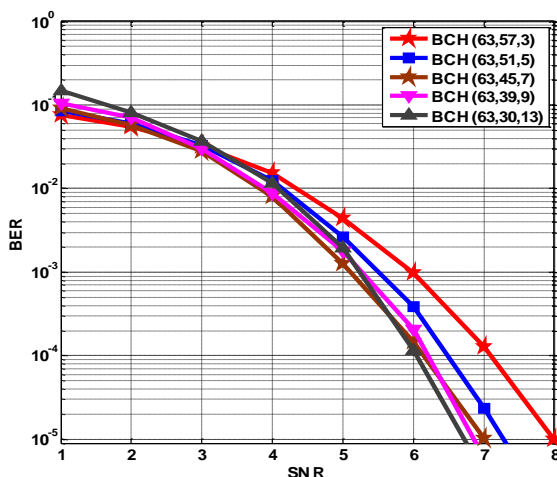


Fig. 2. Performances of GOSMLD for some BCH codes of length 63.

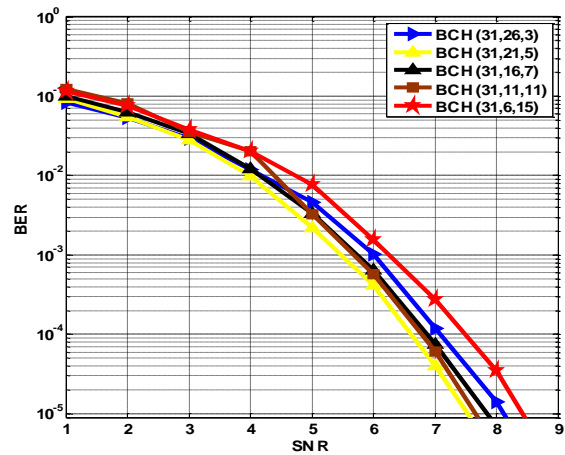


Fig. 3. Performances of GOSMLD for some BCH codes of length 31.

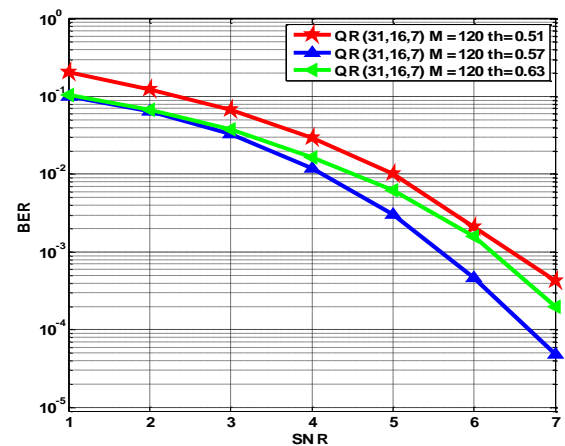


Fig. 4. Impact of the threshold at the BER for QR(31,16,7).

The figures 5, 6, 7, and 8, represent a comparison of the performance of GOSMLD and HSDec for some QR code of length up to 47. The obtained results prove that our technique passes relatively the performances of HSDec decoder.

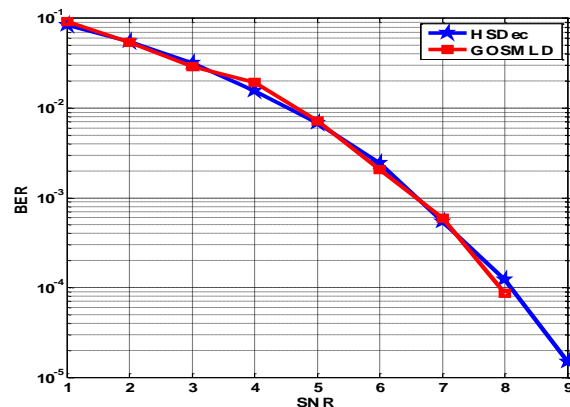


Fig. 5. Comparison of the performances of GOSMLD and HSDec for QR(7,4,3) code.

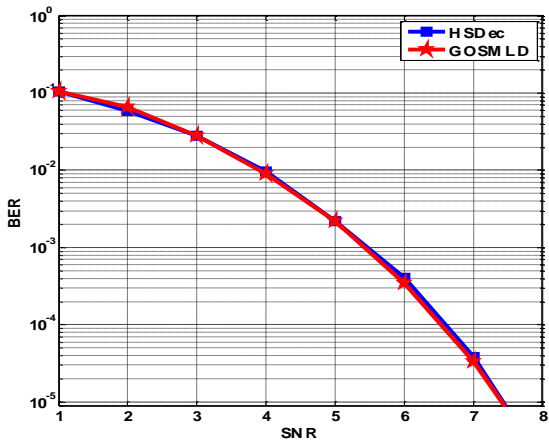


Fig. 6. Comparison of the performances of GOSMLD and HSDec for QR(23,12,7) code.

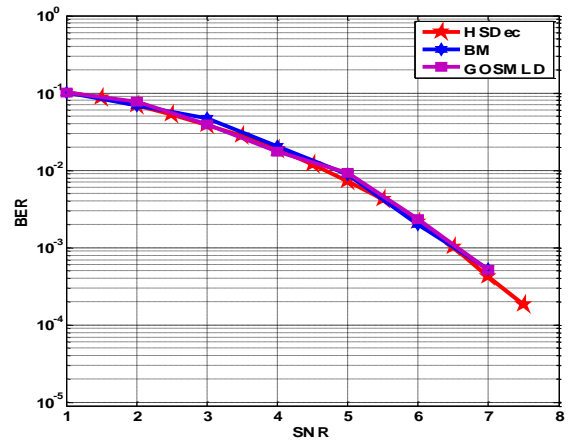


Fig. 9. Comparison of the proposed GOSMLD, HSDec and BM for BCH(15,5,7).

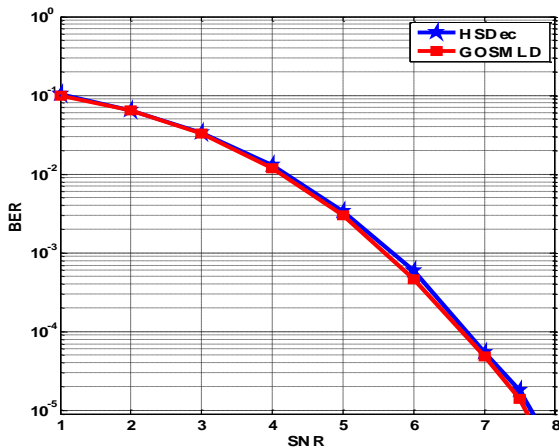


Fig. 7. Comparison of the performances of GOSMLD and HSDec for QR(31,16,7) code.

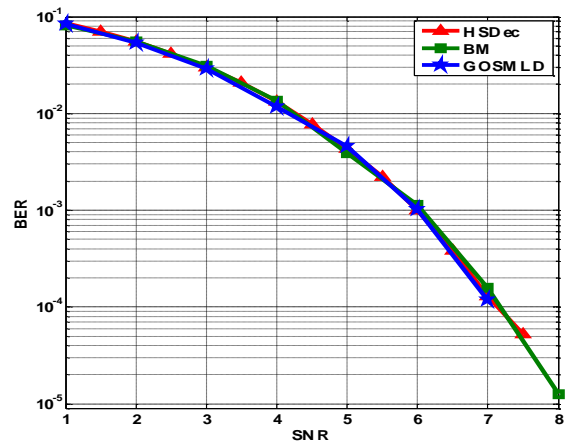


Fig. 10. Comparison of the proposed GOSMLD, HSDec and BM for BCH(31,26,3).

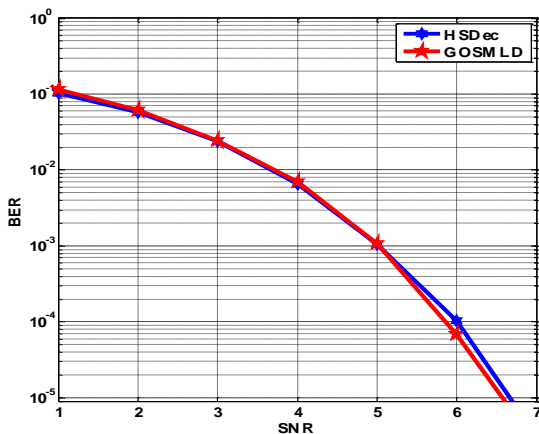


Fig. 8. Comparison of the performances of GOSMLD and HSDec for QR(47,24,11) code.

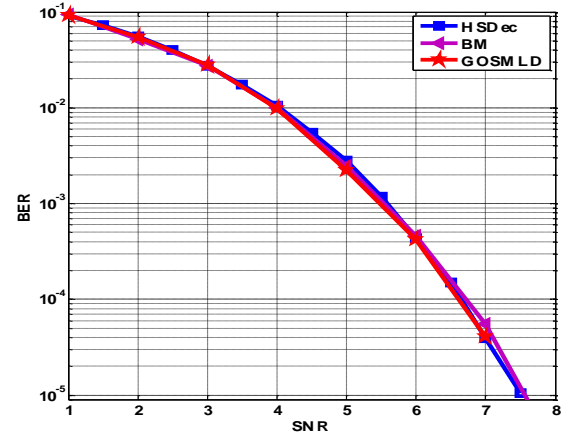


Fig. 11. Comparison of the proposed GOSMLD, HSDec and BM for BCH(31,21,5).

The figures 9, 10, 11, 12, 13, 14, 15 and 16 represent a comparison of the performance of GOSMLD, HSDec and BM for some BCH codes. The obtained results prove that our technique reach the same performance of these decoders.

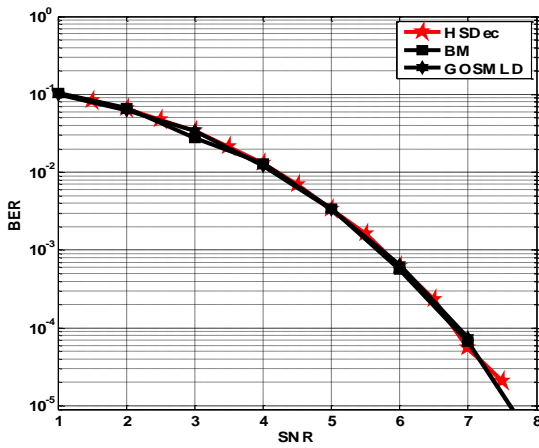


Fig. 12. Comparison of the proposed GOSMLD, HSDec and BM for BCH(31,16,7).

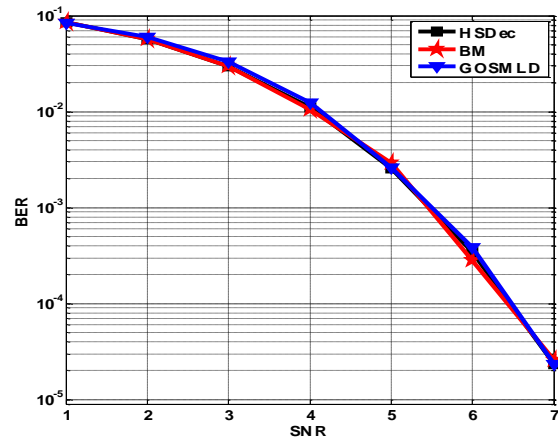


Fig. 15. Comparison of the proposed GOSMLD, HSDec and BM for BCH(63,51,5).

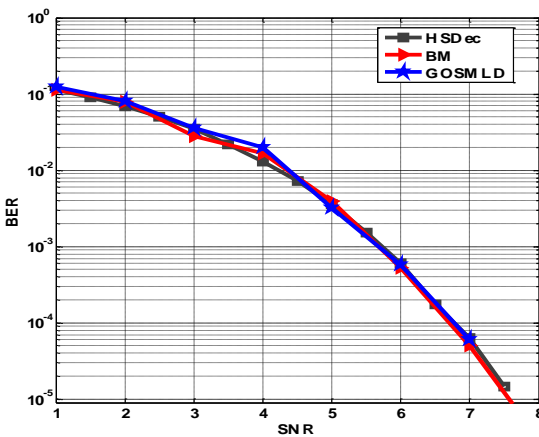


Fig. 13. Comparison of the proposed GOSMLD, HSDec and BM for BCH(31,11,11).

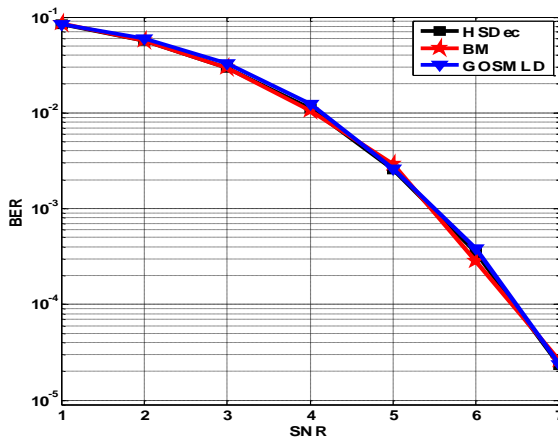


Fig. 16. Comparison of the proposed GOSMLD, HSDec and BM for BCH(127,113,5).

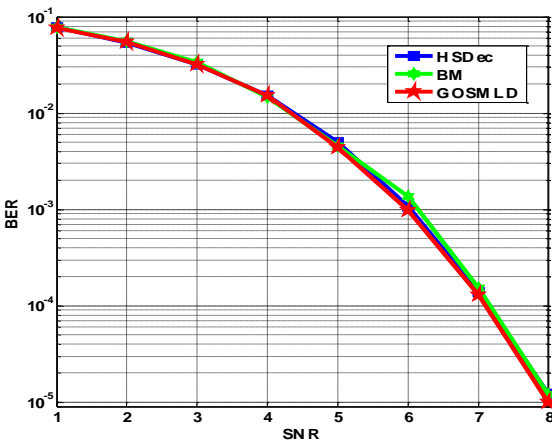


Fig. 14. Comparison of the proposed GOSMLD, HSDec and BM for BCH(63,57,3).

The figure 17, represent a comparison of the performance of GOSMLD, HSDec, ARDecGA and BM for BCH(15,7,5). The obtained results prove that our technique passes relatively the performance of these decoders.

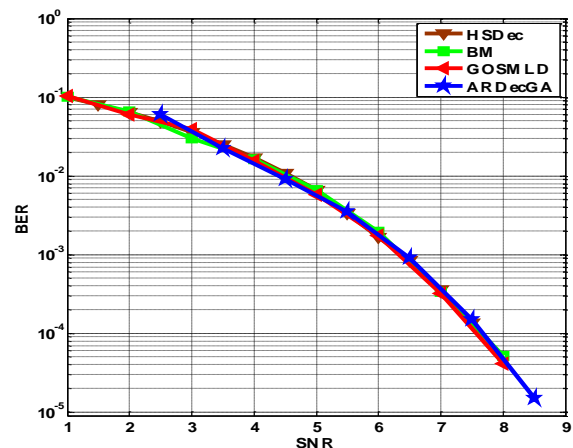


Fig. 17. Comparison of the proposed GOSMLD, HSDec, ARDecGA and BM for BCH(15,7,5).

The figure 18, represent a comparison of the performance of GOSMLD and DND, HDGA, and BM for BCH(63,45,7). The obtained results prove that our technique greatly exceeds HDGA, DND and have relatively the same performance as BM for this code.

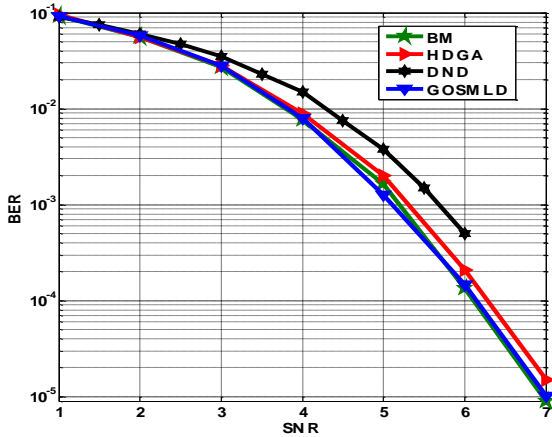


Fig. 18. Comparison of the proposed GOSMLD, DND, HDGA, and BM for BCH(63,45,7)

The figure 19, represent a comparison of the performance of GOSMLD, HDGA and BM for BCH(63,30,13). The obtained results prove that our method greatly exceeds HDGA and BM for this code.

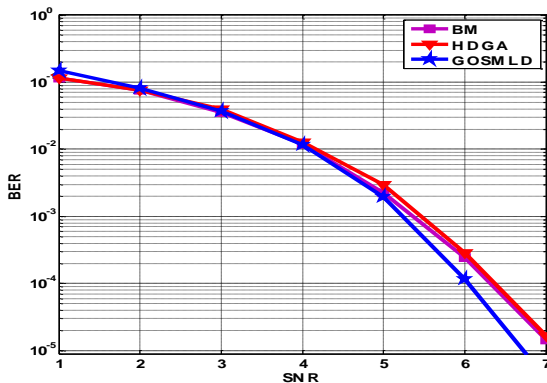


Fig. 19. Comparison of the proposed GOSMLD and HDGA, BM for BCH(63,30,13)

V. CONCLUSION AND PERSPECTIVES

In this paper, we have presented a new efficient hard decision decoding algorithm which generalizes the one step majority-logic decoding for non OSMLD codes. This technique uses a codeword from the dual of the code with last digit equal to one found by MIM method and then estimates the best threshold from which we obtain the best performance in term of the bit error rate (BER). The comparison between performances of the proposed technique and other coder/decoder schemes on AWGN (Additive White Gaussian Noise) channel proves its capacity in correcting more errors. The success of this technique will encourage us to continue to generalize it for other families of error correcting codes.

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