



# Simulation of Boarding Time Prediction using M/M/C/K Queuing Model for Airport Passengers

Sivakami Sundari M, Palaniammal S

**Abstract:** Successfully executing takeoff of an aircraft as per the scheduled time is mainly depends on how effectively various queues of different security systems in the airport is managed. Recently, due to the globalization, air traffic is increasing exponentially. Number of passengers handled by airport authorities in various queues with minimum waiting time of the passenger in queues is need of the hour. Queuing theory concepts and the mathematical models already developed are used to optimize largely in airports. However the congestion in the airports is ever increasing and need computation to arrive at quick decision to handle such situations with minimum cost while not compromising the passengers comfort level. In a typical airport, starting from the entry gate to exit gate, passengers need to experience various types of queues. However the queuing system which is selected for boarding in a aircraft is most critical and aircraft operator need to address the queue properly, to takeoff the aircraft at right time. In this work boarding time is predicted using simulation method and is compared with that of mathematical methods. The simulated results are in good agreement with that of analytical result.

**Keywords :** M/M/C/k queues, finite capacity, single and multi server queuing system.

## I. INTRODUCTION

Queue is very common word now and is experienced in almost all the systems and many times it is pervasive. If it is crowded or there is congestion it means there is a kind of queue exists. Congestion occurs when entities need to be served queue up and is waiting for their turn. Not only people wait for service, in many situations machine, materials and recently data needed to wait at certain time to receive service. Queuing is considered as a system and the purpose of the queuing system is to transfer people, machine, Material and information. Queue is witnessed many places like hospitals, restaurant, bank, supermarket, tollgates, train station, shipyard, airport, call centre, container yard and computer

data. Productivity of any system hinder by queues. Greatest source of inefficiency in production is unattended queues. Queues creates stress and unwanted situation sometimes goes out of control. Organizations scientifically managing the queues are removing congestion with modern computational tools and experiencing reduction of queuing. It means resulting in improved efficiency and significant reduction in operation cost, yielding better profitability. Queuing system consist of customer, servers and the facilities to queue.

Queues can be classified based on the definition of queuing system keeping in view the purpose and nature of the queues between customers and servers.

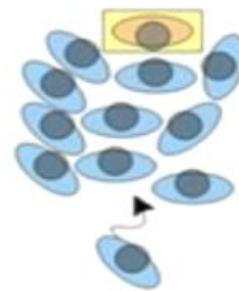


Fig. 1.Schematic of a Bulk Queue



Fig. 2.Example of a Bulk Queue

Queues are categorized in two types based on existence of waiting line. Queues that do not maintain waiting line is classified as bulk queues, which requires minimum space and do not follow any queue discipline. Fig.1 shows the schematic of such a bulk queue and the example of such queue is shown in Fig.2 at the entrance of a train. The properties of bulk queues are randomness, disorganized in nature, creates frustration need muscle power to progress in the queue,

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result to stampede often and may life threatening. But waiting line queues are more disciplined and controllable. The Fig.3 shows the schematic of waiting line queues.

In this queue customers must queue up in the waiting line and wait till their turn to be served. The customers usually arrive at random with certain time distribution, depends on the need of the service and prior knowledge about the property of the service queues. The customers arrive with certain discipline, few cases expects certain prioritization, but mostly first come first serve (FIFO). Servers may be serial or parallel, serve infinite or finite customers and single or multiple.

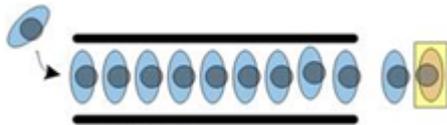


Fig. 3.Example of waiting line queues

## II. CLASSIFICATION OF QUEUES

The objective of Queuing theory or queuing models is to how with minimum resources entities arriving for service with randomness are served. Queuing models estimates waiting time based on certain assumption. Though the customers arrive at random and is transient, it is assumed of steady state. Fig.4 shows the actual transient state and of assumed steady state conditions of queues in terms of Measurement Of Effectiveness (MOE) verses time. Not only customers arrive with randomness, service also experience randomness due to various real time situations and constraints. Fig.5 shows comparison of cumulative service distribution chart of assumed and actual. Service distribution mostly follows poisson process and called Markovian otherwise deterministic[1-3].

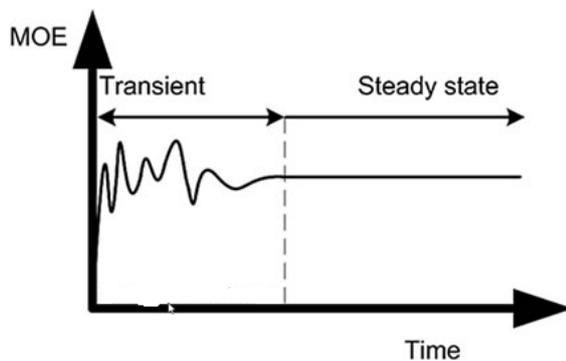


Fig. 4.Transient and Steady state property of queue.

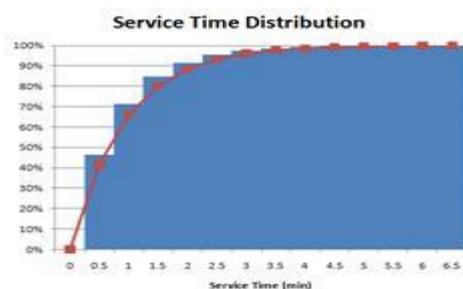


Fig. 5.Trend of service time distribution

## III. M/M/1/∞ QUEUES

D G Kendall(1952) [4-5]classified queues with notation of a/b/c/d/e. The simplest form of queues is M/M/1. The first letter indicates distribution of arrival process, in this case Poisson process means arrivals by Poisson distribution and service by exponential distribution is second letter and both are Markovian hence 'M'. If it is 'E' Erlang distribution, 'D' Deterministic or constant distribution and 'G' General probability distribution with known mean and variance. Fig.6 shows schematic of single server infinite capacity single and Disney queues. The third letter denotes number of servers in this case '1' represents single server and capacity is infinity which is fourth letter of mnemonic representation.

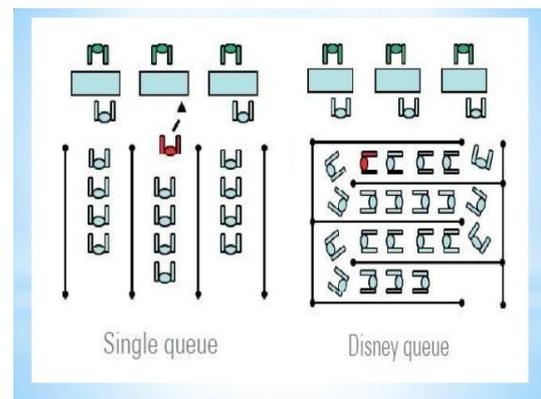


Fig. 6.Schematic of single queues and disney queue

## IV. M/M/C/∞ QUEUES

When traffic density is more and customer waiting time becomes larger it is advisable to go for multi server system in order to optimize waiting cost. Multi server queues increases cost of the services and to decide number of servers optimization of the system to be done. Fig.7 shows trend graph of the multi server optimization using graphical method. From the graph it is clear that the waiting cost is maximum, with that of minimum number of servers and then there is no change in the cost after optimization of number of servers. But the service cost increases against the number of servers increases[6-10]. Total cost of the system is minimum at optimum number of servers and at acceptable waiting cost as shown in the graph. Based on which the number of servers could be based upon.

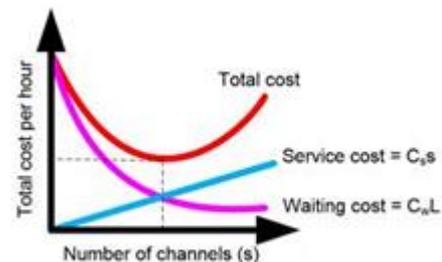


Fig. 7.Graphical method to optimize queuing problem

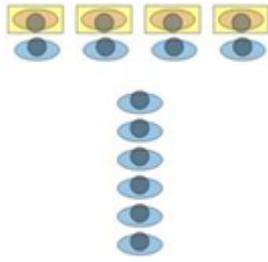


Fig. 8.Schematic of multi server queues

V. M/M/C/K QUEUES ANALYSIS

The previous queuing model discussed is assumed of infinite capacity, however no queue can be infinite capacity practically and is of specific capacity (k). Such queues are experienced in many systems and a real time approach is required to model the queues. Aircraft passengers boarding is following this type of queue. In this model the number of service channels is ‘C’ and the maximum capacity of the system is limited to ‘k’. Behavior and performance of the queue could be predicted mathematically using equations given below [1 – 10].

Probability that the system is idle

$$P_0 = \left[ \sum_{n=0}^{C-1} \frac{1}{n!} \left(\frac{\lambda}{\mu}\right)^n + \frac{1}{C!} \left(\frac{\lambda}{\mu}\right)^C \sum_{n=C}^k \left(\frac{\lambda}{\mu}\right)^{n-C} \right]^{-1} \tag{1}$$

Probability that the system has ‘n’ passengers

$$P_n = \frac{1}{n!} \left(\frac{\lambda}{\mu}\right)^n P_0, \quad n \leq C \tag{2}$$

$$P_n = \frac{1}{C! C^{n-C}} \left(\frac{\lambda}{\mu}\right)^n P_0, \quad \text{for } C < n \leq K \tag{3}$$

Traffic intensity

$$\rho = \frac{\lambda}{\mu C} \tag{4}$$

Average number of passengers in the queue

$$L_q = P_0 \left(\frac{\lambda}{\mu}\right)^C \frac{\rho}{C!(1-\rho)^2} [1 - \rho^{k-C} - (k-C)(1-\rho)\rho^{k-C}] \tag{5}$$

Average number of passengers in the system

$$L_s = L_q + C - \sum_{n=0}^{C-1} (C-n)P_n \tag{6}$$

Average waiting time of customer in the system

$$w_s = \frac{L_s}{\lambda'} \tag{7}$$

Average waiting time of customer in the queue

$$w_q = \frac{L_q}{\lambda'} \tag{8}$$

Effective arrival rate

$$\lambda' = \mu [C - \sum_{n=0}^{C-1} (C-n)P_n] \tag{9}$$

This model is mathematically solved and is simulated by keeping the input values as lambda, μ, number of servers and capacity of the system and the values of (1)- (9) simulated.

VI. AIRCRAFT PASSENGER BOARDING QUEUE

Typical aircraft passengers queue numbers depends on the aircraft model varies like 70,180,300, and 350. The queues formed in this system is depends on the number of passengers booked tickets and maximum capacity of the queue depends on the aircraft model. Fig.9 shows the seating capacity of an aircraft.



Fig. 9.Schematic of aircraft seating capacity

The time available for the aircraft to board the passengers is depends on the number of aircrafts served by the airport. In a busy airport, this is very critical and flight operator is to decide to provide single queue or multiqueu service line[11-14] . However resources available in such situations is to be analysed to take decision on operations. Fig.10 shows the actual picture of passengers boarding in the aircraft. Considering single queue and multiqueues number of customers waiting , waiting time, boarding time simulated.



Fig. 10. Boarding queus discipline multiserver

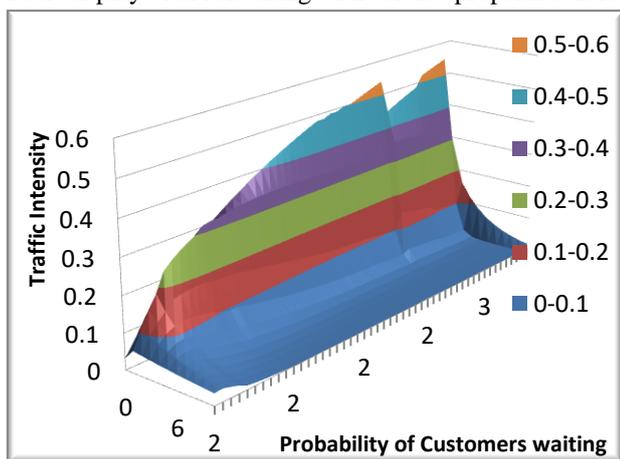
VII. RESULT AND DISCUSSION

The boarding queues involving in passenger aircraft is modeled using analytical method by means of the equations presented here (1) - (9). Mathematically calculating the behavior of the queue is quite tedious manually. The calculation become complex for large number of passengers in the queue and predicting waiting time involved in the queue as well as in the system. Minimum time required to takeoff the flight could be arrived for single and multiple queue system using mathematical analysis. Such calculated data is presented in

**Table- I: Results of Mathematical Modeling**

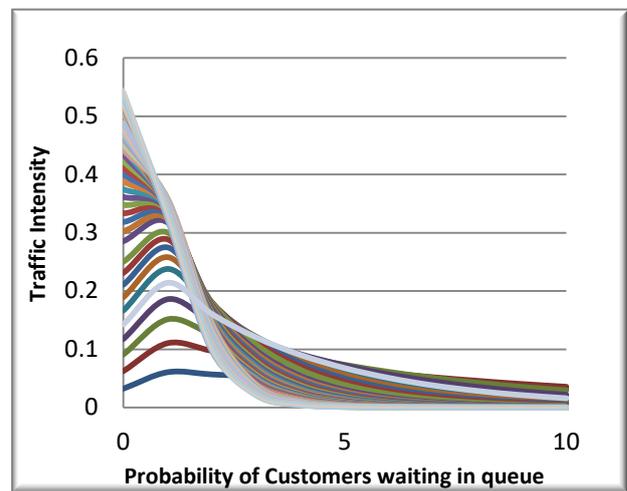
| S.No. | Number of Servers 'C' | Queue Capacity 'k' | Arrival Rate | Service Rate | Ls    | Lq    | Ws   | Wq   | $\rho$ |
|-------|-----------------------|--------------------|--------------|--------------|-------|-------|------|------|--------|
| 1     | 1                     | 180                | 5.0          | 5.0          | 90.0  | 89.0  | 18.1 | 17.9 | 1.0    |
| 2     | 1                     | 180                | 5.0          | 5.3          | 16.7  | 15.7  | 3.3  | 3.1  | 0.9    |
| 3     | 1                     | 180                | 5.0          | 5.9          | 5.6   | 4.7   | 1.1  | 0.9  | 0.8    |
| 4     | 1                     | 180                | 6.3          | 7.4          | 5.7   | 4.9   | 0.9  | 0.8  | 0.9    |
| 5     | 2                     | 180                | 5.0          | 5.0          | 2.0   | 0.3   | 0.4  | 0.2  | 0.5    |
| 6     | 2                     | 180                | 5.0          | 5.6          | 1.9   | 0.2   | 0.4  | 0.2  | 0.4    |
| 7     | 2                     | 180                | 9.9          | 11.0         | 1.9   | 0.2   | 0.2  | 0.1  | 0.5    |
| 8     | 1                     | 300                | 5.0          | 5.0          | 149.8 | 148.8 | 30.1 | 29.9 | 1.0    |
| 9     | 1                     | 300                | 5.0          | 5.3          | 16.7  | 15.7  | 3.3  | 3.1  | 0.9    |
| 10    | 1                     | 300                | 5.0          | 5.9          | 5.6   | 4.7   | 1.1  | 0.9  | 0.8    |
| 11    | 1                     | 300                | 6.3          | 7.4          | 5.7   | 4.9   | 0.9  | 0.8  | 0.9    |
| 12    | 2                     | 300                | 5.0          | 5.0          | 2.0   | 0.3   | 0.4  | 0.2  | 0.5    |
| 13    | 2                     | 300                | 5.0          | 5.6          | 1.9   | 0.2   | 0.4  | 0.2  | 0.4    |
| 14    | 2                     | 300                | 8.6          | 9.7          | 1.9   | 0.2   | 0.2  | 0.1  | 0.4    |

the Table: I It is observed that the traffic density and number of server play vital role along with service properties. It is



**Fig. 11. Simulation of boarding queue**

observed that the service quality is increased with higher server numbers significantly. However the number of passenger increase with the same service and traffic intensity condition not affected the service quality. At any point of time the number passengers waiting for the service is compared with that of analytical method it observed that the values are exactly matching Fig.11. The calculated values of length of queues and waiting time are compared with that of simulated values and difference is negligible. Fig.12 shows Probability of customer waiting in the queue is compared with that of simulated values and the results are well within the limits. Although assumption made to calculate the waiting time and service time based on steady state assumptions the simulated values are in good agreement.



**Fig. 12. Comparison of Simulated queue results**

## VIII. CONCLUSION

Simulation of queuing models using mathematical model available is developed and the results of simulation model and mathematical models converge very well with that of mathematical calculations. Using this simulation if boarding time permitted based on the runway free time available, service facility to be deployed can be calculated. This ensures minimum use of airport facility time which ultimately increases productivity, result to profitability of the aircraft operations.

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