International Journal of Recent Technology and Engineering (IJRTE)
ISSN: 2277-3878 (Online), Volume-8 Issue-4, November 2019

Distance and Similarity Measures Including New Hesitant Degree on Hesitant Fuzzy Set

Biplab Singha, Mausumi Sen, Nidul Sinha

Abstract: ***Hesitant degree plays an important role for finding the distance and similarity measures between two objects. Many researchers have developed many distance and similarity measures so far but in real life some situations arises where these measures fail to achieve the satisfactory result. In this paper, a new hesitant degree is introduced in the distance and similarity measures so that the limitations which are found can be easily handled with a satisfactory outcome. Finally, the validity of the proposed distance measure is illustrated with a suitable example..***

Keywords: ***Hesitant fuzzy set, Hesitant fuzzy elements, Distance measure, Similarity measure.***

I. INTRODUCTION

Distance and similarity measure is an important tool for finding the differences between two objects. Distance and similarity measure can be applied in many areas such as decision making, pattern recognition, image processing, and machine learning etc. Initially Wang [11] introduced the concept of fuzzy sets' similarity measure with a computational formula. Since then many researchers started following this topic and extended further. There are many distance and similarity measures proposed for fuzzy set, intuitionistic fuzzy set and fuzzy multiset etc. The Hamming distance, the Euclidean distance and Housdorff distance are three popular and widely used distance measure. The relationship measure, the similarity measure and the fuzziness of fuzzy set are investigated by Zeng and Li [17]. Szmidt and Kacprzyk [8] studied a new distance between two intuitionistic fuzzy set. Torra and Narukawa [10] introduced weighted distance measure for intuitionistic fuzzy sets based on the Choquet integral with respect to the non-monotonic fuzzy measure. Xu and Xia [13, 14] extended the distance and similarity measure based on hesitant fuzzy set. Peng [7] proposed the generalized hesitant fuzzy synergetic weighted distance measure and applied it to multi criteria decision making problem. Deqing Li et al. [4] also studied new distance and similarity measure on hesitant fuzzy set and their application in multi criteria decision making problem.

When the distance between one sample object with another two objects equal then the existing distance measures cannot distinguish the similarity. To solve these types of problems, a new hesitant degree is introduced and added to the distance measures so that it comes out with a satisfactory result. The existing hesitant degree depends on the length of the hesitant fuzzy elements. In this paper, we have pointed out the limitations of the hesitant degree and to overcome this, another hesitant degree is studied with a proper explanation and with some examples it is shown that the proposed hesitant degree works better than the existing one in decision making problems. The rest of the paper is organized as follows:

Section 2 gives the preliminaries definitions of hesitant fuzzy set and its related distance measures. In section 3, we have proposed the distance measure including a new hesitant degree. In section 4, the proposed distance measure is applied on some existing examples and validity is studied. Section 5 consists of the conclusion.

II. PRELIMINARIES

Throughout the paper, we use $X = \{x_1, x_2, x_3 \dots, x_n\}$ to denote the universal set; HFS and HFE stand for hesitant fuzzy set and hesitant fuzzy element respectively. A stands for a HFS and $A(x)$ stands for a HFE, $\tilde{A}$ stands for the set of all hesitant fuzzy sets in X. $l(A(x))$ stands for the total number of elements in $A(x)$.

Definition 1: [9]
Given a fixed set X, then a hesitant fuzzy set (HFS) on X is in terms of a function that when applied to X returns a subset of [0, 1]. For convenience, the HFS is often expressed simply by mathematical symbol $E = (< x, A_E(x) > x \in X)$, where $A_E(x)$ is a set of some values in [0, 1], denoting the possible membership degree of the element $x \in X$ to the set E. $A(x) = A_E(x)\mu_x$ is called a hesitant fuzzy element (HFE). The concept of hesitant fuzzy set and the graphical representation

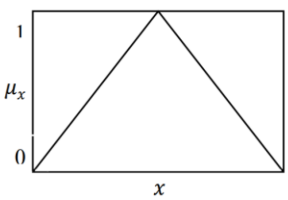

Fig (a): Type-I fuzzy set

Manuscript published on November 30, 2019.
* Correspondence Author
Biplab Singha*, Department of Mathematics, National Institute of Technology Silchar, Silchar, India. Email: sbiplab217@gmail.com
Mausumi Sen, , Department of Mathematics, National Institute of Technology Silchar, Silchar, India. Email: senmausumi@gmail.com
Nidul Sinha, , Department of Electrical Engineering, National Institute of Technology Silchar, Silchar, India. Email: nidul.sinha@gmail.com

of fuzzy set are given as follows:

Fig (a) describes the concept of fuzzy set, where μ_x is the membership value from [0, 1] and x is an element of the crisp set. When decision maker gives his own view about the belongingness of a crisp value of x to the perception about the fact represented by x that means he shares his feelings towards the state of that problem. If he is fully satisfied and certain about belongingness of the value of x in the particular state represented by a membership function or if he confidently disagrees, he assigns the membership value 1 or 0 respectively. And for these types of un-ambiguous situations there is no hesitancy. However, if the designer is partially satisfied or has reservation in judging the belongingness of the value x in the state represented by the membership function then the membership value lies between 0 and 1. In case of fuzzy set we get only one membership value as shown in triangular membership function in figure (a).

Further, there are some cases where the decision maker is confused to assign only one membership value because of higher hesitancy. So instead of giving only one membership value, if a set of values are considered then the representation will be closer to his feeling and more realistic. Moreover, some other cases are also there where more than one experts are engaged to evaluate a particular problem then each and every decision maker may suggest his own opinion as a value for the degree of belongingness. So we get a number of membership values against a single crisp value and this extension is called hesitant fuzzy set. For decision making problems hesitant fuzzy set works as an important tool.

Definition 2: [13]
Let A_1 and A_2 be two hesitant fuzzy sets on $X = \{x_1, x_2, x_3 \dots, x_n\}$, then the distance measure between A_1 and A_2 is denoted as $d(A_1, A_2)$, which satisfies the following properties:

(D_1) $0 \le d(A_1, A_2) \le 1$;

(D_2) $d(A_1, A_2) = 0$ iff $A_1 = A_2$;

(D_3) $d(A_1, A_2) = d(A_2, A_1)$;

(D_4) For three hesitant fuzzy elements $A_1(x), A_2(x)$ and $A_3(x)$ which have the same length l and $A_k(x) = \{A_k^1, A_k^2, \dots, A_k^l\}$, $k = 1,2,3$ if $A_1^i \le A_2^i \le A_3^i, i = \{1,2,3 \dots, l\}$ the $d(A_1(x), A_2(x)) \le d(A_1(x), A_3(x))$, $d(A_2(x), A_3(x)) \le d(A_1(x), A_3(x))$.

Definition 3:
Let A_1 and A_2 be two hesitant fuzzy sets on $X = \{x_1, x_2, x_3 \dots, x_n\}$, then the similarity measure between A_1 and A_2 is denoted as $s(A_1, A_2)$, which satisfies the following properties:

(S_1) $0 \le s(A_1, A_2) \le 1$;

(S_2) $s(A_1, A_2) = 1$ iff $A_1 = A_2$;

(S_3) $s(A_1, A_2) = s(A_2, A_1)$;

Therefore we have the following properties:

Property 1: If d is the distance measure between hesitant fuzzy sets $A_1$and A_2, then $s(A_1, A_2) = 1 - d(A_1, A_2)$ is the similarity measure between hesitant fuzzy sets $A_1$and A_2.

Property 2: If s is the similarity measure between hesitant fuzzy sets $A_1$and A_2, then $d(A_1, A_2) = 1 - s(A_1, A_2)$ is the distance measure between hesitant fuzzy sets $A_1$and A_2.

Sometimes it can be happened that the number of elements in different hesitant fuzzy elements may be different i.e. if $A_1$and A_2 be two HFEs then in most of the cases $l(A_1(x)) \neq l(A_2(x))$. According to Xia and Xu [13], the optimist expert can extend the shorter one by adding the maximum value while pessimist expert add the minimum value to make the lengths equal. This selection of the values mainly depends on the decision makers' risk preferences. In this paper, the shorter one is extended by adding minimum value.

The hesitant normalized Hamming distance, Euclidean distance and generalized hesitant normalized distance given by Xia and Xu [13] as follows:

$$d_h(A_1, A_2) = \frac{1}{n}\sum_{i=1}^{n}[\frac{1}{l_{x_i}}\sum_{j=1}^{l_{x_i}} |A_1^j(x_i) - A_2^j(x_i)|] \quad (1)$$

And

$$d_e(A_1, A_2) = [\frac{1}{n}\sum_{i=1}^{n}(\frac{1}{l_{x_i}}\sum_{j=1}^{l_{x_i}} |A_1^j(x_i) - A_2^j(x_i)|^2)]^{\frac{1}{2}} \quad (2)$$

And

$$d_g(A_1, A_2) = [\frac{1}{n}\sum_{i=1}^{n}(\frac{1}{l_{x_i}}\sum_{j=1}^{l_{x_i}} |A_1^j(x_i) - A_2^j(x_i)|^\lambda)]^{\frac{1}{\lambda}} \quad (3)$$

Where $\lambda > 0, A_1^j(x_i)$ and $A_2^j(x_i)$ are the j^{th} values in $A_1(x_i)$ and $A_2(x_i)$ respectively and l_{x_i}= max $\{l(A_1(x_i)), l(A_2(x_i))\}$. If the weight w_i of each element $x_i \epsilon X$ is taken into account, Xia and Xu [13] defined the generalized hesitant weighted distance as follows:

$$d_{wg}(A_1, A_2) = [\sum_{i=1}^{n} w_i(\frac{1}{l_{x_i}}\sum_{j=1}^{l_{x_i}} |A_1^j(x_i) - A_2^j(x_i)|^\lambda)]^{\frac{1}{\lambda}} \quad (4)$$

For two HFEs $A_1(x)$ and $A_2(x)$ on $X = \{x_1, x_2, x_3 \dots, x_n\}$ Xia and Xu [14] proposed several distance between $A_1(x)$ and $A_2(x)$ as follows:

$$d_1(A_1, A_2) = [\frac{1}{l_{x_i}}\sum_{j=1}^{l_{x_i}} |A_1^j(x_i) - A_2^j(x_i)|] \quad (5)$$

And

$$d_2(A_1, A_2) = [\frac{1}{l_{x_i}}\sum_{j=1}^{l_{x_i}} |A_1^j(x_i) - A_2^j(x_i)|^2]^{\frac{1}{2}} \quad (6)$$

Where $A_1^j(x_i)$ and $A_2^j(x_i)$ are the j^{th} values in $A_1(x_i)$ and $A_2(x_i)$ respectively and l_{x_i}= max $\{l(A_1(x_i)), l(A_2(x_i))\}$.

Definition 4: [4]
Let A be a hesitant fuzzy on $X = \{x_1, x_2, x_3 \dots, x_n\}$ and for any $x_i \epsilon X$, $l(A(x_i))$ be the length of $A(x_i)$. Denote $u(A(x_i)) = 1 - \frac{1}{l(A(x_i))}$ and $u(A) = \frac{1}{n}\sum_{i=1}^{n} u(A(x_i))$, where $u(A(x_i))$ is the hesitant degree of $A(x_i)$ and $u(A)$ is the hesitant degree of A.

For two HFSs $A_1(x)$ and $A_2(x)$ on $X = \{x_1, x_2, x_3 \dots, x_n\}$, Deqing Li et al. [4] proposed the normalized Hamming distance, normalized Euclidean distance, normalized Generalized distance, generalized hesitant weighted distance, all including hesitant degree as follows:

$$d_{hh}(A_1, A_2) = \frac{1}{2n}\sum_{i=1}^{n}[|u(A_1(x_i)) - u(A_2(x_i))| + \frac{1}{l_{x_i}}\sum_{j=1}^{l_{x_i}} |A_1^j(x_i) - A_2^j(x_i)|] \quad (7)$$

Retrieval Number: D8809118419/2019©BEIESP
DOI:10.35940/ijrte.D8809.118419
Journal Website: www.ijrte.org

Published By:
Blue Eyes Intelligence Engineering
& Sciences Publication

And

$d_{he}(A_1, A_2) = [\frac{1}{2n}\sum_{i=1}^{n}(\left|u(A_1(x_i)) - u(A_2(x_i))\right|^2 +$
1𝑙𝑥𝑖𝑗=1𝑙𝑥𝑖|𝐴1𝑗(𝑥𝑖)−𝐴2𝑗(𝑥𝑖)|2)]12
(8)

And

$d_{hg}(A_1, A_2) = [\frac{1}{2n}\sum_{i=1}^{n}(\left|u(A_1(x_i)) - u(A_2(x_i))\right|^\lambda +$
1𝑙𝑥𝑖𝑗=1𝑙𝑥𝑖|𝐴1𝑗(𝑥𝑖)−𝐴2𝑗(𝑥𝑖)|𝜆)]1𝜆 (9)

And

$d_{whg}(A_1, A_2) = [\frac{1}{2}\sum_{i=1}^{n} w_i(\left|u(A_1(x_i)) - u(A_2(x_i))\right|^\lambda +$
1𝑙𝑥𝑖𝑗=1𝑙𝑥𝑖|𝐴1𝑗(𝑥𝑖)−𝐴2𝑗(𝑥𝑖)|𝜆)]1𝜆
(10)

$d_{wphg}(A_1, A_2) = [\sum_{i=1}^{n} w_i(\alpha\left|u(A_1(x_i)) - u(A_2(x_i))\right|^\lambda +$
𝛽1𝑙𝑥𝑖𝑗=1𝑙𝑥𝑖|𝐴1𝑗(𝑥𝑖)−𝐴2𝑗(𝑥𝑖)|𝜆)]1𝜆
(11)

Where $\lambda > 0, A_1^j(x_i)$ and $A_2^j(x_i)$ are the j^{th} values in $A_1(x_i)$ and $A_2(x_i)$ respectively and $l_{x_i}= \max\{l(A_1(x_i)), l(A_2(x_i))\}$, $w_i$is the weight of each $x_i \epsilon X\ s.t. 0 \le w_i \le 1$ and $\sum_{i=1}^{n} w_i$=1, 0≤ $\alpha, \beta \le 1$ and $\alpha + \beta = 1$.

In hesitant fuzzy set, when the similarity between two objects is calculated, the divergence of HFSs is taken into account. The divergence of HFSs includes the divergence of HFEs. More closely if we notice, the similarity is measured according to the divergence of HFEs which consists of their lengths and values. Deqing Li et al. [4] introduced the hesitant degree where he considered the divergence of the length because divergence of the values cannot solve some special types of problems. But real life problems are so complicated that the existing distance measures also fails to give any reasonable results.

Example 1:

Let $X = \{x\}$. Assume that there exists two patterns which are represented by hesitant fuzzy sets $A_1 = \{0.9, 0.8, 0.7\}$ and $A_2 = \{0.7, 0.6, 0.2\}$. Now there is a sample to be recognized which is represented by a hesitant set $A = \{0.7, 0.65, 0.6\}$, the principle of minimum distance measure of hesitant fuzzy set is given by:

$d(A_{i0}, A) = \min_{1\le i\le 2}\{d(A_1, A), d(A_2, A)\}$

which means the sample A belongs to the pattern A_{i0}.

Now applying the above mentioned distance measures equation (1) and (7), then we get,

$d_h(A_1, A) = 0.15$, $d_h(A_2, A) = 0.15$, $d_{hh}(A_1, A) = 0.075$, $d_{hh}(A_2, A) = 0.075$.

From equation (1) we cannot get the minimum distance and after applying hesitant degree in
equation (7), again the problem remains unsolved.

The difference between the membership values of A_2 is very large. This shows that the pattern A_2 is unique. Finally we can conclude that the sample A belongs to the pattern A_1. But by calculating the above distance measures we cannot say that A belongs to the pattern A_1.

Therefore, to solve this type of problems we need to modify the existing distance measure. In the following section we have proposed a new distance measure for overcoming the above limitations.

III. DISTANCE AND SIMILARITY MEASURES CONSIDERING NEW HESITANT DEGREE

In this section, we propose a new hesitant degree and then new distance and similarity measures have been studied.

Definition 5:

Let A be a hesitant fuzzy set on $X = \{x_1, x_2, ..., x_n\}$, and for each $x_i \in X$, $A(x_i)$ be the hesitant fuzzy element. Denote

$u(A(x_i)) = (A(x_i)^+ - A(x_i)^-)$

$u(A) = \frac{1}{n}\sum_{i=1}^{n}\left(u(A(x_i))\right)$, where,

$A(x_i)^+ = max(A(x_i))$ and $A(x_i)^- = min(A(x_i))$.

We call $u(A(x_i))$ the hesitant degree of $A(x_i)$ and $u(A)$ the hesitant degree of A, respectively.

For any hesitant fuzzy element $A(x)$, the value of $u(A(x))$ reflects the difference between the highest and lowest membership value of $A(x)$. If the value of $u(A(x))$ is very large that means the decision maker is fully hesitant while giving the preferences in the process of decision making. And if the value of $u(A(x))$ is very small, it indicates that the decision maker can determine the precise value of the membership confidently.

Therefore, when $A(x_i)^+ = A(x_i)^-$then $u(A(x_i)) = 0$. That means the decision maker is fully confident and can determine the membership value without any hesitancy. When $A(x_i)^+ = 1$ and $A(x_i)^- = 0$, then we have $u(A(x_i)) = 1$. That means the decision maker is fully hesitant and cannot determine the membership value confidently.

Example 2:

Let $A_1(x) = \{0.9, 0.85, 0.8\}$ and $A_2(x) = \{0.8, 0.2\}$, then $u(A(x_1)) = 0.9 - 0.8 = 0.1$ and $u(A(x_2)) = 0.8-0.2 = 0.6$.

If we apply the concept of hesitant degree introduced by Deqing Li et al. [4], we get $u(A(x_1))$=1-$\frac{1}{3}$ =0.67 and $u(A(x_2))$ = $1 - \frac{1}{2}$= 0.5. According to Deqing Li et al. [4], the hesitant degree depends on the length of the HFE. If length is very small, the hesitant degree is near to 1, it indicates that the decision maker is confident while assigning the membership values. Here, the length of $A_2(x)$ is smaller than that of $A_1(x)$. That means decision maker should be confident enough while assigning the membership values of $A_2(x)$ than $A_1(x)$. But this is incorrect because the difference between the membership values $A_2(x)$ is larger than that of $A_1(x)$. Since the proposed hesitant degree is nothing but the difference between the highest and lowest value of the HFE, so it comes out with an accurate explanation when decision makers are involved in the process of decision making problems. This proves that the proposed hesitant degree is more reasonable than the existing one.

New distance and similarity measures including the newly proposed hesitant degree are presented in the following below:

Definition 6:

For two HFSs $A_1(x)$ and $A_2(x)$ on $X = \{x_1, x_2, x_3, ..., x_n\}$, the normalized Hamming distance including new hesitant degree between $A_1(x)$ and $A_2(x)$ is defined as follows:

$d_{nhh}(A_1, A_2) = \frac{1}{2n}\sum_{i=1}^{n}\left[\left|u(A_1(x_i)) - u(A_2(x_i))\right| +\right.$
1𝑙𝑥𝑖𝑗=1𝑙𝑥𝑖𝐴1𝑗𝑥𝑖−𝐴2𝑗𝑥𝑖 (12)

And

$d_{nhe}(A_1, A_2) = \left[\frac{1}{2n}\sum_{i=1}^{n}\left(\left|u(A_1(x_i)) - u(A_2(x_i))\right|^2 + \right.\right.$
1lxij=1lxiA1jxi−A2jxi212 (13)

And

$d_{nhg}(A_1, A_2) = \left[\frac{1}{2n}\sum_{i=1}^{n}(\left|u(A_1(x_i)) - u(A_2(x_i))\right|^\lambda + \right.$
1lxij=1lxiA1jxi−A2jxiλ)1λ (14)

Where $\lambda > 0$, $A_1^j(x_i)$ and $A_2^j(x_i)$ are the j^{th} values in $A_1(x_i)$ and $A_2(x_i)$, respectively, and $l_{x_i} = \max\{l(A_1(x_i)), l(A_2(x_i))\}$.

If we have different preference between the hesitant degree and the membership values, then the distance measures with preferences follows:

$d_{pnhh}(A_1, A_2) = \frac{1}{n}\sum_{i=1}^{n}\left[\alpha\left|u(A_1(x_i)) - u(A_2(x_i))\right| + \right.$
β1lxij=1lxiA1jxi−A2jxi (15)

And

$d_{pnhe}(A_1, A_2) = \left[\frac{1}{n}\sum_{i=1}^{n}\left(\alpha\left|u(A_1(x_i)) - u(A_2(x_i))\right|^2 + \right.\right.$
β1lxij=1lxiA1jxi−A2jxi212 (16)

And

$d_{pnhg}(A_1, A_2) = \left[\frac{1}{n}\sum_{i=1}^{n}(\alpha\left|u(A_1(x_i)) - u(A_2(x_i))\right|^\lambda + \right.$
β1lxij=1lxiA1jxi−A2jxiλ) 1λ (17)

Where $\lambda > 0$, $0 \leq \alpha, \beta \leq 1$, and $\alpha + \beta = 1$.

If we ignore the influence of hesitant degree of HFE, i.e. $\alpha = 0$ then d_{pnhh}, d_{pnhe} and d_{pnhg} become the distance measure d_h , d_e and d_g proposed in Xu and Xia [13] respectively.

Theorem 1. $d_{nhh}(A_1, A_2)$, $d_{nhe}(A_1, A_2)$ and $d_{nhg}(A_1, A_2)$ satisfy the properties *(D1)-(D4).*

The proof is easily obtained.

Usually, the weight of the element $x \in X$ should be taken into account. The weighted distance measures for HFSs are presented as follows:

Assume that the weight of $x_i \in X$ is w_i $(i = 1, 2, ..., n)$, where $0 \leq w_i \leq 1$ and $\sum_{i=1}^{n} w_i = 1$, then we have the following weighted distance measures:

$d_{wnhh}(A_1, A_2) = \frac{1}{2}\sum_{i=1}^{n} w_i\left[\left|u(A_1(x_i)) - u(A_2(x_i))\right| + \right.$
1lxij=1lxiA1jxi−A2jxi (18)

And

$d_{wnhe}(A_1, A_2) = \left[\frac{1}{2}\sum_{i=1}^{n} w_i\left(\left|u(A_1(x_i)) - u(A_2(x_i))\right|^2 + \right.\right.$
1lxij=1lxiA1jxi−A2jxi212 (19)

And

$d_{wnhg}(A_1, A_2) = \left[\frac{1}{2}\sum_{i=1}^{n} w_i(\left|u(A_1(x_i)) - u(A_2(x_i))\right|^\lambda + \right.$
1lxij=1lxiA1jxi−A2jxiλ) 1λ (20)

Where $\lambda > 0$.

If we use both the weight w of each element $x \in X$ and different preference between the influence of hesitant degree and membership value, then we have the weighted distance measure with preference as follows:

$d_{wpnhh}(A_1, A_2) = \sum_{i=1}^{n} w_i\left[\alpha\left|u(A_1(x_i)) - u(A_2(x_i))\right| + \right.$
β1lxij=1lxiA1jxi−A2jxi (21)

And

$d_{wpnhe}(A_1, A_2) = \left[\sum_{i=1}^{n} w_i\left(\alpha\left|u(A_1(x_i)) - u(A_2(x_i))\right|^2 + \right.\right.$
β1lxij=1lxiA1jxi−A2jxi212 (22)

And

$d_{wpnhg}(A_1, A_2) = \left[\sum_{i=1}^{n} w_i(\alpha\left|u(A_1(x_i)) - u(A_2(x_i))\right|^\lambda + \right.$
β1lxij=1lxiA1jxi−A2jxiλ) 1λ (23)

Where $\lambda > 0$, $0 \leq \alpha, \beta \leq 1$, and $\alpha + \beta = 1$.

If $\alpha = 0$, then d_{wpnhg} becomes d_{wg} proposed in Xu and Xia [13]. Similarity measures can also be obtained by using Property 1.

IV. APPLICATION

To validate the proposed distance measure, the following example is taken into account.

Example 3: *(Deqing Li et al. [4])*

Let $X = \{x\}$. Assume that there exist two patterns which are represented by hesitant fuzzy sets $h_1 = \{0.97, 0.95, 0.88, 0.86, 0.82, 0.8\}$ and $h_2 = \{0.45\}$. Now there is a sample to be recognized which is represented by a hesitant set $h = \{0.75, 0.73, 0.7, 0.65, 0.6, 0.55\}$. Firstly, we extend h_2 as $h_2 = \{0.45, 0.45, 0.45, 0.45, 0.45, 0.45\}$, and apply the proposed distance measure, d_{nhh} $(h, h_1) = 0.12335$, d_{mh} $(h, h_2) = 0.20665$. Since the distance between h and h_1 is less than that of h and h_2. This shows that h belongs to the pattern h_1. According to Deqing Li et al. [4], d_{hh} $(h, h_1) = 0.10835$, d_{hh} $(h, h_2) = 0.52165$. That means h belongs to the pattern h_1 which is exactly similar with our result. Therefore, this proves that the proposed distance measure is valid.

The proposed distance measure can solve the following example which is not solved by the existing measures.

Example 4:

Suppose that E denotes the set of all equilateral triangle, where $E = \{\alpha, \beta, \gamma | \alpha = \beta = \gamma = 60^o\}$. For every triangle T, then T is considered as a fuzzy set in E, thus the membership degree of the fuzzy set T is used to reflect the degree that the triangle (fuzzy set) T is related to the equilateral triangle.

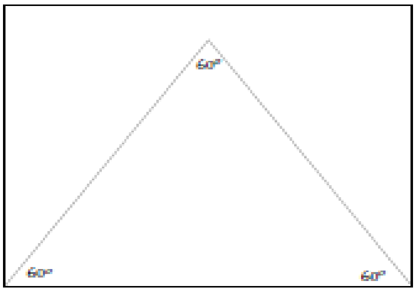

Fig 1: Graphical representation of the equilateral triangle *E*

Let us consider a real life example. Let E_1be a triangle such that E_1= {α, β, γ| $\alpha = 70^o$, $\beta = 55^o$, $\gamma = 55^o$} which is similar to the equilateral triangle.
Graphically,

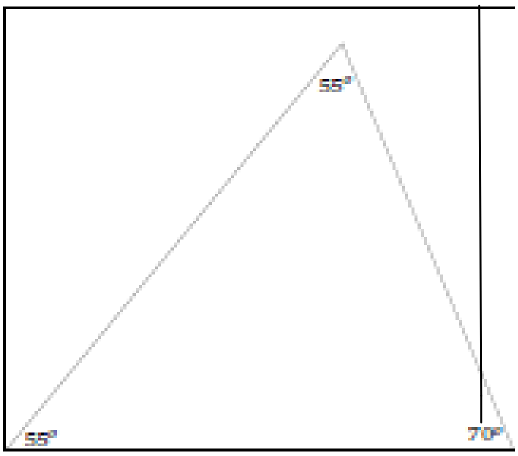

Fig 2: Graphical representation of the triangle E_1

Now consider another two triangles T_1 and T_1 such that T_1= {α, β, γ| $\alpha = 75^o$, $\beta = 55^o$, $\gamma = 50^o$} and T_2= {α, β, γ| $\alpha = 80^o$, $\beta = 50^o$, $\gamma = 50^o$} respectively.

Graphically,

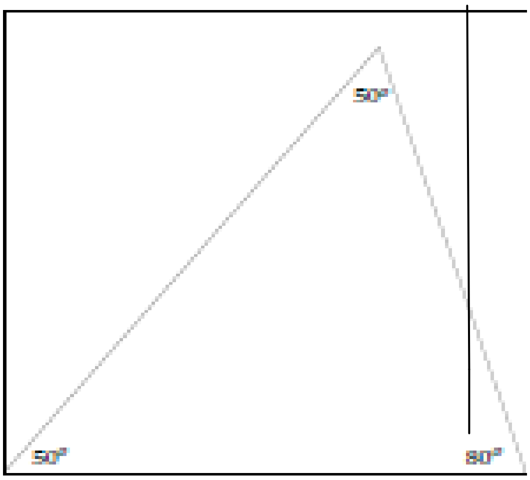

Fig 3: Graphical representation of the triangle T_1

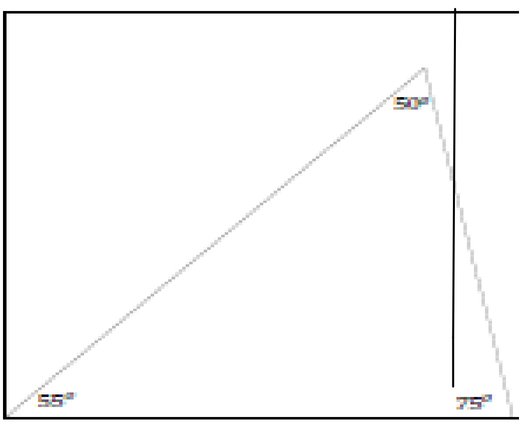

Fig 4: Graphical representation of the triangle T_2

Suppose the membership values of E_1, T_1 and T_2 given by the decision makers are as follows:
A = {(E_1, h), (T_1, h_1), (T_2, h_2)} i.e. A = {(E_1, {0.65, 0.6}), (T_1, {0.7, 0.35}), (T_2, {0.85, 0.7})}.
According to the expert's decision, none of the triangles belongs to the set of equilateral triangles. Now, if we compare the similarity among these three triangles E_1, T_1 and T_2, we have to decide a sample image among these three. Initially let us consider E_1 as the sample image. Using distance measures, the minimum distance between (E_1, T_1) and (E_1, T_2) can be calculated.
We have HFEs as h_1 = {0.7, 0.35} and h_2 = {0.85, 0.7}, and h = {0.65, 0.6}. If we think logically again we get that the difference between the values of h_1 is very large, so the shape of the triangle h_1 is unique and never similar to h. This means h belongs to h_2. When we apply the Hamming distance measure equation (1), then we have d_h (h, h_1) = 0.15, d_h (h, h_2) = 0.15. If we apply the Hamming distance measure including hesitant degree equation (7), then d_{hh} (h, h_1) = 0.075, d_{hh} (h, h_2) = 0.075. That means none of the existing distance measures can solve this problem. Now applying the proposed distance measure equation (12) and (13), then
we have d_{nhh} (h, h_1) = 0.225, d_{nhh} (h, h_2) = 0.125, d_{nhe} (h, h_1) = 0.2475, d_{nhe} (h, h_2) = 0.1323. That means the sample h belongs to the pattern h_2, which is exactly matched with our initial assumption. It indicates that d_{nhh} and d_{nhe} are reasonable.
The following example is a special type of example which is not solved by the existing distance measure including hesitant degree but it is solved by the standard distance measures and the proposed distance measure including new hesitant degree.

Example 5:
Let us consider another real life example where E_1 is almost similar to equilateral triangle such that E_1 = { α, β, γ | $\alpha = 62^o$, $\beta = 59^o$, $\gamma = 59^o$}.
Graphically,

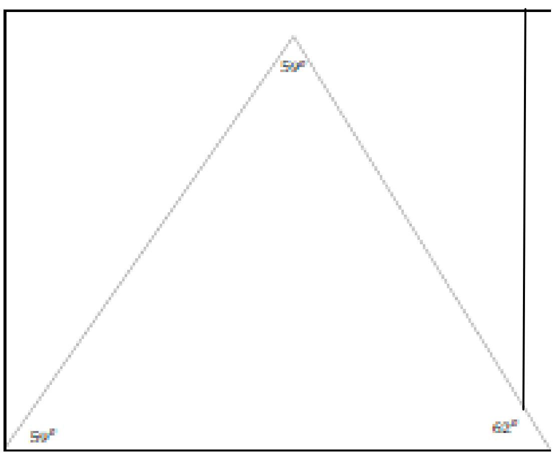

Fig 5: Graphical representation of the triangle E_1

Also let us consider another two triangles T_1 and T_1 such that T_1= {α, β, γ| $\alpha = 64^o$, $\beta = 58^o$, $\gamma = 58^o$} and T_2= {α, β, γ| $\alpha = 82^o$, $\beta = 50^o$, $\gamma = 48^o$} respectively.
Graphically,

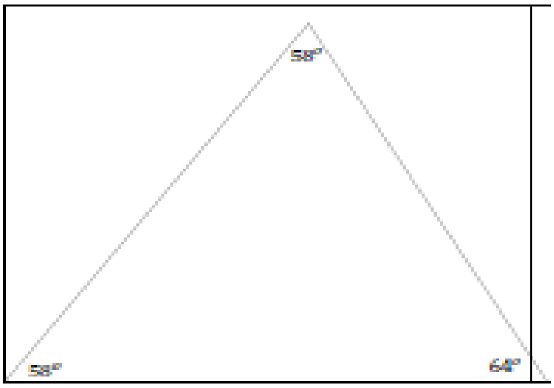

Fig 6: Graphical representation of the triangle T_1

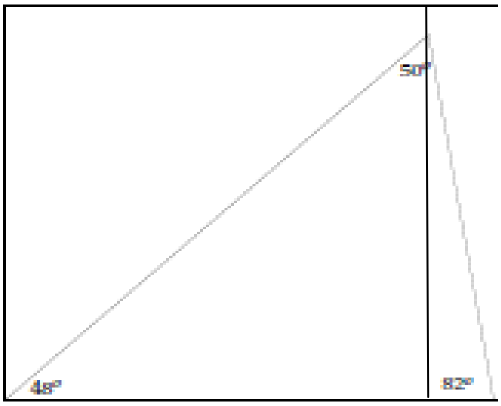

Fig 7: Graphical representation of the triangle T_2

Suppose the membership values of E_1, T_1 and T_2 given by the decision makers are as follows:

A = {(E_1, h), (T_1, h_1), (T_2, h_2)} i.e. A = {(E_1, {0.9}), (T_1, {0.95, 0.9}), (T_2, {0.4})}.

According to the expert's decision, the triangles E_1 and T_1 are almost similar to the equilateral triangle. Using distance measures the similarity among these triangles E_1, T_1 and T_2 can be compared. Initially let the sample image be E_1. The minimum distance between (E_1, T_1) and (E_1, T_2) can be calculated as follows. For calculation we have, h_1 = {0.95, 0.9} and h_2 = {0.4}, and a triangle h = {0.9} to be recognized. If we think logically again we get that the difference between the values of h_1 is very small and this indicates that the shape of the triangle is almost equivalent to the equilateral triangle, so the shape of the triangle h_1 may be similar to h. This means h belongs to h_1. But the membership value of h_2 is 0.4, which indicates that the shape is not exactly similar to the equilateral triangle because the decision maker is fully confident that the shape is unique and with this confident he has assigned the membership value of h_2. So the shape of h_2 is not matched with h.

If we apply the Hamming distance measure including existing hesitant degree equation (7), then d_{hh} (h, h_1) = 0.2625, d_{hh} (h, h_2) = 0.25. That means if we add the existing hesitant degree to the distance measure the result becomes different, which is beyond our intuition. But when we apply the Hamming distance measure equation (1), then we have d_h (h, h_1) = 0.025, d_h (h,h_2) = 0.5. If weapply the Euclidean distance measure equation (2), then d_e (h, h_1) = 0.035, d_e (h, h_2) = 0.5. Now applying the proposed distance measure including new hesitant degree equation (12) and (13) ,then we have d_{nhh} (h, h_1) = 0.0375, d_{nhh} (h, h_2) = 0.25, d_{nhe} (h, h_1) = 0.0433, d_{nhe} (h, h_2) = 0.3536.That means the sample h belongs to the pattern h_1, which is exactly matched with our initial assumption. This indicates that the proposed distance measure including new hesitant degree works better than the existing distance measure with hesitant degree.

The proposed distance measure is applied in multi criteria decision making problem with some special assumption.

Example 6: *(Xia and Xu [13], Alternative selection)*

Energy is an indispensable factor for the socio-economic development of societies. Suppose that there are five alternatives (energy projects) P_i(i = 1, 2, 3, 4, 5) to be invested, and four attributes to be considered: c_1: technological; c_2: environmental; c_3: socio-political; c_4: economic. The attribute weight vector is w = (0.15, 0.3, 0.2, 0.35). Several decision makers are invited to evaluate the performance of the five alternatives. To get a more reasonable result, it is better that the decision makers give their evaluations anonymously. Thus, each value provided only means that it is possible value, but its importance is unknown. So, it is reasonable to allow these values repeated many times appear only once, and all possible evaluations for an alternative under the attributes can be considered as HFS. The result evaluated by the decision makers are contained in a hesitant fuzzy decision matrix, shown in Table 1.

Suppose that the ideal alternative is P^*= {1} seen as a special HFS, we can select the best alternative by calculating the distance between each alternative and the ideal alternative. In order to understand more the effect of parameters α and β in d_{wphg} and d_{wpnhg} , we consider parameter α(α= 0.1, 0.2, 0.5, 0.6, 0.8), respectively. Now the generalized hesitant weighted distance d_{wg} proposed by Xia and Xu [13] equation (4) and the distance measure d_{wphg} proposed by Deqing Li et al. [4] are shown in Table 2 to Table 7, respectively and the proposed distance measure including new hesitant degree d_{wpnhg} with parameters a and b are shown in the Table 8-12 to calculate the deviations between each alternative and the ideal alternative as follows:

Table 1: Hesitant fuzzy decision matrix

	C_1	C_2	C_3	C_4
P_1	{0.5, 0.4, 0.3}	{0.9, 0.8, 0.7, 0.1}	{0.5, 0.4, 0.2}	{0.9, 0.6, 0.5, 0.3}
P_2	{0.5, 0.3}	{0.9, 0.7, 0.6, 0.5, 0.2}	{0.8, 0.6, 0.5, 0.1}	{0.7, 0.4, 0.3}
P_3	{0.7, 0.6}	{0.9, 0.6}	{0.7, 0.5, 0.3}	{0.6, 0.4}
P_4	{0.8, 0.7, 0.4, 0.3}	{0.7, 0.4, 0.2}	{0.8, 0.1}	{0.9, 0.8, 0.6}
P_5	{0.9, 0.7, 0.6, 0.3, 0.1}	{0.8, 0.7, 0.6, 0.4}	{0.9, 0.8, 0.7}	{0.9, 0.7, 0.6, 0.3}

Table 2: ***Results obtain by distance measure d_{wg}***

	P_1	P_2	P_3	P_4	P_5	Rankings
λ=1	0.4779	0.5027	0.4025	0.4292	0.3558	$P_5>P_3>P_4>P_1>P_2$
λ=2	0.5378	0.5451	0.4366	0.5052	0.4129	$P_5>P_3>P_4>P_1>P_2$
λ=6	0.6599	0.6476	0.5156	0.6704	0.5699	$P_3>P_5>P_2>P_1>P_4$

Retrieval Number: D8809118419/2019©BEIESP
DOI:10.35940/ijrte.D8809.118419
Journal Website: www.ijrte.org

Published By:
Blue Eyes Intelligence Engineering
& Sciences Publication

Table 3: ***Results obtain by distance measure*** d_{wphg} ***with*** α **= 0.1,** β **= 0.9**

	P_1	P_2	P_3	P_4	P_5	Rankings
λ= 1	0.5022	0.5222	0.4156	0.4508	0.3943	$P_5>P_3>P_4>P_1>P_2$
λ= 2	0.5589	0.5632	0.4477	0.5216	0.4566	$P_3>P_5>P_4>P_1>P_2$
λ= 6	0.6681	0.6579	0.5206	0.6699	0.6030	$P_3>P_5>P_2>P_1>P_4$
λ=10	0.7221	0.7091	0.5625	0.7330	0.6708	$P_3>P_5>P_2>P_1>P_4$

Table 4: ***Results obtain by distance measure*** d_{wphg} ***with*** α **= 0.2,** β **= 0.8**

	P_1	P_2	P_3	P_4	P_5	Rankings
λ= 1	0.5265	0.5418	0.4287	0.4725	0.4328	$P_3>P_5>P_4>P_1>P_2$
λ= 2	0.5793	0.5807	0.4585	0.5375	0.4965	$P_3>P_5>P_4>P_1>P_2$
λ= 6	0.6759	0.6675	0.5254	0.6694	0.6290	$P_3>P_5>P_2>P_4>P_1$
λ= 10	0.7230	0.7133	0.5646	0.7283	0.6847	$P_3>P_5>P_2>P_1>P_4$

Table 5: ***Results obtain by distance measure*** d_{wphg} ***with*** α **= 0.5, β= 0.5**

	P_1	P_2	P_3	P_4	P_5	Rankings
λ= 1	0.5994	0.6005	0.4679	0.5375	0.5483	$P_3>P_4>P_5>P_1>P_2$
λ= 2	0.6365	0.6304	0.4896	0.5825	0.6004	$P_3>P_4>P_5>P_2>P_1$
λ= 6	0.6968	0.6928	0.5386	0.6680	0.6851	$P_3>P_4>P_5>P_2>P_1$
λ= 10	0.7255	0.7247	0.5705	0.7125	0.7156	$P_3>P_4>P_5>P_2>P_1$

Table 6: ***Results obtain by distance measure*** d_{wphg} ***with*** α ***= 0.6,*** β ***= 0.4***

	P_1	P_2	P_3	P_4	P_5	Rankings
λ= 1	0.6237	0.6201	0.4810	0.5592	0.5868	$P_3>P_4>P_5>P_2>P_1$
λ= 2	0.6545	0.6461	0.4996	0.5967	0.6313	$P_3>P_4>P_5>P_2>P_1$
λ= 6	0.7032	0.7002	0.5427	0.6675	0.6996	$P_3>P_4>P_5>P_2>P_1$
λ= 10	0.7263	0.7281	0.5723	0.7064	0.7237	$P_3>P_4>P_5>P_1>P_2$

Table 7: ***Results obtain by distance measure*** d_{wphg} ***with*** α ***= 0.8,*** β ***= 0.2***

	P_1	P_2	P_3	P_4	P_5	Rankings
λ= 1	0.6723	0.6592	0.5072	0.6025	0.6638	$P_3>P_4>P_2>P_5>P_1$
λ= 2	0.6890	0.6764	0.5189	0.6242	0.6888	$P_3>P_4>P_2>P_5>P_1$
λ= 6	0.7150	0.7141	0.5504	0.6665	0.7247	$P_3>P_4>P_2>P_1>P_5$
λ= 10	0.7279	0.7346	0.5758	0.6927	0.7378	$P_3>P_4>P_1>P_2>P_5$

Table 8: ***Results obtain by distance measure*** d_{wpnhg} ***with*** α ***= 0.1,*** β ***= 0.9***

	P_1	P_2	P_3	P_4	P_5	Rankings
λ= 1	0.4763	0.5044	0.3878	0.4333	0.3692	$P_5>P_3>P_4>P_1>P_2$
λ= 2	0.5427	0.5459	0.4231	0.5021	0.4258	$P_3>P_5>P_4>P_1>P_2$
λ= 6	0.6615	0.6454	0.5072	0.6629	0.5762	$P_3>P_5>P_2>P_1>P_4$
λ= 10	0.7205	0.7006	0.5545	0.7324	0.6561	$P_3>P_5>P_2>P_1>P_4$

Table 9: ***Results obtain by distance measure*** d_{wpnhg} ***with*** α **= 0.2,** β **= 0.8**

	P_1	P_2	P_3	P_4	P_5	Rankings
λ= 1	0.4897	0.5056	0.3725	0.4371	0.3824	$P_3>P_5>P_4>P_1>P_2$
λ= 2	0.5471	0.5466	0.4088	0.5025	0.4381	$P_3>P_5>P_4>P_2>P_1$
λ= 6	0.6629	0.6435	0.4972	0.6548	0.5819	$P_3>P_5>P_2>P_4>P_1$
λ=10	0.7199	0.6968	0.5478	0.7234	0.6574	$P_3>P_5>P_2>P_1>P_4$

Table 10: ***Results obtain by distance measure*** d_{wpnhg} ***with*** α ***= 0.5,*** β ***= 0.5***

	P_1	P_2	P_3	P_4	P_5	Rankings
λ= 1	0.5084	0.5113	0.3283	0.4496	0.4227	$P_3>P_5>P_4>P_1>P_2$
λ= 2	0.5614	0.5496	0.3639	0.4986	0.4735	$P_3>P_5>P_4>P_2>P_1$
λ= 6	0.6432	0.6371	0.4628	0.6271	0.5967	$P_3>P_5>P_4>P_2>P_1$
λ=10	0.7177	0.6833	0.5229	0.6962	0.6621	$P_3>P_5>P_2>P_4>P_1$

Table 11: ***Results obtain by distance measure*** d_{wpnhg} ***with*** α ***= 0.6,*** β ***= 0.4***

	P_1	P_2	P_3	P_4	P_5	Rankings
λ= 1	0.5158	0.5136	0.1615	0.4539	0.4365	$P_3>P_5>P_4>P_2>P_1$
λ= 2	0.5668	0.5508	0.3482	0.4974	0.4848	$P_3>P_5>P_4>P_2>P_1$
λ= 6	0.6698	0.6354	0.4496	0.6165	0.6024	$P_3>P_5>P_4>P_2>P_1$
λ= 10	0.7171	0.6782	0.5123	0.685	0.6642	$P_3>P_5>P_2>P_4>P_1$

Table 12: Results obtain by distance measure d_{wpnhg} ***with*** α ***= 0.8,*** β ***= 0.2***

	P_1	P_2	P_3	P_4	P_5	Rankings
λ= 1	0.5269	0.516	0.284	0.4616	0.4629	$P_3>P_4>P_5>P_2>P_1$
λ= 2	0.5753	0.552	0.3122	0.4946	0.5064	$P_3>P_4>P_5>P_2>P_1$
λ= 6	0.6725	0.6306	0.4075	0.5915	0.6118	$P_3>P_4>P_5>P_2>P_1$
λ= 10	0.7153	0.6666	0.478	0.6547	0.6672	$P_3>P_4>P_2>P_5>P_1$

Table 13: ***Existing hesitant degree of HFEs and synthetic divergence of hesitant degree between*** P_i ***and*** P^*

	C_1	C_2	C_3	C_4	Divergence of hesitant degree between P_i and P^*
P_1	0.6667	0.75	0.6667	0.75	0.7208
P_2	0.5	0.8	0.75	0.6667	0.6983
P_3	0.5	0.5	0.6667	0.5	0.5333
P_4	0.75	0.6667	0.5	0.6667	0.6459
P_5	0.8	0.75	0.6667	0.75	0.7408
P^*	0	0	0	0	

Table 14: ***The proposed new hesitant degree of HFEs and synthetic divergence of hesitant degree between*** P_i ***and*** P^*

	C_1	C_2	C_3	C_4	Divergence of hesitant degree between P_i and P^*
P_1	0.2	0.8	0.3	0.6	0.54
P_2	0.2	0.7	0.7	0.4	0.52
P_3	0.1	0.3	0.4	0.2	0.255
P_4	0.5	0.5	0.7	0.3	0.47
P_5	0.8	0.4	0.2	0.6	0.49
P^*	0	0	0	0	

Retrieval Number: D8809118419/2019©BEIESP
DOI:10.35940/ijrte.D8809.118419
Journal Website: www.ijrte.org

Published By:
Blue Eyes Intelligence Engineering
& Sciences Publication

The results obtained by d_{wpnhg} are different from that of d_{wg} and d_{wphg} because the hesitant degree of HFE has different influence on the calculation. To investigate in detail, we only consider the case λ= 1 in the following. Whenever we calculate the distance measure d_{wg}, the hesitant degree is not considered and the result depends on the divergence of the values only, so P_5 is the best alternative. However, we use the formula d_{wphg} and d_{wpnhg}, hesitant degree is considered as the difference between the values of HFEs. For convenience to analyze, we calculate the hesitant degree of HFEs and the synthetic difference between P_i and P^* calculated by $\sum_{j=1}^{4} w_i \left|u\left(P_i(C_j)\right) - u\left(P^*(C_j)\right)\right|$. The results of the existing hesitant degree and the proposed new hesitant degree are listed in Table 13 and Table 14, respectively.

If we apply the proposed new hesitant degree, then the synthetic difference between P_i and P^* is totally different from that of existing one. According to the hesitant degree proposed by Deqing Li et al. [4], $u(P_4(C_3))$ = 0.5000 and $u(P_5(C_3))$ = 0.6667. That means while assigning the membership values of the HFE P_4, the decision maker is more confident than that of the HFE P_5,when the criteria C_3 is concerned. But if we see the membership values of $P_4(C_3)$ = {0.8, 0.1},it indicates that the decision maker is not confident while assigning the membership values because the difference between the values is very large and again if we see the membership values of $P_5(C_3)$ = {0.9, 0.8, 0.7}, it indicates that decision maker is more confident while assigning the membership values. So the existing hesitant degree could not properly explain about the membership values because it depends on the length of the HFEs and the synthetic difference between P_i and P^* is not reasonable.

When we apply the proposed new hesitant degree, then we get $u(P_4(C_3))$ = 0.7000 and $u(P_5(C_3))$ =0.2000. That means while assigning the membership values of the HFE P_4, the decision maker is less confident than that of the HFE P_5 and this is logically correct. For example, while α= β= 0.5, the ranking of P_3>P_4>P_5 in Table 5 is changed as P_3>P_5>P_4 in Table 10. Since the existing hesitant degree has some limitations so its influence has an adverse effect on the final decision. This indicates that the proposed new hesitant degree gives the accurate final decision. However, the ranking of P_3, P_1, P_2 does not change because the difference of the values of P_3 and P^* is much more smaller than that between P_1 and P^* or P_2 and P^*. This indicates that the divergence of the values pays more attention than the divergence of the hesitant degrees. While increasing the value of α, for example α= 0.8, the ranking P_2>P_5>P_1 in Table 7 is changed as P_5>P_2 >P_1 in Table 12. In this case, we pay more attentions on the divergence of hesitant degree than the divergence of the values. According to the Table 13 and Table 14, we see that the ranking of the synthetic divergence of the hesitant degree is P_3>P_4>P_2>P_1>P_5 and P_3>P_4>P_5>P_2>P_1, respectively. Since the proposed synthetic divergence of the hesitant degree is accurately measured than the existing hesitant degree, so it comes out with an accurate ranking in decision making problem. Hence the results obtained by the proposed distance measure d_{wpnhg} are more reasonable than that calculated by d_{wg} and d_{wphg}.

V. RESULTS AND DISCUSSION

Suppose a sample object (i.e. equilateral triangle) is given and two more objects (i.e. similar to equilateral triangle) are to be judged by experts that which one of the given objects belongs to the sample object. Then the experts give their own feelings in terms of membership value according to the best of their knowledge. After getting their views, we can easily reach to the final solution. All these events are based on our perception that which objects belongs to the given sample. To make the method of decision making based on the perceptions more scientific some mathematical indices like distance measures are proposed. That is why many researchers have proposed distance measure based on some logic. But in real life all the problems we face are not always similar. In this paper we have modified the formula of distance measure including new hesitant degree to overcome the limitation that we have faced in some examples. All the formulae of distance measure depend upon the differences of the membership values given by experts. But in some cases it may happen that after measuring all the distances, the value of the outcome is either equal or incorrect. We have proposed the Example 4 where the distance measures given by all the traditional methods failed to give the final solution because the distance measure given by them is equal for all the cases. Again in Example 5 we have seen that the outcome given by the existing distance measure Deqing Li et al. [4] is incorrect. In these cases our proposed distance measure works properly and has given an accurate result. Examples 3, 4, 5, 6 are enough to show that our proposed distance measure is better than the existing measures. We can apply our proposed distance measure successfully in the field of image classification, pattern recognition, image processing, machine learning, market prediction, power plant site selection, many areas in engineering and medical science etc. where decision making method plays an important role. In this paper, the proposed method is applied in Example 6 which is a multi criteria decision making problem and the results what we have discussed are fully logical and more appropriate.

VI. CONCLUSION

Using hesitant fuzzy set many distance and similarity measures have been proposed so far. In this paper, we have introduced a new hesitant degree. The distance and similarity measures including the proposed hesitant degree overcome the situations where the existing measures cannot give any constructive result. Since hesitant degree plays an important role in distance measures, so we have to be conscious while calculating the hesitant degree and with the help of this we can able to get the exact result without any hesitancy. In our day to day life we come across different situations like pattern recognition i.e. if doctors want to diagnose a patient by drawing similarity of symptoms of two patients based on their perceptions/views on different parameters, some situations may arise due to higher hesitancy leading to failure of the decision making process to come out with clear verdict about the disease if the existing distance measures are used. But if the proposed measures are used then they can handle not only the normal situations which can be handled with the existing measures but also the intricate situations with higher hesitancy wherein the existing measures failed.

In these situations our method comes out with a satisfactory solution better than the existing ones because after mathematical calculation the existing formulas give equal value in some cases and that is why we cannot conclude the solution of the problem properly but the proposed method gives clear distinctive value so we can easily reach to a clear verdict or solution. We have applied the proposed measure in some problems and have shown that our result gives the most satisfactory answer than other measures.

REFERENCES

1. K. Atanassov, Intuitionistic fuzzy sets, Fuzzy Sets Syst 20(1) (1986) 87-96.
2. K. Atanasov, Gargov G. Interval valued intuitionistic fuzzy sets, Fuzzy Sets and Systems 31(1989) 343-349.
3. G. J. Klir, B. Yuan, Fuzzy Sets And Fuzzy Logic, Theory and application.
4. D. Li, W. Zeng, J. Li, New distance and similarity measures on hesitant fuzzy sets and theirapplications in multi-criteria decision making, Engineering application of artificial Intelligence40 (2015) 11-16.
5. D. Li, W. Zeng, Y. Zhao, Note on distance measure of hesitant fuzzy sets, Information Sciences321 (2015) 103-115.
6. H. Liu, R. M. Rodriguez, A fuzzy envelop for hesitant fuzzy linguistic term set and itsapplication to multi criteria decision making, Inform. Science 258 (2014) 220-238.
7. D. H. Peng, C. Y. Gao, Z. F. Gao, Generalized hesitant fuzzy synergetic weighted distancemeasures and their application to multi-criteria decision making, Appl. Math. Modell 37(2013) 5837-5850.
8. E. Szmidt, J. Kacprzyk, Distance between intuitionistic fuzzy sets, Fuzzy sets and system 114(2000) 505-518.
9. V. Torra, Hesitant fuzzy sets, International Journal of Intelligent Systems 25 (2010) 529-539.
10. V. Torra, Y. Narukawa, On hesitant fuzzy sets and decision, The 18th IEEE InternationalConference on Fuzzy Systems Jehu Island Korea (2009) 1378-1382.
11. W. J. Wang, New similarity measures on fuzzy sets and on elements, Fuzzy sets and system85 (1997) 305-309.
12. M. Xia, Z. Xu, Hesitant fuzzy information aggregation in decision making, International Journal of Approximate Reasoning 52 (2011) 395-407.
13. Z. S. Xu, M. M. Xia, Distance and similarity measures for hesitant fuzzy sets, Inf. Sci. 181(2011) 2128-2138.
14. Z. S. Xu, M. M. Xia, On distance and correlation measures of hesitant fuzzy information, Int.J. Intell. Syst. 26 (2011) 410-425.
15. Z. Xu, Hesitant Fuzzy Sets Theory (2014).
16. L. Zadeh, Fuzzy Sets, Inf.Control 8 (1965) 338-353.
17. W. Y. Zeng, H. X. Li, Inclusion measure, similarity measure, and the fuzziness of fuzzy setsand their relations, Int. J. Intell. Syst. 21 (2006) 639-653.

AUTHORS PROFILE

Biplab Singha is the research scholar in the Department of Mathematics, National Institute of Technology Silchar. He completed his master degree from Gauhati University in 2011 and his research area is fuzzy set theory and its application.

Mausumi Sen is the Assistant Professor in the Department of Mathematics, National Institute of Technology Silchar. She has completed her PhD from Institute of Advanced Study in Science &Technology (IASST), Gauhati. Her research areas are Functional Analysis, Fuzzy sets and fuzzy logic, Linear Algebra . She has a number of publications in reputed National and International journals.

Nidul Sinha is the Professor in the Department of Electrical Engineering, National Institute of Technology Silchar. He has completed his PhD from Jadavpur University, Kolkata. His research areas are Power system optimization, Automatic generation control under conventional and deregulated environment, Application of soft computing techniques, technical forecasting, Case base reasoning, intelligent instrumentation. He has a number of publications in reputed National and International journals.