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Group S_3 Cordial Remainder Labeling

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Abstract**:** ***Let*** $G=(V(G),E(G))$ ***be a graph and let*** $g:V(G)\to S_3$ ***be a function. For each edge*** xy***, assign the label*** r ***where*** r ***is the remainder when*** $o(g(x))$ ***is divided by*** $o(g(y))$ ***or*** $o(g(y))$ ***is divided by*** $o(g(x))$ ***according as*** $o(g(x))\geq o(g(y))$ ***or*** $o(g(y))\geq o(g(x))$***. The function*** g ***is called a group*** S_3 ***cordial remainder labeling of*** G ***if*** $|v_g(x)-v_g(y)|\leq 1$ ***and*** $|e_g(0)-e_g(1)|\leq 1$***, where*** $v_g(x)$ ***denotes the number of vertices labeled with*** x ***and*** $e_g(i)$ ***denotes the number of edges labeled with*** i $(i=0,1)$***. A graph*** G ***which admits a group*** S_3 ***cordial remainder labeling is called a group*** S_3 ***cordial remainder graph. In this paper, we introduce the concept of group*** S_3 ***cordial remainder labeling. We prove that some standard graphs admit a group*** S_3 ***cordial remainder labeling.***

Keywords **:** ***Group*** S_3 ***cordial remainder labeling, Group*** S_3 ***cordial remainder graph, path, cycle.***

I. INTRODUCTION

By a graph we mean finite, simple and undirected one. The vertex set and the edge set of a graph G are denoted by $V(G)$ and $E(G)$ so that the order and size of G are $|V(G)|$ and $|E(G)|$ respectively. Terms are not defined here follows from Harary [4]. Graph labeling was first introduced in 1960's. Most of the graph labeling trace their origins in the paper presented by Alex Rosa in 1967 [6]. The complete survey of graph labeling is in [3]. Cordial labeling is a weaker version of graceful labeling and harmonious labeling introduced by I. Cahit in [1]. Ponraj et al. introduced the concept of remainder cordial labeling in [5]. Chandra et al. introduced the concept of group S_3 cordial prime labeling in [2]. Motivated by these concepts, we introduce the concept of group S_3 cordial remainder labeling.

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II. PRELIMINARIES

Definition 1.1. The join of two graphs G_1 and G_2 is denoted by G_1+G_2 and whose vertex set is $V(G_1+G_2)=V(G_1)\cup V(G_2)$ and edge set is $E(G_1+G_2)=E(G_1)\cup E(G_2)\cup\{uv: u\in V(G_1),\ v\in V(G_2)\}$.

Definition 1.2. The wheel W_n is defined as the join C_n+K_1.

Definition 1.3. The graph $F_n=P_n+K_1$ is called a fan.

Definition 1.4. The bistar $B_{n,n}$ is the graph obtained by attaching the apex vertices of two copies of star $K_{1,n}$ by an edge.

Definition 1.5. The corona $G_1\odot G_2$ of two graphs G_1 and G_2 is defined as the graph obtained by taking one copy of G_1 (with p_1 vertices) and p_1 copies of G_2 and then joining the i^{th} vertex of G_1 with an edge to every vertex in the i^{th} copy of G_2. The graph $P_n\odot K_1$ is called a Comb. The graph $C_n\odot K_1$ is called a Crown.

Definition 1.6. Let A be a group. The order of $a\in A$ is the least positive integer n such that $a^n=e$. We denote the order of a by $o(a)$.

In this paper, we prove that path, cycle, star, bistar, complete bipartite, wheel, fan, comb and crown graphs admit a group S_3 cordial remainder labeling.

III. MAIN RESULTS

Definition 2.1. Consider the symmetric group S_3. Let the elements of S_3 be e,a,b,c,d,f where

$$e=\begin{pmatrix}1&2&3\\1&2&3\end{pmatrix}\quad a=\begin{pmatrix}1&2&3\\1&3&2\end{pmatrix}\quad b=\begin{pmatrix}1&2&3\\3&2&1\end{pmatrix}$$
$$c=\begin{pmatrix}1&2&3\\2&1&3\end{pmatrix}\quad d=\begin{pmatrix}1&2&3\\2&3&1\end{pmatrix}\quad f=\begin{pmatrix}1&2&3\\3&1&2\end{pmatrix}$$

We have $o(e)=1,\ o(a)=o(b)=o(c)=2,\ o(d)=o(f)=3.$

Definition 2.2. Let $G=(V(G),E(G))$ be a graph and let $g:V(G)\to S_3$ be a function. For each edge xy assign the label r where r is the remainder when $o(g(x))$ is divided by $o(g(y))$ or $o(g(y))$ is divided by $o(g(x))$ according as $o(g(x))\geq o(g(y))$ or $o(g(y))\geq o(g(x))$. The function g is called a group S_3 cordial remainder labeling of G if $|v_g(x)-v_g(y)|\leq 1$ and $|e_g(0)-e_g(1)|\leq 1$, where $v_g(x)$ denotes the number of vertices labeled with x and $e_g(i)$

denotes the number of edges labeled with i $(i = 0,1)$. A graph G which admits a group S_3 cordial remainder labeling is called a group S_3 cordial remainder graph.

Theorem 2.3. Path P_n is a group S_3 cordial remainder graph.

Proof. Let $v_1, v_2, ..., v_n$ denote the vertices of P_n. Fig. 1 gives a group S_3 cordial remainder labeling of P_n for $n \le 5$.

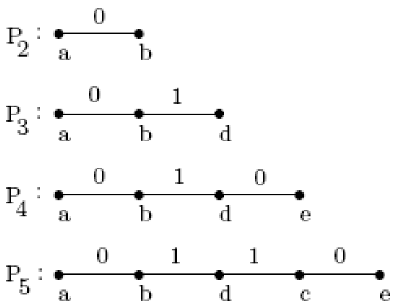

Fig. 1. Path P_n for $n \le 5$.

Suppose $n \ge 6$. Define a function $g : V(P_n) \to S_3$ as follows.

Case 1. $n \equiv 0 \pmod 6$.

Let $n = 6k$ and $k \ge 1$.

$$g(v_i) = \begin{cases} a & \text{if } i \equiv 1 \ (mod\ 6) \text{ and } 1 \le i \le 6k \\ d & \text{if } i \equiv 2 \ (mod\ 6) \text{ and } 1 \le i \le 6k \\ b & \text{if } i \equiv 3 \ (mod\ 6) \text{ and } 1 \le i \le 6k \\ c & \text{if } i \equiv 4 \ (mod\ 6) \text{ and } 1 \le i \le 6k \\ e & \text{if } i \equiv 5 \ (mod\ 6) \text{ and } 1 \le i \le 6k \\ f & \text{if } i \equiv 0 \ (mod\ 6) \text{ and } 1 \le i \le 6k. \end{cases}$$

Here $v_g(a) = v_g(b) = v_g(c) = v_g(d) = v_g(e) = v_g(f) = k$ and $e_g(0) = 3k$, $e_g(1) = 3k - 1$. Therefore $|v_g(i) - v_g(j)| \le 1$ for $i, j \in S_3$ and $|e_g(0) - e_g(1)| \le 1$. Hence g is a group S_3 cordial remainder labeling.

Case 2. $n \equiv 5 \pmod 6$.

Let $n = 6k + 5$ and $k \ge 1$. Assign the label to the vertices v_i for $1 \le i \le 6k$ as in Case (1) and for the remaining vertices assign the following labels:

$$g(v_i) = \begin{cases} a & \text{if } i = 6k+1 \\ b & \text{if } i = 6k+2 \\ d & \text{if } i = 6k+3 \\ c & \text{if } i = 6k+4 \\ e & \text{if } i = 6k+5. \end{cases}$$

Here $v_g(a) = v_g(b) = v_g(c) = v_g(d) = v_g(e) = k+1, v_g(f) = k$ and $e_g(0) = e_g(1) = 3k+2$. Clearly $|v_g(i) - v_g(j)| \le 1$ for $i, j \in S_3$ and $|e_g(0) - e_g(1)| \le 1$. Hence g is a group S_3 cordial remainder labeling.

Case 3. $n \equiv 4 \pmod 6$.

Let $n = 6k + 4$ and $k \ge 1$. Assign the label to the vertices v_i for $1 \le i \le 6k$ as in Case (1) and for the remaining vertices assign the following labels:

$$g(v_i) = \begin{cases} a & \text{if } i = 6k+1 \\ b & \text{if } i = 6k+2 \\ d & \text{if } i = 6k+3 \\ f & \text{if } i = 6k+4. \end{cases}$$

Here $v_g(a) = v_g(b) = v_g(d) = v_g(f) = k+1, v_g(c) = v_g(e) = k$ and $e_g(0) = 3k+2$, $e_g(1) = 3k+1$. Clearly $|v_g(i) - v_g(j)| \le 1$ for $i, j \in S_3$ and $|e_g(0) - e_g(1)| \le 1$. Hence g is a group S_3 cordial remainder labeling.

Case 4. $n \equiv 3 \pmod 6$.

Let $n = 6k + 3$ and $k \ge 1$. Assign the label to the vertices v_i for $1 \le i \le 6k$ as in Case (1) and for the remaining vertices assign the following labels:

$$g(v_i) = \begin{cases} a & \text{if } i = 6k+1 \\ d & \text{if } i = 6k+2 \\ f & \text{if } i = 6k+3. \end{cases}$$

Here $v_g(a) = v_g(d) = v_g(f) = k+1, v_g(b) = v_g(c) = v_g(e) = k$ and $e_g(0) = e_g(1) = 3k+1$. Clearly $|v_g(i) - v_g(j)| \le 1$ for $i, j \in S_3$ and $|e_g(0) - e_g(1)| \le 1$. Hence g is a group S_3 cordial remainder labeling.

Case 5. $n \equiv 2 \pmod 6$.

Let $n = 6k + 2$ and $k \ge 1$. Assign the label to the vertices v_i for $1 \le i \le 6k$ as in the Case (1) and for the remaining vertices assign the following labels:

$$g(v_i) = \begin{cases} a & \text{if } i = 6k+1 \\ b & \text{if } i = 6k+2. \end{cases}$$

Here $v_g(a) = v_g(b) = k+1, v_g(c) = v_g(d) = v_g(e) = v_g(f) = k$ and $e_g(0) = 3k+1$, $e_g(1) = 3k$. Therefore $|v_g(i) - v_g(j)| \le 1$ for $i, j \in S_3$ and $|e_g(0) - e_g(1)| \le 1$. Hence g is a group S_3 cordial remainder labeling.

Case 6. $n \equiv 1 \pmod 6$.

Let $n = 6k + 1$ and $k \ge 1$. Assign the label to the vertices v_i for $1 \le i \le 6k$ as in the Case (1) and $g(v_{6k+1}) = a$. Here $v_g(a) = k+1$, $v_g(b) = v_g(c) = v_g(d) = v_g(e) = v_g(f) = k$ and $e_g(0) = e_g(1) = 3k$. Therefore $|v_g(i) - v_g(j)| \le 1$ for $i, j \in S_3$ and $|e_g(0) - e_g(1)| \le 1$. Hence g is a group S_3 cordial remainder labeling.

Hence path P_n is a group S_3 cordial remainder graph. □

Theorem 2.4. Cycle C_n is a group S_3 cordial remainder graph.

Proof. Let $v_1, v_2, ..., v_n$ denote the vertices of C_n. The same labeling pattern is followed as in Theorem 2.3, except for $n \equiv 2 \pmod 6$.

Suppose $n \equiv 2 \pmod 6$. Define a function $g : V(C_n) \to S_3$ as follows. Let $n = 6k + 2$ and $k \ge 1$.

$$g(v_i)=\begin{cases} a \text{ if } i\equiv 1 \pmod 6 \text{ and } 1\le i\le 6k \\ d \text{ if } i\equiv 2 \pmod 6 \text{ and } 1\le i\le 6k \\ b \text{ if } i\equiv 3 \pmod 6 \text{ and } 1\le i\le 6k \\ c \text{ if } i\equiv 4 \pmod 6 \text{ and } 1\le i\le 6k \\ f \text{ if } i\equiv 5 \pmod 6 \text{ and } 1\le i\le 6k \\ e \text{ if } i\equiv 0 \pmod 6 \text{ and } 1\le i\le 6k \\ f \text{ if } i=6k+1 \\ a \text{ if } i=6k+2 \end{cases}.$$

Here $v_g(a)=v_g(f)=k+1, v_g(b)=v_g(c)=v_g(d)=v_g(e)=k$ and $e_g(0)=e_g(1)=3k+1$. Therefore $|v_g(i)-v_g(j)|\le 1$ for $i,j\in S_3$ and $|e_g(0)-e_g(1)|\le 1$. Hence g is a group S_3 cordial remainder labeling.

Hence cycle C_n is a group S_3 cordial remainder graph. □

Theorem 2.5. Star $K_{1,n}$ is a group S_3 cordial remainder graph for every n.

Proof. Let v be the apex vertex and $v_1,v_2,...,v_n$ be the pendent vertices of $K_{1,n}$. Then $K_{1,n}$ is of order $n+1$ and size n. We define $g:V(K_{1,n})\to S_3$ as follows:

$g(v)=d$;

$$g(v_i)=\begin{cases} a \text{ if } i\equiv 1 \pmod 6 \text{ and } 1\le i\le n \\ e \text{ if } i\equiv 2 \pmod 6 \text{ and } 1\le i\le n \\ b \text{ if } i\equiv 3 \pmod 6 \text{ and } 1\le i\le n \\ f \text{ if } i\equiv 4 \pmod 6 \text{ and } 1\le i\le n \\ c \text{ if } i\equiv 5 \pmod 6 \text{ and } 1\le i\le n \\ d \text{ if } i\equiv 0 \pmod 6 \text{ and } 1\le i\le n \end{cases}.$$

It is easy to verify that $|v_g(i)-v_g(j)|\le 1$ for $i,j\in S_3$ and $|e_g(0)-e_g(1)|\le 1$. Hence g is a group S_3 cordial remainder labeling. □

Theorem 2.6. Bistar $B_{n,n}$ is a group S_3 cordial remainder graph for every n.

Proof. Let v and u be the apex vertices and $v_1,v_2,...,v_n,u_1,u_2...,u_n$ be the pendent vertices of $K_{1,n}$. Then $B_{n,n}$ is of order $2n+2$ and size $2n+1$. We define $g:V(B_{n,n})\to S_3$ as follows:

$g(u)=a$; $g(u_1)=b$; $g(u_2)=f$;

$g(v)=d$; $g(v_1)=c$; $g(v_2)=e$;

$$g(u_i)=\begin{cases} e \text{ if } i\equiv 0 \pmod 3 \text{ and } 3\le i\le n \\ d \text{ if } i\equiv 1 \pmod 3 \text{ and } 3\le i\le n \\ b \text{ if } i\equiv 2 \pmod 3 \text{ and } 3\le i\le n; \end{cases}$$

$$g(v_i)=\begin{cases} a \text{ if } i\equiv 0 \pmod 3 \text{ and } 3\le i\le n \\ f \text{ if } i\equiv 1 \pmod 3 \text{ and } 3\le i\le n \\ c \text{ if } i\equiv 2 \pmod 3 \text{ and } 3\le i\le n. \end{cases}$$

From Table - I, it is clear that g is a group S_3 cordial remainder labeling.

Table- I: Bistar

Nature of n	$v_g(a)$	$v_g(b)$	$v_g(c)$	$v_g(d)$	$v_g(e)$	$v_g(f)$
1	1	1	1	1	0	0
2	1	1	1	1	1	1
$3k$ ($k\ge 1$)	$k+1$	k	k	k	$k+1$	k
$3k+1$ ($k\ge 1$)	$k+1$	k	k	$k+1$	$k+1$	$k+1$
$3k+2$ ($k\ge 1$)	$k+1$	$k+1$	$k+1$	$k+1$	$k+1$	$k+1$

Hence bistar $B_{n,n}$ is a group S_3 cordial remainder graph. □

Theorem 2.7. Wheel W_n is a group S_3 cordial remainder graph for $n\ge 3$.

Proof. Let W_n be the wheel C_n+K_1. Let u be the apex of the wheel and $u_1,u_2,...,u_n$ be the vertices on the cycle C_n.

Case 1. $n=3$.

Define g *by* $g(u)=d, g(u_1)=a$, $g(u_2)=b$ and $g(u_3)=c$. It is easy to verify that $|v_g(i)-v_g(j)|\le 1$ for $i,j\in S_3$ and $|e_g(0)-e_g(1)|\le 1$. Hence g is a group S_3 cordial remainder labeling.

Case 2. $n=4$.

Define g *by* $g(u)=d$, $g(u_1)=a$, $g(u_2)=f$, $g(u_3)=b$ and $g(u_4)=e$. It is easy to verify that $|v_g(i)-v_g(j)|\le 1$ for $i,j\in S_3$ and $|e_g(0)-e_g(1)|\le 1$. Hence g is a group S_3 cordial remainder labeling.

Case 3. $n=5$.

Define g *by* $g(u)=d$, $g(u_1)=a$, $g(u_2)=b$, $g(u_3)=f$, $g(u_4)=c$ and $g(u_5)=e$. It is easy to verify that $|v_g(i)-v_g(j)|\le 1$ for $i,j\in S_3$ and $|e_g(0)-e_g(1)|\le 1$. Hence g is a group S_3 cordial remainder labeling.

Case 4. $n\ge 6$. Define $g:V(W_n)\to S_3$ as follows.

Subcase (i). $n\equiv 0 \pmod 6$.

Let $n=6k$ and $k\ge 1$.

$g(u)=d$;

$$g(u_i)=\begin{cases} a \text{ if } i\equiv 1 \pmod 6 \text{ and } 1\le i\le 6k \\ d \text{ if } i\equiv 2 \pmod 6 \text{ and } 1\le i\le 6k \\ b \text{ if } i\equiv 3 \pmod 6 \text{ and } 1\le i\le 6k \\ c \text{ if } i\equiv 4 \pmod 6 \text{ and } 1\le i\le 6k \\ f \text{ if } i\equiv 5 \pmod 6 \text{ and } 1\le i\le 6k \\ e \text{ if } i\equiv 0 \pmod 6 \text{ and } 1\le i\le 6k. \end{cases}$$

Here $v_g(a)=v_g(b)=v_g(c)=v_g(e)=v_g(f)=k, v_g(d)=k+1$ and $e_g(0)=e_g(1)=6k$. Therefore $|v_g(i)-v_g(j)|\le 1$ for $i,j\in S_3$ and $|e_g(0)-e_g(1)|\le 1$. Hence g is a group S_3 cordial remainder labeling.

Subcase (ii). $n\equiv 5 \pmod 6$.

Let $n = 6k+5$ and $k \geq 1$. Assign the label to the vertices u and u_i for $1 \leq i \leq 6k$ as in Subcase (i) and for the remaining vertices assign the following labels:

$$g(u_i) = \begin{cases} a & \text{if } i = 6k+1 \\ b & \text{if } i = 6k+2 \\ f & \text{if } i = 6k+3 \\ c & \text{if } i = 6k+4 \\ e & \text{if } i = 6k+5. \end{cases}$$

Here $v_g(a) = v_g(b) = v_g(c) = v_g(d) = v_g(e) = v_g(f) = k+1$ and $e_g(0) = e_g(1) = 6k+5$. Clearly $|v_g(i) - v_g(j)| \leq 1$ for $i, j \in S_3$ and $|e_g(0) - e_g(1)| \leq 1$. Hence g is a group S_3 cordial remainder labeling.

Subcase (iii). $n \equiv 4 \pmod 6$.

Let $n = 6k+4$ and $k \geq 1$. Assign the label to the vertices u and u_i for $1 \leq i \leq 6k$ as in Subcase (i) and for the remaining vertices assign the following labels:

$$g(u_i) = \begin{cases} a & \text{if } i = 6k+1 \\ f & \text{if } i = 6k+2 \\ b & \text{if } i = 6k+3 \\ e & \text{if } i = 6k+4. \end{cases}$$

Here $v_g(a) = v_g(b) = v_g(d) = v_g(e) = v_g(f) = k+1, v_g(c) = k$ and $e_g(0) = e_g(1) = 6k+4$. Clearly $|v_g(i) - v_g(j)| \leq 1$ for $i, j \in S_3$ and $|e_g(0) - e_g(1)| \leq 1$. Hence g is a group S_3 cordial remainder labeling.

Subcase (iv). $n \equiv 3 \pmod 6$.

Let $n = 6k+3$ and $k \geq 1$. Assign the label to the vertices u and u_i for $1 \leq i \leq 6k$ as in Subcase (i) and for the remaining vertices assign the following labels:

$$g(u_i) = \begin{cases} a & \text{if } i = 6k+1 \\ b & \text{if } i = 6k+2 \\ c & \text{if } i = 6k+3. \end{cases}$$

Here $v_g(a) = v_g(b) = v_g(c) = v_g(d) = k+1, v_g(e) = v_g(f) = k$ and $e_g(0) = e_g(1) = 6k+3$. Clearly $|v_g(i) - v_g(j)| \leq 1$ for $i, j \in S_3$ and $|e_g(0) - e_g(1)| \leq 1$. Hence g is a group S_3 cordial remainder labeling.

Case 5. $n \equiv 2 \pmod 6$.

Let $n = 6k+2$ and $k \geq 1$. Assign the label to the vertices u and u_i for $1 \leq i \leq 6k$ as in Subcase (i) and for the remaining vertices assign the following labels:

$$g(u_i) = \begin{cases} f & \text{if } i = 6k+1 \\ a & \text{if } i = 6k+2. \end{cases}$$

Here $v_g(a) = v_g(f) = v_g(d) = k+1, v_g(b) = v_g(c) = v_g(e) = k$ and $e_g(0) = e_g(1) = 6k+2$. Therefore $|v_g(i) - v_g(j)| \leq 1$ for $i, j \in S_3$ and $|e_g(0) - e_g(1)| \leq 1$. Hence g is a group S_3 cordial remainder labeling.

Case 6. $n \equiv 1 \pmod 6$.

Let $n = 6k+1$ and $k \geq 1$. Assign the label to the vertices u and u_i for $1 \leq i \leq 6k$ as in Subcase (i) and $g(u_{6k+1}) = f$. Here $v_g(a) = v_g(b) = v_g(c) = v_g(e) = k$, $v_g(d) = v_g(f) = k+1$ and $e_g(0) = e_g(1) = 6k+1$. Therefore $|v_g(i) - v_g(j)| \leq 1$ for $i, j \in S_3$ and $|e_g(0) - e_g(1)| \leq 1$. Hence g is a group S_3 cordial remainder labeling.

Hence W_n is a group S_3 cordial remainder graph for $n \geq 3$. □

Corollary 2.8. Fan F_n is a group S_3 cordial remainder graph for $n \geq 2$.

Proof. Let u be the apex vertex and $u_1, u_2, ..., u_n$ be the vertices of path P_n. Let $E(G) = \{u_i u_{i+1} : 1 \leq i \leq n-1;$ $uu_i : 1 \leq i \leq n\}$. Therefore F_n is of order $n+1$ and size $2n-1$. Define $g : V(F_n) \to S_3$ as follows. Suppose $n = 2$, we assign the labels d, a, b to the vertices u, u_1, u_2 respectively. Clearly F_2 is a group S_3 cordial remainder.

For $n \geq 3$, the fan graph obtained by removing the edge $u_1 u_n$. Then the same labeling pattern as in Theorem 2.7 is followed. Clearly, g is a group S_3 cordial remainder labeling for $n \geq 3$.

Hence, F_n admits a group S_3 cordial remainder labeling for $n \geq 2$. □

Theorem 2.9. The complete bipartite graph $K_{2,n}$ is a group S_3 cordial remainder graph if and only if $n = 1, 2, 3, 4$ or $n \equiv 0 \pmod 6$.

Proof. Let (V_1, V_2) be the bipartition of $K_{2,n}$ with $V_1 = \{v_1, v_2\}$ and $V_2 = \{u_1, u_2, ..., u_n\}$.

Suppose $n = 1, 2, 3$ or 4. Table - II gives a group S_3 cordial remainder labeling of $K_{2,n}$ for $1 \leq i \leq 4$.

Table - II: $K_{2,n}$ for $1 \leq i \leq 4$

Nature of n	v_1	v_2	u_1	u_2	u_3	u_4
1	a	e	d			
2	a	e	d	f		
3	d	e	a	b	c	
4	a	b	c	d	e	f

Suppose $n \equiv 0 \pmod 6$. Let $n = 6k$ and $k \geq 1$.

Define $g : V(K_{2,n}) \to S_3$ as follows.

$g(v_1) = d; \quad g(v_2) = f;$

$$g(u_i) = \begin{cases} a & \text{if } i \equiv 1 \ (mod\ 6) \text{ and } 1 \leq i \leq 6k \\ b & \text{if } i \equiv 2 \ (mod\ 6) \text{ and } 1 \leq i \leq 6k \\ c & \text{if } i \equiv 3 \ (mod\ 6) \text{ and } 1 \leq i \leq 6k \\ d & \text{if } i \equiv 4 \ (mod\ 6) \text{ and } 1 \leq i \leq 6k \\ e & \text{if } i \equiv 5 \ (mod\ 6) \text{ and } 1 \leq i \leq 6k \\ f & \text{if } i \equiv 0 \ (mod\ 6) \text{ and } 1 \leq i \leq 6k. \end{cases}$$

Here $v_g(a) = v_g(b) = v_g(c) = v_g(e) = k, v_g(d) = v_g(f) = k+1$ and $e_g(0) = e_g(1) = 6k$. Therefore $|v_g(i) - v_g(j)| \leq 1$ for $i, j \in S_3$ and $|e_g(0) - e_g(1)| \leq 1$. Hence g is a group S_3 cordial remainder labeling of $K_{2,n}$ if $n \equiv 0 \pmod 6$.

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Conversely, assume that the complete bipartite graph $K_{2,n}$ is a group S_3 cordial remainder graph. Let g be a group S_3 cordial remainder labeling of $K_{2,n}$.

Case 1. $g(v_1)=e$ or $g(v_2)=e$.

Without loss of generality, let $g(v_1)=e$. The n edges incident with v_1 get label 0. So the remaining n edges should get label 1. If $g(v_2)=d$, then only a, b and c can be used to label the remaining vertices. Hence $n=3$. If $g(v_2)=a$, then only d and f can be used to label the remaining vertices. Hence $n=2$. If $g(v_2)=a$, then only d can be used to label the remaining vertices. Hence $n=1$.

Case 2. $g(v_i)\neq e$ for $i=1,2$.

It is enough to consider the following subcases.

Subcase (i). $g(v_1)=d,\ g(v_2)=f$.

Assign labels d, e, f to any three vertices. We get six edges with label 0. Next we assign labels a, b, c to another set of three vertices and in this process we get six edges with label 1. Thus for $n\equiv 0 \pmod 6$, we assign the labels d, e, f for every 3 vertices and we need 3 vertices with labels a, b, c. Hence $n\equiv 0 \pmod 6$.

Subcase (ii). $g(v_1)=a,\ g(v_2)=b$.

Assign labels e, c to any two vertices. We get four edges with label 0. Next we assign labels d, f to another set of two vertices and in this process we get four edges with label 1. Hence $n=4$.

Subcase (iii). $g(v_1)=d,\ g(v_2)=a$.

Assign the label e to a vertex. We get 2 edges with label 0. Vertices with every other label give one edge with label 0 and another edge with label 1. So, $|e_g(0)-e_g(1)|>1$. Hence this is impossible. □

Theorem 2.10. The comb $P_n \odot K_1$ is a group S_3 cordial remainder graph for every n.

Proof. Let $V(P_n\odot K_1)=\{u_i,v_i:1\le i\le n\}$ and $E(P_n\odot K_1)=\{u_iu_{i+1}:1\le i\le n-1\}\cup\{u_iv_i:1\le i\le n\}$. Then $P_n\odot K_1$ is of order $2n$ and size $2n-1$.

Define $f:V(P_n\odot K_1)\to S_3$ as follows:

For $1\le i\le n$,

$$g(u_i)=\begin{cases} a \text{ if } i\equiv 1 \pmod 6\\ b \text{ if } i\equiv 2 \pmod 6\\ d \text{ if } i\equiv 3 \pmod 6\\ c \text{ if } i\equiv 4 \pmod 6\\ e \text{ if } i\equiv 5 \pmod 6\\ f \text{ if } i\equiv 0 \pmod 6;\end{cases}$$

$$g(v_i)=\begin{cases} c \text{ if } i\equiv 1 \pmod 6\\ f \text{ if } i\equiv 2 \pmod 6\\ e \text{ if } i\equiv 3 \pmod 6\\ d \text{ if } i\equiv 4 \pmod 6\\ b \text{ if } i\equiv 5 \pmod 6\\ a \text{ if } i\equiv 0 \pmod 6.\end{cases}$$

Table- III: Comb graph

Nature of n	$v_g(a)$	$v_g(b)$	$v_g(c)$	$v_g(d)$	$v_g(e)$	$v_g(f)$
$6k-5$ ($k\ge 1$)	$2k-1$	$2k-2$	$2k-1$	$2k-2$	$2k-2$	$2k-2$
$6k-4$ ($k\ge 1$)	$2k-1$	$2k-1$	$2k-1$	$2k-2$	$2k-2$	$2k-1$
$6k-3$ ($k\ge 1$)	$2k-1$	$2k-1$	$2k-1$	$2k-1$	$2k-1$	$2k-1$
$6k-2$ ($k\ge 1$)	$2k-1$	$2k-1$	$2k$	$2k$	$2k-1$	$2k-1$
$6k-1$ ($k\ge 1$)	$2k-1$	$2k$	$2k$	$2k$	$2k$	$2k-1$
$6k$ ($k\ge 1$)	$2k$	$2k$	$2k$	$2k$	$2k$	$2k$

From Table III, it is clear that g is a group S_3 cordial remainder labeling. Hence comb is a group S_3 cordial remainder graph. □

Corollary 2.11. The crown $C_n\odot K_1$ is a group S_3 cordial remainder graph for every n.

Proof. Let $V(C_n\odot K_1)=\{u_i,v_i:1\le i\le n\}$ where $u_1,u_2,\dots,u_n$ are the vertices of the cycle and $v_1,v_2,\dots,v_n$ are the pendent vertices adjacent to $u_1,u_2,\dots,u_n$ respectively. Assign the labels to the vertices u_i and v_i as in the Theorem 2.10, except by interchanging the labels of the vertices v_1 and v_3 for $n\equiv 1, 2, 5 \pmod 6$. It is easy to verify that $|v_g(i)-v_g(j)|\le 1$ for $i,j\in S_3$ and $|e_g(0)-e_g(1)|\le 1$. Hence g is a group S_3 cordial remainder labeling. □

Theorem 2.12. Let g be a group S_3 cordial remainder labeling of a graph G_1 of order $6m$ and size $2n$ and let h be any group S_3 cordial remainder labeling of a graph G_2. Then $G_1\cup G_2$ is also a group S_3 cordial remainder graph.

Proof. Let g, h be a group S_3 cordial remainder labeling of G_1 and G_2 respectively. Since G_1 has $6m$ vertices and $2n$ edges, we have $v_g(a)=v_g(b)=v_g(c)=v_g(d)=v_g(e)=v_g(f)=m$ and $e_g(0)=e_g(1)=n$. We define a vertex labeling l of $G_1\cup G_2$ such that

$$l(u)=\begin{cases} g(u) & \text{if } u\in V(G_1)\\ h(u) & \text{if } u\in V(G_2).\end{cases}$$

Hence, $v_l(i) = v_g(i) + v_h(i)$ for $i \in S_3$ and $e_l(i) = e_g(i) + e_h(i)$ for $i = 0, 1$. Therefore $|v_l(i) - v_l(j)| \le 1$ for $i, j \in S_3$ and $|e_l(0) - e_l(1)| \le 1$. Hence l is a group S_3 cordial remainder labeling. Thus $G_1 \cup G_2$ is also a group S_3 cordial remainder graph. □

IV. CONCLUSION

Labeled graphs serve as useful models for a broad range of applications such as: coding theory, radar, astronomy, circuit design and communication network. In this paper, we have introduced the concept of group S_3 cordial remainder labeling. Researchers can explore the possibility of applying this concept to the above mentioned areas. We have proved that path, cycle, star, bistar, complete bipartite, wheel, fan, comb and crown graphs are group S_3 cordial remainder graphs.

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