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The Square of A Directed Graph

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Abstract: *The square of an oriented graph $D^2 = (V, E^2)$ is an oriented graph $D = (V, E)$ such that $(v_i, v_j) \in E^2$ if and only if for some $v \in V$, both $(v_i, v_k) \in E$ and $(v_k, v_j) \in E$ exist. According to the square of oriented graph conjecture (SOGC), there exists a vertex $v \in V$ such that $d^+_{D^2}(v) \geq 2d^+_D(v)$. It is a special case of a more general Seymour's second neighborhood conjecture (SSNC) which states for every oriented graph D, there exists a vertex $v \in V$ such that $|N^{++}_D(v)| \geq |N^+_D(v)|$. In this study, the methods to square a directed graph and verify its correctness were introduced. Moreover, some lemmas were introduced to prove some classes of oriented graph including regular oriented graph, directed cycle graph and directed path graphs are satisfied the SOGC. Besides that, the relationship between SOGC and SSNC are also proved in this study. As a result, the verification of the SOGC in turn implies partial results for SSNC.*

Keywords *: square of directed graph, oriented graph, Seymour's second neighborhood conjecture.*

I. INTRODUCTION

Throughout this paper, all considered oriented graph are directed graphs with no loops, multiple or bidirected edges. Let the outdegree of a vertex v in oriented graph D denoted as $d^+_D(v)$ and $d^{++}_D(v)$ for distance of 1 and 2 respectively. The out-neighborhood set of v in D at distance 1 is denoted as $N^+_D(v)$ while $N^{++}_D(v)$ for distance of 2. We also denote $d^+_D(v) = |N^+_D(v)|$ and $d^{++}_D(v) = |N^{++}_D(v)|$. The square of an oriented graph D^2 contains an edge (v_i, v_j) if there is a path $v_i \to v_k \to v_j$ in the oriented graph D. Seymour posed the conjecture concerning the square of oriented graph (SOGC) which stated that every oriented graph D has a vertex v such that $d^+_{D^2}(v) \geq 2d^+_D(v)$ [1]. It is a special case of a more general Seymour's second neighborhood conjecture (SSNC) which stated that every oriented graph D has a vertex v such that $|N^{++}_D(v)| \geq |N^+_D(v)|$ [2]. Dean and Latka verified that the SSNC holds for regular tournaments, almost regular tournament, and tournaments that contain minimum $d^+_T \leq 5$

Fisher proved the Dean's conjecture concerning the proof of SSNC for tournament T such that the minimum $d^+_T \leq 5$ by using a certain probability distribution over vertices called losing density and a basic Markovian argument [2]. Havet and Thomassé proved the Dean's conjecture by using another tool called median order. They showed that a tournament with no sink (a vertex with outdegree zero) contains at least two satisfactory vertices [3]. Beside that, Kaneko and Locke showed that SSNC to be true for the digraph D with minimum $d^+ \leq 6$ [4]. Chen, Shen and Yuster showed that in every digraph D there contains around 0.678 satisfactory vertex [5]. Besides, another approach was applied by Fidler and Yuster which is weighted median orders. Furthermore, they proved that SSNC holds for digraphs D with minimum degree $|V(D)| - 2$, tournaments missing a star and tournaments missing a subtournament [6-13].
Cohn, Wright and God bolen showed that the SSNC is true for almost all random digraphs. However, Brantner, Kay and Snively focused on implications of SSNC being false and the properties of counter example to SSNC are presented [14-20].

II. METHODOLOGY

Steps to square a directed graph

A. Steps to Square a Directed Graph

First of all, consider the list of the vertices in the digraph. Secondly, a new edge is added to the digraph between two distinct vertices (assuming it does not already exist) if there exists a path of distance at exactly two edges between the mentioned vertices. Then list down the new edge. Lastly, examine each of the vertices in the digraph until a complete list is done and get the squared digraph.

B. Adjacency Matrix

Adjacency matrix A is a square matrix and can be used to represent a digraph D. In $A(D)$, the element a_{ij} represents the number of edges that directed from vertex i to j. For an oriented graph, the adjacency matrix is a $(0,1)$-matrix with zeros on its diagonal (no loop) and the symmetric elements a_{ij} and a_{ji} for $i \neq j$ could not be bigger than zero at the same time to avoid bidirected edges. The sum of the row i is the outdegree of the vertex i.

C. Lemmas

Some supporting lemmas are introduced to prove the truth of SOGC for regular oriented graph, directed cycle graph and directed path graph.

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The supporting lemmas also used to verify the relationship between the SOGC and SSNC.

D. Software

We make use of MATLAB (matrix laboratory) to create the MATLAB code for squaring a directed graph. Beyond that, Maple software was also used to draw the digraph and its square.

III. RESULTS AND DISCUSSION

In Fig. 1 explain the code used to a directed graph.

MATLAB code for squaring process

```
%To square a directed graph
n=input('Please enter the number of vertices: ');
A=input('Please input the adjacency matrix of digraph, D: \n');
for i= 1:n
    for j= 1:n
        A2(i,j)=0;
        for k= 1:n
            if A(i,k)>0 && A(k,j)>0 && A(i,j)==0;
                A2(i,j)=1;
                break;
            end
        end
    end
end
disp('Adjacency matrix of digraph, A(D)=')
disp(A)
disp('New edge(s) added=')
disp(A2)
disp('Adjacency matrix of squared digraph, A(D^2)=')
disp(A+A2)
```

Fig. 1. MATLAB code used to square a directed graph

Example 1: Consider an oriented graph D and its adjacency matrix $A(D)$. The results obtained by the MATLAB code are the adjacency matrix for new edges added and the adjacency matrix $A(D^2)$ of its squared oriented graph D^2 in Figure 2.

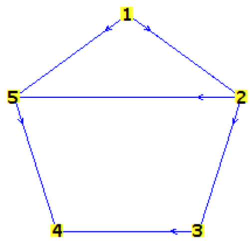

Fig. 2. Oriented graph D

$$A(D) = \begin{bmatrix} 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

In Fig. 3 new edges added graph and Figure 4 squard D^2

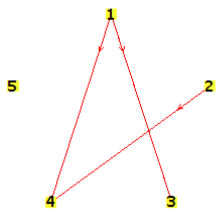

Fig. 3. New edges added

$$A(new\ edges) = \begin{bmatrix} 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

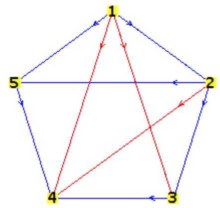

Fig. 4. Squared D^2

$$A(D^2) = \begin{bmatrix} 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

IV. VERACITY OF SQUARED GRAPH BY ITS ADJACENCY MATRIX

The equation $A(D^2) = [A(D)]^2 \mid A(D)$ is used to define the correctness of squared digraph that obtained from previous method. The element a_{ij} in $A(D^2)$ will return to 1 if either a_{ij} in $[A(D)]^2$ or $A(D)$ is bigger than 0; otherwise returns to 0. There is the result as follow by using the Example 1.

$$A(D^2) = [A(D)]^2 \mid A(D)$$

$$= \left(\begin{bmatrix} 0&1&1&1&1 \\ 0&0&0&2&0 \\ 0&0&0&0&0 \\ 0&0&0&0&0 \\ 0&0&0&0&0 \end{bmatrix} \middle| \begin{bmatrix} 0&1&0&0&1 \\ 0&0&1&0&1 \\ 0&0&0&1&0 \\ 0&0&0&0&0 \\ 0&0&0&1&0 \end{bmatrix} \right)$$

$$= \begin{bmatrix} 0&1&1&1&1 \\ 0&0&1&1&1 \\ 0&0&0&1&0 \\ 0&0&0&0&0 \\ 0&0&0&1&0 \end{bmatrix}$$

V. THE PROOF OF SQUARE OF ORIENTED GRAPH CONJECTURE (SOGC)

There are several preconditions to show that a digraph D is satisfies the SOGC. First of all, the digraph must be a proper oriented graph with 2-distance path (a path that contains exactly 2 directed edges). Consequently, it can be proceed to squaring process. After squaring, there must be a vertex v with out degree at least doubles in its square D^2 compared to original digraph D. The following lemmas are used to prove our main results.

A. Lemma

Let $D = (V, E)$ be a k-regular oriented graph with n vertices, then we have $k \leq \frac{n-1}{2}$.

Proof. Each vertex can direct to the other $(n-1)$ vertices. Therefore, there are maximum $n(n-1)$ edges can be formed in D. However, $n(n-1)$ need to be divided by 2 to avoid forming of bidirected edges. As D is a regular digraph, therefore there must be the same number of directed edge(s) that directed from each vertex. To fulfill this situation, $\frac{n(n-1)}{2}$ must be divided by n. Consequently, we have $\frac{n-1}{2}$. To ensure that $\frac{n-1}{2}$ is a whole number, so we have $\left\lfloor \frac{n-1}{2} \right\rfloor$.

B. Lemma

Let $D = (V, E)$ be a k-regular oriented graph with n vertices. Then there are k^2 2-distance paths for every vertex $v \in V(D)$.

Proof. Since it is a k-regular digraph which means that there are k outdegree, $d^+(v) = k$ for every vertex $v \in V$. Note that there is no loop for an oriented graph; therefore it is impossible for a vertex v directed to itself. Consequently, each vertex v is directed to other k vertices and forms the path(s) with minimum distance of 2. Hence, there must be k^2 2-distance paths for every vertex $v \in V(D)$.

C. Lemma

Let $D = (V, E)$ be a k-regular oriented graph with n vertices. Then we get at least k new edge(s) formed which direct from each vertex during squaring process.

Proof. In this proof, we want to show that there are at least k new edge(s) can be formed even though there is maximum number of 1-distance path that represents the 2-distance path in the original digraph D since the existence of the mentioned 1-distance path causes the failure of new edge formed in D^2. In the other word, we try to form $v_i \rightarrow v_j$ if $v_i \rightarrow v_k \rightarrow v_j$ is exist in the original digraph D so that the possibility of new edge(s) formed during the squaring process can be reduced.

Consider a source vertex v_0 directed to k vertices $v_1, v_2, \dots, v_k$ which denoted as $N_D^+(v_0)$. Then, the vertex $v_i \in N_D^+(v_0)$ may direct to $k - i$ vertices within the same set for $i = 1, 2, \dots, k$ to increase the possibility of 1-distance path formed. The $N_D^{++}(v_0)$ set that represents $v_{a1}, v_{a2}, \dots, v_{ak}$ is introduced. Therefore, the vertices in $N_D^+(v_0)$ set may direct to the vertices of $N_D^{++}(v_0)$ set to make sure that each vertex in $N_D^+(v_0)$ is directed to k vertices Fig. 5.

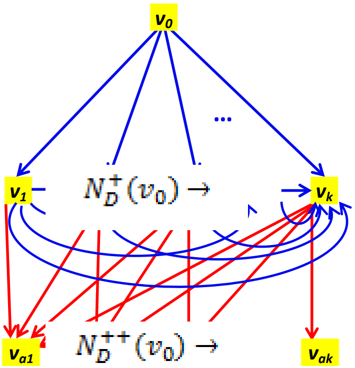

Fig. 5. A source vertex v_0 directed to k vertices $v_i \in N_D^+(v_0)$ and consequently directed to k vertices $v_i \in N_D^{++}(v_0)$.

Table- I: List of new edge(s) formed during squaring process

2-distance paths	*New edge(s) formed during squaring*	*Number of new edge(s) formed*
$v_0 \rightarrow v_1 \rightarrow v_{a1}$ $v_0 \rightarrow v_2 \rightarrow v_{a1}$ $v_0 \rightarrow v_3 \rightarrow v_{a1}$ $\vdots$ $v_0 \rightarrow v_k \rightarrow v_{a1}$	$v_0 \rightarrow v_{a1}$	1
$v_0 \rightarrow v_2 \rightarrow v_{a2}$ $v_0 \rightarrow v_3 \rightarrow v_{a2}$ $\vdots$ $v_0 \rightarrow v_k \rightarrow v_{a2}$		
$v_0 \rightarrow v_2 \rightarrow v_{a2}$ $v_0 \rightarrow v_3 \rightarrow v_{a2}$ $\vdots$ $v_0 \rightarrow v_k \rightarrow v_{a2}$	$v_0 \rightarrow v_{a2}$	1
$v_0 \rightarrow v_3 \rightarrow v_{a3}$ $\vdots$ $v_0 \rightarrow v_k \rightarrow v_{a3}$	$v_0 \rightarrow v_{a3}$	1
$\vdots$	$\vdots$	$\vdots$
$v_0 \rightarrow v_k \rightarrow v_{ak}$	$v_0 \rightarrow v_{ak}$	1
Total number of new edge(s) formed		k

In table I there are still k new edges that directed from v_0 can be formed during squaring process even though we try to reduce its possibility of new edge(s) formed. As a result, there is at least k new edges which direct from each vertex can be formed during square a k-regular oriented graph.

D. Theorem

For every k-regular oriented graph D, there exists a vertex whose out degree at least doubles in its squared regular oriented graph D^2. This implies that there exists a vertex $v \in V(D)$ such that $d_{D^2}^+(v) \geq 2d_D^+(v)$.

Proof. A proper k-regular oriented graph D can only be formed by 3 or more vertices $(n \geq 3)$ with $1 \leq k \leq \left\lfloor \frac{n-1}{2} \right\rfloor$. Since there are k^2 2-distance paths for each vertex of original k-regular oriented graph D by Lemma 1, therefore it can be proceeding for squaring process. By Lemma 2, there are at least k new edge(s) formed which direct from each vertex after squaring, so the out degree of each vertex is at least doubles in its squared graph D^2. Hence it is satisfies the square of oriented graph conjecture.

E. Theorem

Let $C = (V, E)$ be a directed cycle graph with n vertices. Then there exists a vertex v such that $d_{C^2}^+(v) \geq 2d_C^+(v)$.

Proof. A directed cycle graph C is a cycle graph with all of the directed edges oriented in the same direction. It has uniform indegree and out degree of 1. The graph is formed by the edges $v_i \rightarrow v_{i+1}$ where $i = 1, 2, \dots, n-1$ and a directed edge $v_n \rightarrow v_1$ in Fig. 6. It also is an oriented graph since there is no loop or bidirected edge in directed cycle graph. Let us consider $n \geq 3$ since an oriented cycle graph could not be formed for $n < 3$.

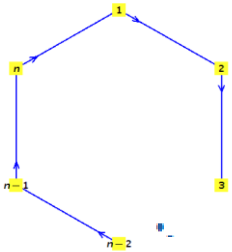

Fig. 6. Directed cycle graph C

Since there is $d_C^+(v) = 1$ for each vertex $v \in V(C)$, therefore it must have a 2-distance path for every vertex v. Consequently, a new edge that directed from each vertex v can be formed in its square C^2. Thus, there is $d_{C^2}^+(v) = 2$ for each vertex v in C^2 and it is satisfied $d_{C^2}^+(v) \geq 2d_C^+(v)$.

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Therefore, a directed cycle graph is satisfied the square of oriented graph conjecture in Fig. 6.

F. Lemma

Let $D = (V, E)$ be an oriented graph with n vertices such that $d^+(v) = 0$ for some $v \in V(D)$. Then the square of oriented graph conjecture is satisfied by the vertex v.

Proof. If there is a vertex v with $d_D^+(v) = 0$ in D, then there is no new edge that direct from vertex v can be formed in D^2 since there is no 2-distance path that direct from vertex v. As a result, $d_{D^2}^+(v) = 0$ for vertex v in D^2 and it is satisfied $d_{D^2}^+(v) \geq 2d_D^+(v)$. Consequently, the square of oriented graph conjecture is satisfied by the vertex v with outdegree 0.

G. Theorem

Let $P = (V, E)$ be a directed path graph with n vertices. Then there exists a vertex v such that $d_{P^2}^+(v) \geq 2d_P^+(v)$.

Proof. A directed path graph P is a sequence of directed edges which connect a sequence of vertices in the same direction. The vertices can be listed in the order $v_1, v_2, \ldots, v_n$ such that the edges are $v_i \rightarrow v_{i+1}$ where $i = 1, 2, \ldots, n-1$. There are two terminal vertices v_1 (with outdegree 1) and v_n (with indegree 1) while all others (if any) have indegree 1 and outdegree 1 at the same time (see Fig. 7). It also is an oriented graph since there is no loop or bidirected edge allowed. Let consider $n \geq 2$ since an oriented path graph could not be formed for $n < 2$ in Figure 7.

$$1 \rightarrow 2 \rightarrow 3 \; \ldots \; n \rightarrow n \rightarrow n$$

Fig. 7. Directed path graph P

There must be a sink v_n with $d_P^+(v_n) = 0$ in a directed path graph P. Thus, by Lemma F, the square of oriented graph conjecture is satisfied by the vertex v_n.

H. Lemma

Let $D = (V, E)$ be an oriented graph. Then for every $v \in V(D)$, we have $|N_{D^2}^+(v)| = |N_D^+(v)| + |N_D^{++}(v)|$.

Proof. According to the definition of the square of an oriented graph, the first out-neighborhood of v denoted as $N_D^+(v)$ cannot have a common element in the second out-neighborhood of v denoted by $N_D^{++}(v)$. In the other saying, the new edge(s) formed when we square the graph could not presently exist in original graph D. Therefore, we have $|N_{D^2}^+(v)| = |N_D^+(v)| + |N_D^{++}(v)|$.

I. Theorem

Let $D = (V, E)$ be an oriented graph. If Seymour's second neighborhood conjecture (SSNC) is true, then the square of oriented graph conjecture (SOGC) is true.

Proof. If the SSNC is true, then for all vertex $v \in V(D)$ such that $|N_D^{++}(v)| \geq |N_D^+(v)|$. Let $|N_D^+(v)| = m$, then we have $|N_D^{++}(v)| \geq m$. From Lemma H, we get $|N_{D^2}^+(v)| = |N_D^+(v)| + |N_D^{++}(v)| \geq 2m$. Since, $d_D^+(v) = |N_D^+(v)|$ and $d_{D^2}^+(v) = |N_{D^2}^+(v)|$. Thus $d_{D^2}^+(v) \geq 2d_D^+(v)$ and the SOGC is true if the SSNC is true.

VI. CONCLUSIONS

At the end of this study, the square of oriented graph conjecture (SOGC) is verified for the regular oriented graph, directed cycle graph and directed path graph. The relationship between Seymour's second neighborhood conjecture (SSNC) and SOGC is also being verified. The proof showed that the SOGC is true if SSNC is true for oriented graph. Beyond that, the verification of SOGC in turn implies partial results for SSNC and consequently the Caccetta-Häggkvist conjecture and Behzad-Chartrand-Wall conjecture.

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