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Longitudinal Wave in a Thermally Conducting Elastic Medium

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Abstract: ***The longitudinal wave propagation in a thermally conducting elastic medium has been investigated. Considering the equations of motions of longitudinal wave in displacement and temperature field, the frequency equation has been derived. The dispersion and damping equations have been derived for the propagation of longitudinal wave in four materials i.e Copper, Steel, Aluminum, and Lead. Effect of Phase velocity and damping coefficient are shown graphically. It is found that the increase in wave number results the decrease in Phase velocity and increase in damping coefficient.***

Keywords: Longitudinal wave, Thermally conducting, Phase velocity, Damping coefficient.

I. INTRODUCTION

When an earthquake occurred artificially or naturally, two types of body waves Longitudinal wave (P-wave) and Transverse wave (S-wave) are propagated through different types of medium present in the internal part of the earth. It is assumed that the earth is made up of elastic, non-elastic, isotropic and anisotropic thermally conducting materials. Nowacki [1] has described clearly the dynamics of thermo-elasticity and presented numerous problems in his book. Beevers [2] studied the stability of wave and shock structure in generalized thermo-elasticity at low temperature. Manacorda [3] discussed the propagation of discontinuity wave in a thermoelastic incompressible materials. Puri [4] studied plain wave in generalized thermo-elasticity. Nayfeh, et al.[5], described the propagation of thermoelastic wave in solid material. Similarly Singh B. [6] gave the concept of wave propagation in anisotropic generalized thermo-elastic solid. Singh, H. et al. [7] generalized the thermoelastic wave in transversely isotropic media. Chadwick, P. et al. [8], discussed the wave propagation in transversely isotropic thermally conducting elastic materials.

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In this paper, Propagation of Longitudinal wave in a thermally conducting medium has been studied. Longitudinal wave, generated due to an earthquake travel along the x-axis in a thermally conducting elastic medium. Equations of motions are taken in elastic field and thermal field and solved analytically to get phase velocity and damping coefficient for four materials i.e Copper, Aluminum, Steel and Lead.

II. GEOMETRY OF THE PROBLEM

We consider the longitudinal wave in a thermally conducting medium has been studied. Longitudinal wave, generated due to an earthquake travel along the x-axis in a thermally conducting elastic medium. Equations of motions are taken in elastic field and thermal field and solved analytically to get phase velocity and damping coefficient for four materials i.e Copper, Aluminum, Steel and Lead.

A. EQUATION OF MOTION AND SOLUTION FOR THE THERMOELASTIC MEDIUM

The equations of motions of wave in thermoelastic medium are, Nowacki [1].

$$\frac{\partial^2 u}{\partial x^2} - \frac{1}{c_T{}^2}\frac{\partial^2 u}{\partial t^2} = m\frac{\partial \theta}{\partial x} \tag{2.1}$$

$$\frac{\partial^2 \theta}{\partial x^2} - \frac{1}{\aleph}\frac{\partial^2 \theta}{\partial t^2} = \frac{\partial^2 u}{\partial x\, \partial t} \tag{2.2}$$

Where $\eta = 3T_0K\alpha_T$, $m = \dfrac{K\alpha_T}{\lambda+2\mu}$

α_T is the thermal expansion coefficient, c_T is the longitudinal wave velocity, $\aleph$ is the thermal diffusivity ,T_0 is the reference temperature , λ and μ are Lame's constants, λ_0 is the coefficient of heat conduction and K is the bulk modulus. Since the causes producing longitudinal wave are assumed to very harmonically in time, both the displacement and temperature vary in the same manner.

We take u and θ in the following form

$$u = u^* e^{-i\omega t} , \theta = \theta^* e^{-i\omega t} \tag{2.2}$$

Using (2.2) in (2.1) ,the equations take the forms

$$\frac{\partial^2 u^*}{\partial x^2} + \sigma^2 u^* = m\theta^* \tag{2.3}$$

$$\frac{\partial^2\theta^*}{\partial x^2} + q\theta^* = \eta\chi q u^* \frac{\partial u^*}{\partial x} \qquad (2.4)$$

Where $\sigma^2 = \frac{w^2}{{c_T}^2}$, $q = \frac{iw}{\chi}$

Introducing into (2.3) and (2.4), the quantities

$$u^* = u^0 e^{ikx} \quad , \quad \theta^* = \theta^0 e^{ikx} \qquad (2.5)$$

we obtain the relation

$$\frac{u^0}{\theta^0} = \frac{mik}{\sigma^2 - k^2} \quad , \quad \frac{\theta^0}{u^0} = \frac{-\eta\chi q i k}{q - k^2} \qquad (2.6)$$

Eliminating $\frac{u^0}{\theta^0}$, (2.6) yield the algebraic equation

$$k^4 - k^2[\sigma^2 + q(1+\epsilon)] + q\sigma^2 = 0 \quad , \quad \epsilon = \eta m \chi$$

Now taking

$$\xi = \frac{c_T k}{\omega^*} \quad , \quad \omega^* = \frac{{c_T}^2}{\chi} \quad , \quad \chi = \frac{\omega}{\omega^*} \qquad (2.7)$$

Then (2.7) becomes

$$\xi^4 - \xi^2[\chi^2 + i\chi(1+\epsilon)] + i\chi^3 = 0 \qquad (2.8)$$

The roots of (2.8) are the following

$$\xi_1 = \frac{1}{2}\sqrt{\chi}[\chi + (1+i)\sqrt{2\chi} + i(1+\epsilon)]^{\frac{1}{2}} + [\chi - (1+i)\sqrt{2\chi} + i(1+\epsilon)]^{\frac{1}{2}}$$

$$\xi_1 = \frac{1}{2}\sqrt{\chi}[\chi + (1+i)\sqrt{2\chi} + i(1+\epsilon)]^{\frac{1}{2}} + [\chi - (1+i)\sqrt{2\chi} + i(1+\epsilon)]^{\frac{1}{2}} \qquad (2.9)$$

These roots depend on the parameters ϵ and $\xi = \frac{\omega}{\omega^*}$. The quantity $\epsilon = \eta m \chi$ which depends on the thermal and mechanical properties of the material, is constant, whereas χ depends only on the frequency ω . The quantity ω^* is a characteristic of the material.

B. NUMERICAL CALCULATIONS

Some numerical calculations have been done directly from equation (2.9) making use of following relation, Nowacki [1]

$$\xi_{1,2} = c_T \left(\frac{\chi}{v_{1,2}} + i\,\frac{\vartheta_{1,2}}{\omega^*}\right) \qquad (2.10)$$

where v_1 and ϑ_1 are phase velocity and damping coefficient respectively.

Table- I: Phase velocity ($\frac{v_1}{c_T}$) with respect to frequency (χ) of four materials

Frequency (χ)	Copper($\frac{v_1}{c_T}$)	Aluminum ($\frac{v_1}{c_T}$)	Steel ($\frac{v_1}{c_T}$)	Lead ($\frac{v_1}{c_T}$)
1	2.00084	2.0189	2.0049	2.03876
2	2.00034	2.00722	2.0019	2.0151
3	2.00017	2.00357	2.0001	2.00739
4	2.0001	2.00209	2.00006	2.0043
5	2.00006	2.00136	2.00004	2.0028
6	2.00005	2.00096	2.00003	2.00196
7	2.000031	2.00071	2.00002	2.00145
8	2.00003	2.00054	2.00001	2.00111

Table-II : Damping coefficient ($\frac{\vartheta_1}{\vartheta_1^\infty}$) with respect to frequency (χ) of four materials.

Frequency (χ)	Copper($\frac{\vartheta_1}{\vartheta_1^\infty}$)	Aluminum($\frac{\vartheta_1}{\vartheta_1^\infty}$)	Steel($\frac{\vartheta_1}{\vartheta_1^\infty}$)	Lead ($\frac{\vartheta_1}{\vartheta_1^\infty}$)
0	0.0	0.0	0.0	0.0
1	0.124947	0.123816	0.12497	0.122392
2	0.200047	0.201017	0.200027	0.202141
3	0.225046	0.225893	0.225024	0.226863
4	0.235324	0.235922	0.235311	0.236596
5	0.240406	0.240829	0.240397	0.241304
6	0.243259	0.241357	0.243252	0.243917
7	0.245012	0.245248	0.245007	0.245511
8	0.24613	0.246348	0.246159	0.246553

Table-I depicts that the phase velocity of Lead (Pb) is higher as compared to other three materials, whereas, Table 2 shows the damping coefficient ϑ_1 of Lead (Pb) is very much less than the other three materials. As well as it can be seen that the phase velocity and damping coefficient of copper and steel in Table 1 and Table 2 respectively are almost same.

For $\chi = \infty$,the damping coefficient $\vartheta_1^\infty = \frac{\epsilon\omega^*}{2c_T}$. Figure 1 and 2 represent the ratio $\frac{v_1}{c_T}$, $\frac{\vartheta_1}{\vartheta_1^\infty}$ for Aluminum (ϵ=3.56×10^{-2}) , copper ($\epsilon = 1.68 \times 10^{-2}$), Steel ($\epsilon$=2.97 ×$10^{-4}$), Lead ($\epsilon = 7.33 \times 10^{-2}$) in terms the variable $\chi = \frac{\omega}{\omega^*}$.

Figure-II indicates the phase velocity v_1 is greater than c_T and approaches twice of latter as $\chi = \infty$.

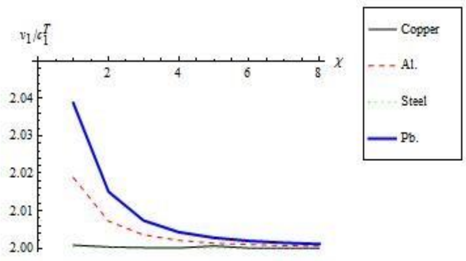

Fig.1: Variation of phase velocity with respect to χ

Here we observe that the phase velocity of the wave of Lead and Aluminum is more than the Copper and steel. As $\chi \to \infty$, all the curves touch to 2. The phase velocity of Copper and Steel is more or less same.

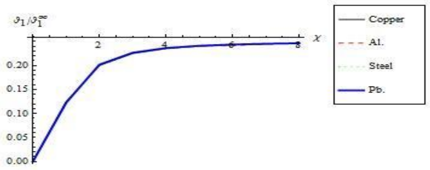

Fig.II: Variation of Damping coefficient with respect to χ

Figure-II shows that the damping coefficient increases as χ increases for Copper, Aluminum, Steel and Lead and it approaches the asymptotic value as $\chi \to \infty$. he damping coefficient is more or less same for all metals.

III. CONCLUSION

Finally, it can be concluded that the phase velocity of lead and aluminum is more than the phase velocity of copper and steel, whereas the damping coefficient of lead and aluminum is less than the damping coefficient of copper and steel.

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