Characterization of Functionally Graded Timoshenko Beams with Variable Rotational Speed

S.N.Padhi, K. S. Raghu Ram, Jasti Kasi Babu, T.Rout

Abstract: *This paper investigates the free vibration characteristics and stability of a functionally graded Timoshenko beam spinning with variable angular speed. Material properties of the beam are assumed to be varied continuously along the thickness of the beam according to a power law and exponential law. The results show that increasing beam rotational speed increases fundamental mode frequency and the beam becomes more and more stable at higher speeds. This paper reports the dynamic behaviour of a rotating FGM beam subjected to axial periodic forces using the finite element method. The numerical results show good agreement with the reported beams models. Effects of static and time dependent components of axial loads on the stability of the FGM beam have been studied.*

Index Terms: Exponential distribution, Timoshenko beam, load factor, Power law, Rotational speed, Stability.

I. INTRODUCTION

The modern engineering application demands the design and analysis of rotating shafts, beams, and gears. The whirling of beams has also been increasingly used in the exploration of space. The vibration analysis and hence the stability of a rotating cantilever beam is performed in this paper because it may represent many of these structures.

Composite materials have been used for several years for their advantage of achieving desired properties. The Metal Matrix Composite includes two metals which includes aluminum, and chromium. Aluminum is chosen because of its superior strength to weight ratio. Chromium is chosen because of its strength-weight ratio. The fabrication of composite material is done through stir casting method. Deformation of composite done using manual rolling after the casting of composite. Further analysis of composite includes microstructural study, hardness values and machinability results. The specimens are collected for every work to analyses the composite[1,2]. The determination of designing MMC is to enhance the required characteristic of metals and porcelains to the base metals. Aluminium Metal Matrix Composites (AMMCs) are significantly important in the

structural, aerospace, medicine, marine and automobile applications. [3]. The fiber orientation in composites has a significant effect on the properties of natural fiber-reinforced polymer composites.Longitudinally aligned fiber composites show maximum strength along the direction of the fiber reinforcement and they are orthotropic in nature. In the transverse direction, composite properties are lower than those in unidirectional, which may also be less than that of a neat polymer matrix sample[4]. Friction Stir Processing (FSP) is a new method by which the surface modification can be done for alloys and composites. Friction Stir Processing can enhance the mechanical properties of the composite[5]. Vibrating structures under rotation such as compressors, motors, pumps and micro-electro-mechanical systems is a naturally occurring phenomenon and results severe vibration in a structural resonant mode with an excitation by harmonic loading because of imbalanced rotor or variable fluid dynamic force, which causes heavy mechanical damage. Thus, the understanding of stability and dynamic response of rotating structures inservice is highly important to avoid the risk of such resonance problems. In real life, the above mentioned rotating structures are normally pretwisted and the cross-section is asymmetric in nature. However, Prismatic beams under rotation may be used as a sample model and compared at par with the actual rotating structures for investigation of stability and dynamic response. The research on functionally graded materials (FGMs) is rapidly growing because of its ability to meet desired material properties in contrast to the conventional homogeneous and layered composite materials which suffer from debonding, huge residual stress, locally large plastic deformations etc. An FGM can be a good replacement for the material of rotating beams. Timoshenko beam theory and classical Ritz method is employed to derive the governing equations. In order to solve the nonlinear governing equations, direct substitution iterative technique is used. Effects of various parameters such as rotating speeds, radius of hub, depth of crack, location of crack, and different functionally graded material properties on linear and nonlinear vibration characteristics are studied [6-8]. From a mechanics viewpoint, the main advantages of material property grading appear to be improving bonding strength, toughness, wear and corrosion resistance, and reduced residual and thermal stresses. Therefore, now-a-days, an FGM has been a promising candidate for many engineering applications where a high temperature gradient field is the main concern [9-11]. Effects of rotary inertia and shear deformation are not negligible for thick beams or even thin beams that are vibrating at high frequencies. Due to their dimensions, resonance frequencies of micro- and nano-scale resonators are extremely high, namely in the range of kHz to GHz [12].

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Retrieval Number: D8974118419/2019©BEIESP
DOI:10.35940/ijrte.D8974.118419
Journal Website: www.ijrte.org

5926

Published By:
Blue Eyes Intelligence Engineering
& Sciences Publication

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The high resistance of the ceramic at various temperature gradients, thermal stress, abrasion and oxidation and the toughness and strength of metal can be taken advantage of at the same time[13].

Finite element method used to study the dynamic stability analysis of functionally graded sandwich (FGSW) rotating cantilever beams reveals that better stability of the beam under vibration can be obtained following exponential law[14]. The vibration analysis of a rotating tapered shaft in Functionally Graded Material (FGM) is modeled using the Timoshenko beam theory (FSDBT) with consideration of gyro-scopic effect and rotary inertia. The equations of motion are expressed by the hierarchical finite element method based on bi-articulated boundary conditions [15-18]. Usually the FGM beams are subjected to higher rotational speeds, but ahead of their complete break down the incidence of vibration, plastic failure or creep relaxation can create serious damages which finally prevent the increase of the rotational speed. An important aspect in the analyses of heterogeneous FGM discs is the study of their creep relaxation. One of the well-known constitutive equations for the modeling of creep phenomenon is known as the Shery's law. Based on the steady state creep, the behavior of a variety of FGM rotating discs are studied. The analysis considers the conditions in which the distribution of volume fraction follows a power-law pattern [19]. Many machine and structural components can be modeled as rotating beams. These rotating beam structural members may undergo forced vibration and dynamic instability during their service period, which may even lead to their failure. So the study of effect of the system parameters on free vibration behavior of FGO and FGSW beams forms an important aspect of investigation [20-23].

Many researchers have been found to have reported enough number of articles on static and dynamic stability of ordinary beams, the literature on dynamic stability of functionally graded rotating beams are found to be not in adequate number to the best of the authors' knowledge. The present article details on the impact of rotation of a functionally graded Timoshenko beam with fixed-free support condition on the dynamic stability of the beam.

II. FORMULATION

A functionally graded Timoshenko beam with alumina as top skin, steel as bottom skin is shown in Fig. 1(a). The beam is fixed at one end and free at the other end. A pulsating axial force $P(t) = P_s + P_t \cos \Omega \, t$, is applied on the beam and acting along its undeformed axis. Where P_s is the static component of the axial force. P_t and t are respectively the amplitude, and t is time and Ω is the frequency of load. Fig. 1(b) shows a two noded finite element coordinate system used to derive the governing equations of motion. Fig. 1(b) shows the expression for the displacements on (x-y) plane (reference plane) at the centre of the longitudinal axis. The thickness coordinate is measured as 'z' from the reference plane. The axial displacement and the transverse displacement of a point on the reference plane are, and respectively and is rotation of cross-sectional plane with respect to the un-deformed

configuration. Figure 1(c) shows a two-nodded beam finite element having three degrees of freedom per node.
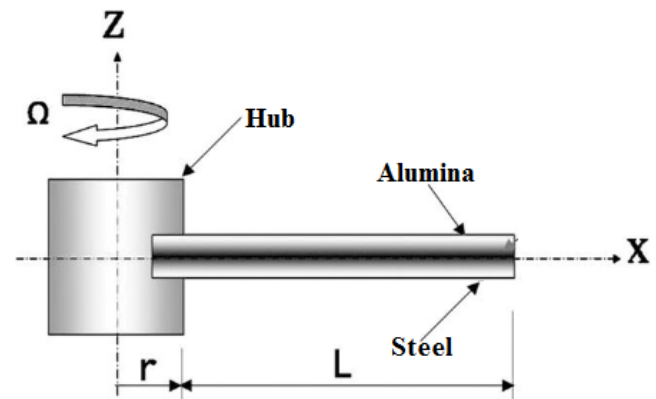

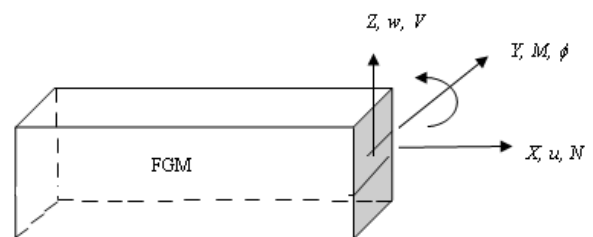**Figure 1(a) Functionally graded ordinary beam subjected to dynamic axial load.**

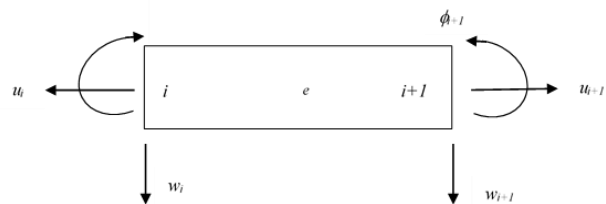**Figure 1(b) The coordinate system with generalized forces and displacements for the FGO beam element.**

Figure 1(c) Beam element showing generalized degrees of freedom for ith element.

A. Shape Functions

According to the first order Timoshenko beam theory the displacement fields are expressed as

$$U(x, y, z, t) = u(x,t) - z\phi(x,t),$$
$$W(x, y, z, t) = w(x,t), \tag{1}$$

Where U = axial displacement and W = transverse displacement of a material point. The respective linear strains are

$$\varepsilon_{xx} = \frac{\partial u}{\partial x} - z\frac{\partial \phi}{\partial x}, \quad \gamma_{xz} = -\phi + \frac{\partial w}{\partial x} \tag{2}$$

The matrix form of stress-strain relation is

$$\{\sigma\} = \begin{Bmatrix} \sigma_{xx} \\ \tau_{xz} \end{Bmatrix} = \begin{bmatrix} E(z) & 0 \\ 0 & kG(z) \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx} \\ \gamma_{xz} \end{Bmatrix} \tag{3}$$

Where $\sigma_{xx}, \tau_{xz}, E(z), G(z)$ and k are the normal stress on x-x plane, shear stress in x-z plane, Young's modulus, shear modulus and shear correction factor respectively. The variation of material properties along the thickness of the FGM beam governed by

(i) Exponential law is given by

$$M(z) = M_t \exp(-e(1 - 2z/h)) \qquad (4)$$

$$e = \frac{1}{2} \log\left(\frac{M_t}{M_b}\right), \text{ and}$$

(ii) Power law is given by

$$M(z) = (M_t - M_b)\left(\frac{z}{h} + \frac{1}{2}\right)^n + M_b \qquad (5)$$

Where, $M(z)$ can be any one of the material properties such as, E, G and ρ etc., denote the values of The corresponding properties at top and bottommost layer of the beam are represented by M_t and M_b respectively, and the power index is n. The change in the values of ρ of FGM governed by power law with different indices along the thickness and a comparison between power law and exponential law is shown in Fig. 2(a) and Fig. 2(b) respectively.
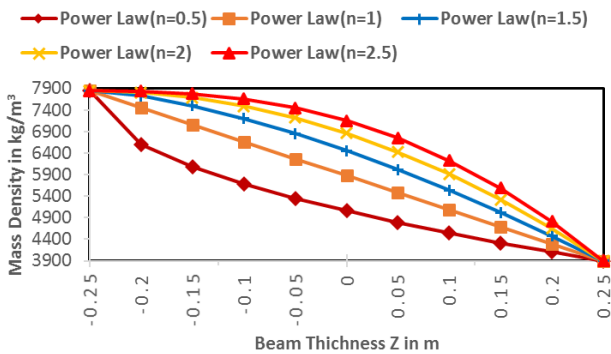

Fig. 2(a) Change in Mass Density (ρ) along depth(Z) of FGM beam with steel-rich bottom and alumina-rich top according to power law with various indices.
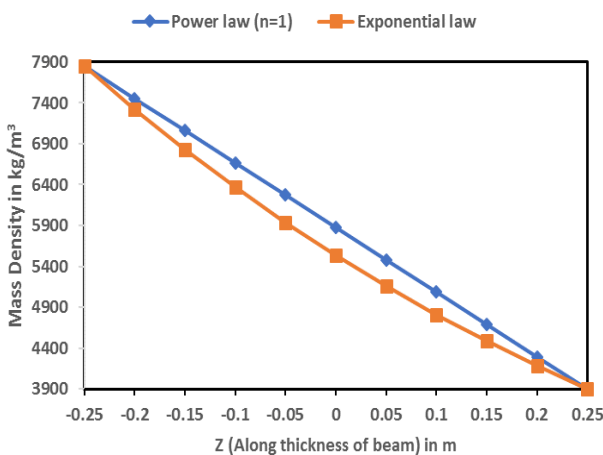

Fig. 2(b) Comparison of change in Mass Density (ρ) along depth(Z) of FGM beam with steel-rich bottom and alumina-rich top according to power law at n = 1 and exponential law.

Now the shape function can be expressed as

$$\aleph(x) = \begin{bmatrix} \aleph_u(x) & \aleph_w(x) & \aleph_\phi(x) \end{bmatrix}^T \qquad (6)$$

where, $\aleph_u(x)$, $\aleph_w(x)$, $\aleph_\phi(x)$ are the shape functions for the axial, transverse and rotational degree of freedom respectively.

B. Element Elastic Stiffness Matrix

The element elastic stiffness matrix is given by the relation

$$[k_e]\{\hat{u}\} = \{F\} \qquad (7)$$

where, $\{F\}$ = nodal load vector and $[k_e]$ = element elastic stiffness matrix.

C. Element Mass Matrix

The element mass matrix is given by

$$T = \frac{1}{2} \{\hat{u}\}^T [m]\{\hat{u}\} \qquad (8)$$

D. Element Centrifugal Stiffness Matrix

The ith element of the beam is subjected to centrifugal force which can be expressed as

$$F_c = \int_{x_i}^{x_i + l} \int_{-h/2}^{h/2} b\rho(z)\tilde{N}^2(R + x)dzdx \qquad (9)$$

Where x_i = the distance between i[th] node and axis of rotation, $\tilde{N}$ and R are the angular velocity and radius of hub. Work due to centrifugal force is

$$W_c = \frac{1}{2}\int_0^l F_c\left(\frac{dw}{dx}\right)^2 dx = \frac{1}{2}\{\hat{u}\}[k_c]\{\hat{u}\} \qquad (10)$$

Where,

$$[k_c] = \int_0^l F_c [\aleph'_w]^T [\aleph'_w]dx \qquad (11)$$

E. Element Geometric Stiffness Matrix

The work done due to axial load P may be written as

$$W_p = \frac{1}{2}\int_0^l P\left(\frac{\partial w}{\partial x}\right)^2 dx \qquad (12)$$

Substituting the value of w from eq. (6) into eq. (12) the work done can be expressed as

$$W_p = \frac{P}{2}\int_0^l \{\hat{u}\}^T [\aleph'_w]^T [\aleph'_w]\{\hat{u}\}dx$$

$$= \frac{P}{2}\{\hat{u}\}[k_g]\{\hat{u}\} \qquad (13)$$

here,

$$[k_g] = \int_0^l [\aleph'_w]^T [\aleph'_w] dx \qquad (14)$$

Where, $[k_g]$ = geometric stiffness matrix of the element.

III. EQUATION OF MOTION

Using Hamilton's principle.

$$\delta \int_{t_1}^{t_2} (T - S + W_p - W_c) dt = 0 \qquad (15)$$

Substituting Eqns (7, 8, 10 and 13) into Eqn (15) and rewritten in Eqn (16)

$$[m]\{\ddot{\hat{u}}\} + [[k_t] - P(t)[k_g]]\{\hat{u}\} = 0 \qquad (16)$$

$$[m]\{\ddot{\hat{u}}\} + [[k_t] - P^\oplus (\alpha + \beta_d \cos\Omega t)[k_g]]\{\hat{u}\} = 0 \qquad (17)$$

$$[k_t] = [k_e] + [k_c] \qquad (18)$$

where, $[k_e]$, $[k_c]$, $[m]$ and $[k_g]$ are elastic stiffness matrix, centrifugal stiffness matrix, mass matrix and geometric stiffness matrix respectively. $[k_t]$ is the total stiffness matrix. Assembling the element matrices as used in eq. (17), the equation of motion in global matrix form for the beam, can be expressed as

$$[M]\{\ddot{\hat{U}}\} + [[K_t] - P^\oplus(\alpha + \beta_d \cos\Omega t)[K_g]]\{\hat{U}\} = 0 \qquad (19)$$

Where $[M]$, $[K_t]$, $[K_g]$ are global mass, total stiffness and geometric stiffness matrices respectively and $[\hat{U}]$ is global displacement vector. Equation (19) represents a system of second order differential equations with periodic coefficients of the MathieuHill type. Floquet Theory has been used to distinguish between the dynamic stability and instability zones as follows. A solution with period 2T which is of practical importance is represented by

$$\hat{U}(t) = c_1 \sin\frac{\Omega t}{2} + d_1 \cos\frac{\Omega t}{2} \qquad (20)$$

Substituting eq. (20) into eq. (19) and solving the boundary solutions with period 2T. The resulting equation is given by

$$\left([K_t] - (\alpha \pm \beta_d / 2) P^\oplus [K_g] - \frac{\Omega^2}{4}[M]\right)\{\hat{U}\} = 0 \quad (21)$$

Equation (21) ends up with an eigenvalue problem with known quantities $P^\oplus$, α, β_d. Where $P^\oplus$ is the critical buckling load.

The plus and minus sign in the eq. (21) results with two sets of eigenvalues (Ω) binding the regions of instability and can be determined from the solution of the above equation

$$\left| [K_t] - (\alpha \pm \beta_d / 2) P^\oplus [K_g] - \frac{\Omega^2}{4}[M] \right| = 0 \qquad (22)$$

A. Free Vibration

The eq. (22) can be written for a problem of free vibration by substituting $\alpha = 0$, $\beta_d = 0$, and $\omega = \frac{\Omega}{2}$

$$\left| [K_t] - \omega^2 [M] \right| = 0 \qquad (23)$$

The values of the natural frequencies $\{\omega\}$ can be obtained by solving eq. (23).

B. Static Stability

The eq. (22) can be written for a problem of static stability by substituting $\alpha = 1$, $\beta_d = 0$, and $\omega = 0$

$$\left| [K_t] - P^\oplus [K_g] \right| = 0 \qquad (24)$$

The values of buckling loads can be obtained by solving eq. (24).

C. Regions of Instability

ω_1 and $P^\oplus$ are calculated from eq. (23) and eq. (24) for an isotropic steel beam with identical geometry and end conditions ignoring the centrifugal force.

Choosing $\Omega = \left(\frac{\Omega}{\omega_1}\right)\omega_1$, eq. (22) can be rewritten as

$$\left| [K_t] - (\alpha \pm \beta_d / 2) P^\oplus [K_g] - \left(\frac{\Omega}{\omega_1}\right)^2 \frac{\omega_1^2}{4}[M] \right| = 0 \qquad (25)$$

For fixed values of α, β_d, $P^\oplus$, and ω_1, the eq. (25) can be solved for two sets of values of $\left(\frac{\Omega}{\omega_1}\right)$ and a plot between β_d and $\left(\frac{\Omega}{\omega_1}\right)$ can be drawn which will give the zone of dynamic instability.

IV. RESULTS AND DISCUSSION

In the present formulation the FGO beam reduces to a homogeneous beam when the power law index (n) for the property distribution is made equal to zero. In order to establish the correctness of calculation, the fundamental non-dimensional natural frequencies of a homogenous rotating steel beam fixed at one end and free at other end are calculated for various rotational speed parameters and compared with [20, 21, 22].

The length of the beam is denoted by L.

Hub radius parameter $\delta = \frac{R}{L}$, Rotary inertia parameter $r = \frac{1}{L}\sqrt{\frac{I}{A}}$,

Frequency parameter $\eta_n = \sqrt{\frac{\rho A L^4 \omega_n^2}{EI}}$

The area moment of inertia of the cross section about the centroidal axis is I, the nth mode frequency of the beam is

ω_n and η_n is the nth mode frequency parameter.

Table-1: Variation of fundamental natural frequency of Timoshenko cantilever beam for different rotational speed parameters ($\delta = 0$, $r = 1/30$, $E/kG = 3.059$)

ν	Fundamental natural frequency η_1			
	present	Ref [20]	Ref [21]	Ref [22]
0	3.4798	3.4798	3.4798	3.4798
1	3.6460	3.6445	3.6445	3.6452
2	4.1025	4.0971	4.0971	4.0994
3	4.7617	4.7516	4.7516	4.7558
4	5.5462	5.5314	5.5314	5.5375
5	6.4048	6.3858	6.3858	6.3934
10	11.0971	11.0643	-	-

The present results are found to be in good agreement as shown in table 1.

The following additional non-dimensional parameters are chosen for the analysis of the beam.

Slenderness parameter $s = h/L$

Rotational speed parameter $\nu = \sqrt{\dfrac{\rho A L^4 \tilde{N}^2}{EI}}$

$\tilde{N}$ is the rotational speed in rad/s

E, G and ρ are the Young's modulus, shear modulus and mass density of steel respectively and their values are given in the following section.

An FGO rotating cantilever beam of steel-alumina with length 1 m and width 100mm is taken for the parametric instability. The beam is steel-rich bottom and alumina-rich top. The mechanical properties of the two phases of the beam are considered as given in the following table [14].

Table 2. Material properties of Steel-Alumina FGO beam.

Material	Properties		
	Young's modulus E	Shear modulus G	Mass density ρ
Steel	2.1×10^{11} Pa	0.8×10^{11} Pa	7.85×10^3 kg/m^3
Alumina	3.9×10^{11} Pa	1.37×10^{11} Pa	3.9×10^3 kg/m^3
Poisson's ratio v is assumed as 0.3, shear correction factor $k = (5+v)/(6+v) = 0.8667$			
Static load factor $\alpha = 0.1$			
Critical buckling load, $P^{\oplus} = 6.49 \times 10^7$ N			
Fundamental natural frequency $\omega_1 = 1253.1$ rad/s			

The effect of rotational speed parameter (ν) on the dynamic stability of a FGO-2.5 beam is presented in fig. 3(a) and fig. 3(b) for first and second mode respectively. The increase in rotational speed increases the stability of the beam for both the modes. Figure 3(c) and 3(d) show the effect of rotation on stability of e-FGO beam for first mode and second mode respectively. The stability, in this case, is also enhanced with the increase in angular speed as the instability regions are shifted away from the dynamic load factor axis and their areas are reduced as well.
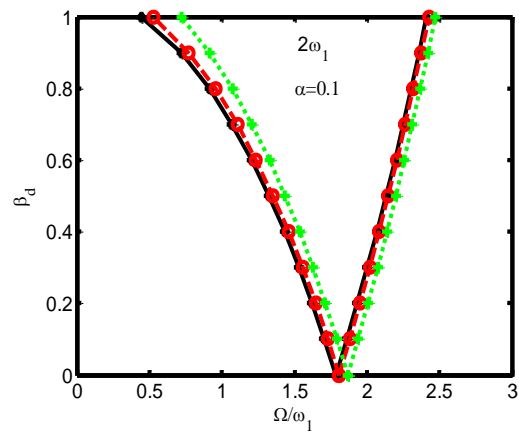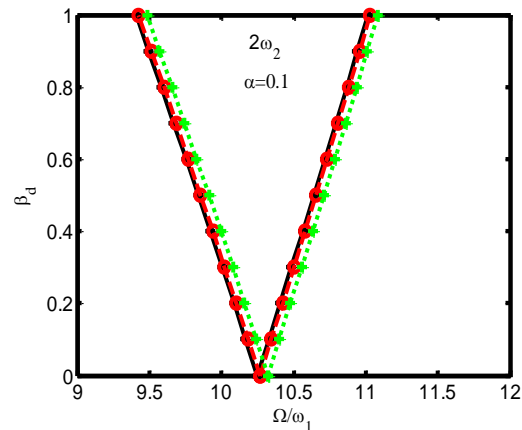

Figure 3(a). Effect of rotational speed parameter on first mode instability region of steel-alumina FGO beam for n=2.5, s =0.2, δ=0.1, (*v =0.1, ^{o} v =0.5, $^+$v=1.0)

Figure 3(b). Effect of rotational speed parameter on second mode instability region of steel-alumina FGO

beam for n=2.5, s =0.2, δ=0.1, (*v =0.1, ^{o} v =0.5, $^+$v=1.0)
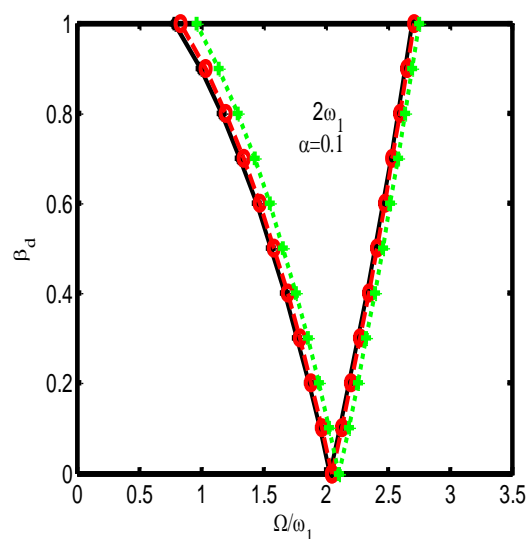

Figure 3(c). Effect of rotational speed parameter on first mode instability region of steel-alumina FGO beam for exp. law, s =0.2, δ=0.1, (*v =0.1, ^{o} v =0.5, $^+$v=1.0)

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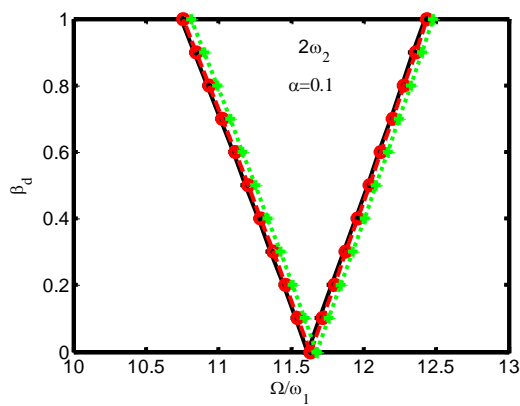

Figure 3(d). Effect of rotational speed parameter on second mode instability region of steel-alumina FGO beam for exp. law, $S =0.2$, $\delta=0.1$, ($^*\nu =0.1$, $^o\nu =0.5$, $^+\nu=1.0$)

V.CONCLUSION

The stability of rotating cantilever Timoshenko beams were analyzed using finite element analysis. The variation of steel and alumina mass density along the thickness of the beam following exponential law or power law from n=0.5 to n=2.5 were found out. The effect of variation in rotational speed of beam on parametric instability of the beam has been investigated.

Increase in rotational speed of Functionally Graded Timoshenko beams enhance their stability. This is attributed as the increase in rotational speed increases the axial force in the beam, which in turn increases its stiffness. Therefore the dynamic stability is increased.

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