

Credit Scoring for Financial Services Institution using Ant Colony Optimization Algorithm under Logistic Regression Model

Ulfa Rahmani, Sukono, Riaman



Abstract: Economic and trade activities are important in a country. All these activities are regulated by financial institutions, such as banks. The process of channeling funds to the public or known as credit is one of the tasks of the banking sector which aims to improve the people's economy. Credit granting is required for credit analysis, which is useful to determine the level of eligibility of a debtor to receive credit. The function of the credit analysis is to reduce the credit risk of prospective debtors who have failed to pay as well as to avoid financial institution losses or charges. The method used to analyze credit risk in this study is the Ant Colony Optimization algorithm in the Logistic Regression model. Past data held by each prospective debtor obtained from one financial institution in Indonesia is used as a feasibility parameter in this analysis. The results of the study showed that eight variables analyzed were five variables including the significant influence (age of debt (X_1), family dependents (X_2), value of the collection (X_4), the number of credit limits (X_6), and the term of the loan (X_8) while the other three variables (the amount of savings (X_3), income per month (X_5), net income (X_7) are not significant to the risk of default.

Index Terms: Credit risk, credit scoring, logistic regression, Ant Colony optimization algorithm.

I. INTRODUCTION

The role of banks is very dominant in the financial system and also an important role holder to support a country's economic progress. A bank is a financial institution whose main business is providing credit. Banking can help the community in business funding and in helping finance through the distribution of funds in the form of loans or loans to people who need funds. In lending, banks get income generated through installments and interest from each loan by the debtor. However, on the implementation of credit, banks are often faced with credit risk. Credit risk occurs because the

debtor does not fulfill the obligation to pay credit at a predetermined maturity. The other thing that causes credit risk or problem loans is the lack of selection from the bank. Basically, each bank has the same working principle, but banks are distinguished by the services that each bank has [1],[2].

Credit risk can be overcome by selecting prospective debtors based on past data held by the prospective debtor in the form of credit assessment given in the form of a credit score given by the credit bureau. A good credit rating model must be able to correctly rank customers from low to high probability, which is the basis of the approval strategy [3],[4]. The score obtained by the prospective debtor is very influential on the lender's decision to give credit and the conditions that apply [5], [6], [7]. Kusi et al. [8],[9], states that to support the quality of bank assets and reduce problem loans or problem loans, namely by sharing credit information reduces adverse bank selection.

The discussion of credit risk uses credit valuation techniques such as the credit assessment model for individuals, logistic regression and discriminant analysis in commercial banks in Pakistan [10], then Jakubik et al. [11] differentiating functional relationships between squared deviations from quarterly credit growth on credit growth based on financial stability. Test the distribution of credit information through private credit bureaus and public credit offices and their effect on bank credit risk in low and high income countries in Africa [9]. Approach methods have been carried out by researchers about credit assessment.

In this paper the author extends the discussion about credit valuation for banking using the Ant Colony Optimization algorithm estimation with a Logistic regression model in one of the financial service cooperatives in Indonesia. Credit assessment analysis is done to minimize credit or non-performing credit risk because the credit assessment analysis has not been applied to financial services..

II. LITERATURE STUDY AND DATA ANALYSIS

A. Data

The data used in this research was obtained from the Financial Services Institution in the period 2001-2011, consisting of 100 samples in 2 categories. Credit is not problematic or said to be feasible is category 0 which amounts to $n_0 = 85$ while non-feasible or non-performing loans are category 1 which amounts to $n_1 = 15$. The influential variables have 8 variables, including the age of debtors (X_1), family dependents (X_2), the amount of savings (X_3), the

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value of collateral (x_4), the amount of income per month (x_5), given the credit limit (x_6), take home pay (x_7), and the loan term (x_8) [12].

After the data is obtained, the normalization test is conducted to determine whether the data is normally distributed or not. This step is not a striking value that can cause errors in data analysis. Data that has been normalized is then carried out standard assessment with a logistic regression model. Furthermore, Ant Colony Optimization programming uses Matlab software.

B. Logistic Regression Model

Logistic regression analysis studied the relationship between categorical dependent variables and a set of independent variables (explanatory). Logistic regression is used when the dependent variable has only two values, such as 0 and 1 or Yes and No [12],[13].

Logistic Regression Analysis is used to predict the probability of an event occurring, by matching the data to the logit function. Logistic regression does not require the assumption of normality, heteroscedasticity, and autocorrelation, because the dependent variable is dichotomic / binary. The parameters of the Binary Logistic Regression model are estimated by the Maximum Likelihood method which is then solved by the Ant Colony Optimization algorithm. In this study the binary logistic regression equation used is:

$$\pi(x_i) = \frac{\exp(\beta_0 + \beta_1 x_{1i} + \dots + \beta_k x_{ki})}{1 + \exp(\beta_0 + \beta_1 x_{1i} + \dots + \beta_k x_{ki})} \quad i = 1, 2, \dots, N \quad (1)$$

$\pi(x_i)$: Chance of a successful event; $0 \leq \pi(x_i) \leq 1$

β_p : Parameter value; $p = 1, 2, \dots, k$.

Then logit transformation $\pi(x_i)$, is carried out, so the simpler equation is obtained, namely:

$$g(x) = \ln \left[\frac{\pi(x)}{1 - \pi(x)} \right] = (\beta_0 + \beta_1 x_1 + \dots + \beta_k x_k) \quad (2)$$

Independence between consecutive observations and the existence of a linear relationship between logit x and predictors x_1, x_2, \dots, x_k are important assumptions of logistic regression. The consideration is to determine whether the relationship between the independent variable and the probability of the event changes its mind or direction, or not before applying the logistic regression model [14],[15].

Logistic regression aims to estimate the parameters β_k ($k = 0, 1, \dots, k$) which affect equation (2) [15]. Suppose there are independent variables x_1, x_2, \dots, x_k , the conditional density function Y to β follows the Bernoulli distribution as follows [14].

$$f(y | \beta) = \prod_{i=1}^N \pi_i^{y_i} (1 - \pi_i)^{1-y_i} \quad y_i = 0, 1 \quad (3)$$

for each pair x_i, y_i the y_i variable is given code 0 and 1. If y_i then the contribution to the likelihood function is $1 - \pi(x_i)$ and if $y_i = 0$ then the contribution to the likelihood function

is $1 - \pi(x_i)$, where $\pi(x_i)$ denotes the value of $\pi(x_i)$ on x_i . So that the likelihood function equation is obtained from pair (x_i, y_i) [16].

The expectation value and variance of the aggregate claim model (3) can be calculated using equations (4) and (5) as follows [6; 7]:

$$l(\beta) = \sum_{i=1}^N \left\{ y_i \sum_{k=0}^K x_{ik} \beta_k - \ln \left(1 + e^{\sum_{k=0}^K x_{ik} \beta_k} \right) \right\} \quad (4)$$

The occurrence of the vector β in equation (4) is estimated to use Ant Colony Optimization.

C. Ant Colony Optimization

The optimization of ant colonies (ACO) is, as the name suggests, based on the natural behavior of their individual worker ants and ant colonies. When ants forage, they naturally seem to find "logical" and "effective" routes between their nests and food sources; in other words, they seem to determine the optimal route. This observed behavior is the basis of ACO [17],[18]. Imagine two ants running from a nest to a food source using two different routes. When ants run, they release pheromones which naturally decay over time. Ants that (randomly) choose a short route will start the journey back faster than other ants, thus strengthening the pheromone trail on shorter routes. Other ants will instinctively follow a pathway of pheromones that is stronger, strengthens and even adds it. According to Al-Behadili et al. [17] and Ning et al. [18], the ACO algorithm contains three main steps that form the central optimization circle of the algorithm:

1. Build Ant Solutions. This is a procedure in which "ants" gradually and stochastically build a path, namely solutions in the context of wider optimization;
2. Evaporating Pheromones. This is the process by which pheromones for certain solutions are reduced using "local" information; therefore, this step is also often referred to as local update. This step is very important to ensure that the ACO algorithm does not prematurely blend into a single solution;
3. Daemon action. This step refers to decisions made based on global information relating to optimization problems. Note the difference between local in step 2 and global in step 3. Analog to step 2, step 3 is also often referred to as global update.

In accordance with the ACO algorithm inspired by the behavior of uni-colonized colonies looking for food. Ants find the shortest distance between ant nests and the source of food. To signify the road they traveled, ants mark with pheromones. Pheromones are chemicals that come from the endocrine glands. The workings of pheromones are spread outside the body and can only affect and be recognized by other similar individuals, this process of pheromone inheritance is known as stigmergy. Ants will smell pheromones and tend to choose paths that have been marked with pheromones. If the ant has found the shortest path, the ants will continue through that path. Pathways marked by old pheromones will fade or evaporate and short pathways have a high probabilistic pheromone thickness, so ants can find the shortest path to return to food sources or to their nests.



- Path selection

An ant will run from point i to point j with probability

$$p_{i,j} = \frac{\left(\tau_{i,j}\right)^\alpha \left(\eta_{i,j}\right)^\beta}{\sum \left(\tau_{i,j}\right)^\alpha \left(\eta_{i,j}\right)^\beta} \quad (5)$$

Where $\tau_{i,j}$: the number of pheromones on the side i, j

α : influence controller parameters $\tau_{i,j}$

$\eta_{i,j}$: side desirability i, j (usually $1/d_{i,j}$, where d is distance)

β : influence controller parameters $\eta_{i,j}$

- Addition and Evaporation of Phomone

$$\tau_{i,j} = (1 - \rho)\tau_{i,j} + \Delta\tau_{i,j} \quad (6)$$

Where ρ : pheromone evaporation rate

$\Delta\tau_{i,j}$: the number of pheromones produced

- Update Phomone

$$\tau(t, v) = (1 - \alpha)\tau(t, v) + \alpha.\Delta\tau(t, v) \quad (7)$$

D. Estimated Significance Test Parameters

Parameter estimation is an estimate used to estimate a population of samples by drawing conclusions on parameter prices. The sample statistics obtained from the population will be estimated to find out the actual price of unknown parameters. In this paper, a significance test of the logistic model is conducted which aims to find out which significant independent variables. The significance test of the logistic regression model in this paper was carried out with several statistical tests, namely: Likelihood Ratio Test, Wald Test, Hosmer & Lemeshow Test, and R-Square (R^2).

1. Likelihood Ratio Test

Maximum Likelihood is a method that can be used to estimate a parameter in a regression in order to get an estimator for unknown parameters from a population with a maximum Likelihood function. Before forming a logistic regression model, a significant parameter test was first carried out. The first test conducted to test the effect of the role of parameters in the overall model, namely the hypothesis as follows:

$H_0 : \beta_1 = \beta_2 = \dots \beta_i = 0$ (the model has no significant effect)

$H_1 : \exists \beta_1 \neq \beta_2 \neq \dots \beta_i \neq 0$ (the model has a significant effect on the model) ; $i = 0, 1, \dots, p$

According to Hosmer and Lemeshow in 1989, the Likelihood Ratio test is to test the significance of all the coefficients of the independent variables in the model shown by the G statistics, whose equations are as follows [12], [19]:

$$G = 2 \left[\sum_{i=1}^n y_i \ln \pi_i + \sum_{i=1}^n (1 - y_i) \ln (1 - \pi_i) - n_1 \ln n_1 - n_0 \ln n_0 + n \ln n \right] \quad (8)$$

These statistics follow the Chi-square distribution with the degree of freedom equal to the number of independent variables. The criteria used are: If $G \geq \chi^2_{(1-\alpha)}(df)$ it H_0 is rejected and if $G < \chi^2_{(1-\alpha)}(df)$ then H_0 is accepted. Where α is the significant level specified, and $df = m - 1$ with m the number of model parameters.

2. Wald Test

Testing the hypothesis is intended to determine whether there is a significant influence between the independent variables to the dependent variable. In this test, the authors set a significant test, with the determination of the null hypothesis H_0 and alternative hypothesis H_1 .

Zero H_0 is a hypothesis which states that there is no significant influence between dependent and independent variables. While the alternative hypothesis H_1 is a hypothesis which states that there is a significant influence between the independent variable and the dependent variable. Tests carried out by the author in testing this hypothesis using Wald test (Wald test). The Wald test uses statistics Z , where these statistics Z follow the Normal Raw distribution. The statistics Z are:

$$Z = \frac{\beta_i}{SE(\beta_i)} \quad (9)$$

β_i : estimator for parameters (β_i)

$SE(\beta_i)$: estimator of standard error / error for coefficient β_i

The hypothesis model used is:

$H_0 : \beta_i = 0$ (does not have a significant effect between the independent variables on the dependent variable)

$H_1 : \beta_i \neq 0$ (has a significant influence between the independent variables on the dependent variable)

The Wald test hypothesis is $H_0 : \beta_i = 0$ (logit coefficients are not significant for the model) and $H_1 : \beta_i \neq 0$ (logit criteria are significant for the model). The criteria used are if $-Z_{\frac{1}{2}(1-\alpha)} < Z < Z_{\frac{1}{2}(1-\alpha)}$ then H_0 is accepted and if

$Z_{\frac{1}{2}(\alpha)} \leq Z \leq Z_{\frac{1}{2}(1-\alpha)}$ then H_0 is rejected. Where $Z_{\frac{1}{2}(\alpha)}$ is

the percentile of a standard normal distribution with level significance α [12], [19]-[20].

3. Test Hosmer dan Lemeshow

According to Hosmer and Lemeshow in 1989, the Hosmer and Lemeshow test is known as the model compatibility test in Logistic Regression of data. The Hosmer-Lemeshow statistics follow the Chi-Square distribution with $df = g - 2$ where g are the number of groups. The equation of this test is as follows [12], [19]:

$$C = \sum_{i=1}^g \frac{(o_i - n_i \bar{\pi}_i)^2}{n_i \bar{\pi}_i (1 - \bar{\pi}_i)} \quad (10)$$

n_i : Total frequency of group observations i th

o_i : Frequency of observation of group i th

$\bar{\pi}_i$: Average estimated group opportunity i th

To test the suitability of the model, the Chi-square value obtained was compared with the Chi-Square value in the Chi-Square table with degrees of freedom $df = g - 2$. In general use $g = 10$.

The hypothesis used is as follows:

H_0 : there is no difference between observations and the model used

H_1 : there is a difference between the results of observations with the model used

Test the criteria used, namely:

H_0 rejected if $C \geq \chi^2_{(1-\alpha)(g)}$ and H_1 accepted if $C < \chi^2_{(1-\alpha)(g)}$.

4. R^2 Test

According to Hosmer and Lemeshow in 1989, the value of R^2 in the Logistic Regression analysis shows the strong relationship between independent variables and free variables. For the value of R^2 it is:

$$R^2 = 1 - \exp \left[- \left(\frac{L^2}{n} \right) \right] \tag{9}$$

where: n : Likelihood log value of the model and n : amount of data. If $R^2 \rightarrow 1$, then the relationship between the independent variable and the dependent variable is strong and if $R^2 \rightarrow 0$, then the relationship is weak [12], [19],[20].

III. RESULTS AND DISCUSSION.

Data analysis is done in the manner described in part II.A, before the data is carried out further, a normality test that applies to multivariate analysis is carried out. In the multivariate analysis the normality test aims to determine whether the data distribution is close to or follows a normal distribution. Data that has a pattern like a normal distribution is good data for multivariate analysis. Data can be used to estimate the parameters of a binary logistic model after data is normally distributed. For this reason, a data normality test is conducted.

A. Result

The vector estimator $\beta = (\beta_0, \beta_1, \dots, \beta_8)$ is obtained using the parameter estimation of Binary Logistic Regression by maximizing the Likelihood function in equation (6). Estimates were made using the Ant Colony Optimizain algorithm using Matlab, while for estimates and Standard Error (SE) values were performed using SPSS. The results obtained are as follows:

Table 1. Estimating Significant Variable Parameters

Parameter Coefficient	Parameter Estimator	Standard Error	Ratio	Significance
Age (X_1)	0.4314	0.974	0.443	Significance
Family dependents (X_2)	0.1555	0.415	0.375	Significance
The value of collateral (X_4)	0.7686	1.918	0.401	Significance
Given the credit limit (X_6)	-0.2139	2.045	-0.105	Significance

The loan term (X_8)	-0.6420	0.646	-0.994	Significance
Constant	0.2668	0.535	0.499	Significance

Maximum Likelihood value $\hat{\beta} = 3.0000$

Based on the table above obtained significant results namely X_1, X_2, X_4, X_6 and X_8 . The parameters that are not significant to the parameters are removed from the model because they have no significant effect on the model. Furthermore, analyzing the suitability of the Logistics model with data, namely the Hosmer & Lemeshow statistical test using equation (8).

The Hosmer & Strength statistical test uses Statistics (8). The significant level used is $\alpha = 0.05$. In this study, obtained was 0.317, which caused no difference between observations and estimator models.

Testing the relationship between the independent variable and the dependent variable using the value of R^2 is done by equation (12). Then, an R^2 value of 0.874 is obtained, this R^2 value is calculated by re-estimating the data obtained, among others, Age, dependents of the family, value of the collateral, number of credit limits, and loan term which is dependent on the variable $\pi(X)$ very significant probability. Therefore, the estimated logistic regression is obtained as follows:

$$\pi(X) = e^{3.000 + 0.443X_1 + 0.375X_2 + 0.401X_3 - 0.105X_4 - 0.994X_8} \tag{11}$$

The Logistic Regression equation above is a default problem obtained from debtor data in the opinion of the borrower credit.

B. Analysis

Credit worthiness decisions are made using risk prediction (valuation) by considering the probability of defaulting on the prospective debtor. Here are the credit risk titles :

Table 2. Predicate Credit Risk

Probability of default (Credit risk)	Predicate	Description
$0.00 < \pi(X) \leq 0.49$	A	Decent
$0.49 < \pi(X) \leq 0.69$	B	Pretty Decent
$0.69 < \pi(X) \leq 1.00$	C	Not Decent

Illustration. a person submits an application for borrowing credit, by providing the following data: age (X_1) 40 years; have dependents (X_2) 4 people; total savings in financial services institutions (X_3) of IDR 750,000; credit loan (X_6) submitted for IDR 11,000,000; and the loan period (X_8) desired for 24 months (or 2 years).

Analysis. Based on the data submitted by someone applying for the loan above, by using equation (11) it can be determined that the probability of default (credit risk) for a prospective loan borrower is 0.5584057.

Based on the credit risk predicate in Table 2, that someone who is applying for a loan belongs to the predicate B (Pretty Decent).



Predicate B Thus, financial service institutions can make the decision to accept or reject an application for a creative loan submitted by a prospective borrower. If there is doubt, the financial services institution can request additional data that can improve the credit worthiness rating. For example, by asking to lower the credit limit (X_6) submitted, so that after analyzing the calculation the default probability can be increased to the predicate A (Decent).

By using the logistic regression equation estimator (11) and the predicate of Probability of default (credit risk) in Table 2, the credit risk above can be used to analyze credit in a Financial Services Institution. Credit assessment analysis aims to reduce the risk of default on potential borrowers who apply for credit. Because prospective borrowers who have a high probability of default, will result in losses for Financial Services Institution. Therefore, Financial Services Institution are able to do a good credit scoring analysis based on debtor data and facts, in order to avoid possible losses.

IV. CONCLUSION

To analyze credit in a Financial Services Institution it is estimated using the Ant Colony Optimization Algorithm with the Logistic Regression model. Age, family dependents, value of collateral, total credit limit, and loan term have a significant effect, while the other three variables are not significant to the risk of default. Furthermore, Logistic Regression is used to estimate the probability of default, the results of which will be taken into account, seen at the feasibility interval to determine the level of eligibility of prospective debtors. The decision of financial services to provide credit is based on the predicate obtained by each debtor.

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