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Construction of Codes from Symmetric Groups

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Abstract: *Let FS_n be semisimple group algebra where S_n denotes the Symmetric group of degree n. We obtain the complete set of irreducible linear idempotents of the group algebra FS_n. We also find the dimension and minimum distance of the group codes over the group S_n.*

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I. INTRODUCTION

The birth of coding theory was inspired by a classic paper of Shannon in 1948. Error correcting codes are now widely used in many practical applications such as returning pictures from deep space, storage of data on magnetic tape and generate of registration numbers etc. A linear code is an ideal of the group algebra FG generated by an idempotent. C.G. Pillado and S. Gonzalez [7] gave the non-abelian codes over F_2 of length 24 dimension 6 and minimum weight 10. Marines [5] presented a survey on the results from group algebra and coding theory. In papers [8],[9],[10] and [11] the authors Manju Pruthi and Sudesh Sehrawat studied the codes over non-abelian groups: group of quaternions; dihedral groups and generalized quaternions; direct product of dihedral and cyclic groups. In this paper we study the semisimple group algebra FS_n.

We structure the paper as follows: Section 2 is devoted to basic definitions and known results. In section 3, we obtain the complete set of irreducible linear idempotents of the group algebra of symmetric group S_n and also find the dimension and minimum distance of the group codes. In section 4,5,6 and 7 we give all the generating idempotents of the group codes of symmetric groups of order 6,24,120, and 720 respectively.

2. Preliminaries

The group algebra FG of group G over the field F is the set of all linear combinations $\sum_{g\in G}\alpha_g g$ where $\alpha_g \in F$ and FG is vector space with basis G over field F, and has the structure of an associative ring with unity.

Let E = $\{e_i\}_{i=1}^{s}$ the set of irreducible idempotents of the group algebra FG and I is any group code generated by $\{e_j\}_{j=1}^{t} \subseteq E$ and μ=E\ $\{e_j\}_{j=1}^{t}$, then I={ u ∈FG : ue = 0 $\forall e \in \mu$ } and we denote I by Iμ (Ref. [2]).

If dimension of Iμ is k and d is the minimum distance and n is the length then Iμ is called an (n,k,d) group code.

Ref.(remark [2]) FG= ($\bigoplus_{ei\in E_L} FG_{e_i}$) $\oplus$ ($\bigoplus_{ej\in E_N} FG_{e_j}$) where EL and EN are the sets of all linear and nonlinear irreducible idempotents of FG respectively. FG_{e_i} is minimal ideal generated by e_i. Now E= EL∪EN, set of irreducible idempotents. Note that if $e_i \in$ EL, then $dim(FG_{e_i}) = 1$; and if $e_j \in$ EN, then $dim\left(FG_{e_j}\right) = m^2$ where $m = \chi_j(1)$ and χ_j is the jth character.

Therefore if μ= μL∪ μN where μL ⊆EL and μN ⊆EN, then

$$dim(I_\mu) = dim(FG) - |\mu_L| dim(FG_{e_i}) - |\mu_N| dim\left(FG_{e_j}\right)$$

where $dim(FG) = |G| = n!$.

Ref.(chap13,14,15, [6])Number of irreducible characters in S_n = number of conjugacy classes in S_n which is equal to number of partition of n. One conjugate class consists of all permutations which have the same cycle decomposition.

Ref(theo.17.11,[6]) The number of distinct linear characters of G is equal to |G/G′| where G′ is derived subgroup of G. Let G = S_n then $S_n' = A_n$ where A_n is the set of all even permutations so Sn/ An = {An, An(1 2)}$\cong$ C2 therefore FS_n has two linear idempotents.

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Now we give the explicit expression of generating idempotents of codes over symmetric groups. Ref. (Example 17.12, 17.13,[6]).

3. Codes over Symmetric groups corresponding to linear characters

Theorem 3.1. The group algebra FSn has exactly two linear irreducible idempotents.

Proof. Symmetric group Sn has exactly two linear irreducible characters, $\chi 1$ and $\chi 2$ which are given by

$\chi 1(g) = 1$ for all $g \in Sn$

$$\chi 2(g) = \begin{cases} 1 & \text{if } g \in An \\ -1 & \text{if } g \notin An \end{cases}$$

Define

$\overline{A_n}$ = sum of all even permutations

$\overline{B_n}$ = sum of all odd permutations

By using the (prop. 14.10 [6]) we have

$$e = \frac{1}{|G|} \sum_{g \in G} \chi(g^{-1}) g \qquad (3.1)$$

where χ is character of FG-module.

The two linear irreducible idempotents are given by

$$e1 = \frac{1}{|G|} \sum_{g \in G} \chi_1(g^{-1}) g = \frac{1}{n!} \sum_{g \in Sn} g$$

$$e2 = \frac{1}{|G|} \sum_{g \in G} \chi_2(g^{-1}) g = \frac{1}{n!} (\overline{A_n} - \overline{B_n})$$

Theorem 3.2. Let e1, e2 be irreducible linear idempotents of FSn then d ($I_{\{e_i\}}$) is 2 and dim ($I_{\{e_i\}}$) is n!−1 for i=1,2.

Proof. Let $u = \sum_{i=1}^{n!} \alpha_i g_i$ be any element of FSn then

$$ue1 = \left(\sum_{i=1}^{n!} \alpha_i g_i\right) \left(\frac{1}{n!} \sum_{g \in Sn} g\right)$$

Since gigj = gk (say)

$$\text{We have } ue1 = \left(\sum_{i=1}^{n!} \alpha_i\right) e1 \qquad (3.2)$$

$$ue2 = \left(\sum \alpha_i - \sum \alpha_j\right) e2 \qquad (3.3)$$

where α_i are the coefficients of even permutation and α_j are the coefficients of odd permutation. As we know in Sn number of odd permutations is equal to number of even permutations.

If we take $u = \alpha g \in$ FSn then $u \notin I_{\{e_1\}}$. Let $u = \alpha_1 g1 + \alpha_2 g2$ be any element of FSn with wt(u) =2 then ue1 = 0 implies $u \in I_{\{e_1\}}$ hence $d(I_{\{e_1\}}) = 2$. Therefore $I_{\{e_1\}}$ is (n!, n!−1,2) MDS group code. Similarly the case i=2 can be proved.

We refer the following results from [4] for computing the character table of Sn

4. Explicit formula for calculating the Characters of Sn

Consider a partition $\lambda = (\lambda_1, \lambda_2, \dots, \lambda_k)$ of n, let Ci denote conjugacy class of Sn. Let j ($1 \le j \le n$) be an index. We write any element in Ci as the $\prod$ disjoint cycles, and i_j denote the number of cycles of length j. We introduce k independent variables $x_1, x_2, \dots, x_k$.

Define $Pj(x) = x_1{}^j + x_2{}^j + \dots + x_k{}^j$. Define discriminant of $\{x_1, x_2, \dots, x_k\}$ to be $\Delta(x) =$

$$\begin{vmatrix} 1 & x_k & \cdots & x_k^{k-1} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & x_1 & \cdots & x_1^{k-1} \end{vmatrix}$$

$$= \prod_{l,m=1,\ l<m}^{k} (x_l - x_m)$$

Let $l_s = \lambda_s + k - s$ for every $1 \le s \le k$. If $f(x)$ is some polynomial of $x_1, x_2, \dots, x_k$, let $[f(x)]_{(l_1, l_2, \dots, l_k)}$ denote the coefficient of the term $x_1{}^{l_1} x_2{}^{l_2} \dots x_k{}^{l_k}$.

"Theorem 4.1 (The Frobenius Formula) The character of the irreducible representation of Sn associated with λ is given by

$$\chi_{(\lambda)}(C_i) = [\,\Delta(x) \prod_j P_j(x)^{i_j}]_{(l_1, l_2, \dots, l_k)}$$ "

(4.1)

Example 4.2. The characters of the irreducible representations of S_3.

We have S_3 = {I, (1 2), (1 3), (2 3), (1 2 3), (1 3 2)} and the Conjugacy classes of S_3 are given by C_1 ={I}, C_2={(1 2), (1 3), (2 3)}, C_3= {(1 2 3), (1 3 2)} and therefore S_3 consist of 3 irreducible characters.

In the above formula if we take the partition $\lambda = (3)$ then k=1 therefore we introduce one independent variable x_1 . So we have $\Delta(x) = 1$.

Identity can be written as product of 1 cycles gives

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i_1=3, $i_2 = 0$, $i_3 = 0$, Further

$\chi_{(3)}(I) = [\Delta(x)P_1(x)^3]_{(3)} = [{x_1}^3]_{(3)} = 1$

In the conjugacy class C_2 each element is product of a 1 cycle and a 2-cycle, thus we have i_1=1, $i_2 = 1$, $i_3 = 0$.

$\chi_{(3)}(C_2) = [\Delta(x)P_1(x)^1P_2(x)^1]_{(3)} = [{x_1}^3]_{(3)} = 1$

In the conjugacy class C_3 each element is a 3-cycle, so we have i_1=0, $i_2 = 0$, $i_3 = 1$.

$\chi_{(3)}(C_3) = [\Delta(x)P_3(x)^1]_{(3)} = [{x_1}^3]_{(3)} = 1$

(ii) Now let the partition λ = (2,1) then k=2 so we introduce 2 independent variables x_1 , x_2.

In this case $\Delta(x) = x_1 - x_2$ and $l_1 = 3$, $l_2 = 1$.

$\chi_{(2,1)}(I) = [(x_1 - x_2)(x_1 + x_2)^3]_{(3,1)} = 2$

In the conjugacy class C_2 every element is product of a 1 cycle and a 2-cycle, so we have i_1=1, $i_2 = 1$, $i_3 = 0$.

$\chi_{(2,1)}(C_2) = [\Delta(x)P_1(x)^1P_2(x)^1]_{(3,1)} = [{x_1}^4 - {x_2}^4]_{(3,1)} = 0,$

In the conjugacy class C_3 each element is a 3-cycle, so we have i_1=0, $i_2 = 0$, $i_3 = 1$.

$\chi_{(2,1)}(C_3) = [\Delta(x)P_3(x)^1]_{(3,1)} = -1$

(iii) let the partition λ = (1,1,1) here k=3 so we introduce 3 independent variable x_1 , x_2, x_3. As above we can compute that

$\Delta(x) = {x_1}^2x_2 - x_1{x_2}^2 - {x_1}^2x_3 + x_3{x_2}^2 + x_1{x_3}^2 - x_2{x_3}^2$. In this case $l_1 = 3$, $l_2 = 2$, $l_3 = 1$.

$\chi_{(1,1,1)}(I) = [\Delta(x)P_1(x)^3]_{(3,2,1)} = 1$

$\chi_{(1,1,1)}(C_2) = [\Delta(x)P_1(x)^1P_2(x)^1]_{(3,2,1)} = -1$

$\chi_{(1,1,1)}(C_3) = [\Delta(x)P_3(x)^1]_{(3,2,1)} = 1$

We have $\chi_{(3)}$ and $\chi_{(1,1,1)}$ are two linear characters and $\chi_{(2,1)}$ is non linear character.

4.3 Character table for S_3

g_i	I	C_2	C_3
$\chi_{(3)}$	1	1	1
$\chi_{(1,1,1)}$	1	−1	1
$\chi_{(2,1)}$	2	0	−1

4.4 Idempotents in the group algebra FS_3::

Using Table 4.3 and formula 3.1, irreducible idempotents in the group algebra FS_3 are given by

$e_1 = \frac{1}{6}[1 + \overline{C_2} + \overline{C_3}]$

$e_2 = \frac{1}{6}[1 - \overline{C_2} + \overline{C_3}]$

$e_3 = \frac{1}{3}(2 - \overline{C_3})$ where $\overline{C_2}$ and $\overline{C_3}$ are the sum of 2 and 3 cycles respectively.

Example 4.5. Let $F=F_5$ be the finite field such that gcd(char(F), 6)=1 then the irreducible idempotents in FS_3 are given by

$e_1 = 1 + \overline{C_2} + \overline{C_3}$

$e_2 = 1 + 4\overline{C_2} + \overline{C_3}$

$e_3 = 4[1 + 2\overline{C_3}]$

The irreducible idempotents in F_7S_3 are given by

$e_1 = 6(1 + \overline{C_2} + \overline{C_3})$

$e_2 = 6(1 + 6\overline{C_2} + \overline{C_3})$

$e_3 = 5(2 + 6\overline{C_3})$

Now E = $\{e_1, e_2, e_3\}$ is the set of all irreducible idempotents of the group algebra FS_3. If $\mu \subseteq E$ define $I_\mu = \{u \in FS_3 : ue = 0 \ \forall e \in \mu\}$ is a group code generated by the set of idempotents $\{E \setminus \mu\}$ e.g. $I_{\{e_3\}} = \{u \in FS_3 : ue_3 = 0\}$ is group code generated by $\{e_1, e_2\}$.

Proposition 4.6. If I_μ is the group code as defined above then dimension and minimum distance of the codes are given by

(i) $d(I_{\{e_i\}}) = 2$ and $dim(I_{\{e_i\}}) = 5$

(ii) $d(I_{\{e_3\}}) = 3$ and $dim(I_{\{e_3\}}) = 2$

(iii) $d(I_{\{e_i, e_3\}}) = 2$ and $dim(I_{\{e_i, e_3\}}) = 4$

(iv) $d(I_{\{e_i, e_3\}}) = 6$ and $dim(I_{\{e_i, e_3\}}) = 1$

for i=1,2.

Proof. We prove (i) Let u = $\alpha_1 + \alpha_2(1\ 2) + \alpha_3(1\ 3) + \alpha_4(2\ 3) + \alpha_5(1\ 2\ 3) + \alpha_6(1\ 3\ 2)$ be any element of FS_3 where $\alpha_i \in F$ then

$$ue_1 = \left(\sum_{i=1}^{6} \alpha_i\right) e_1 \quad (4.2)$$

$$ue_2 = (\alpha_1 - \alpha_2 - \alpha_3 - \alpha_4 + \alpha_5 + \alpha_6) e_2 \quad (4.3)$$

$$ue_3 = \frac{1}{3}[(2\alpha_1 - \alpha_5 - \alpha_6) + (2\alpha_2 - \alpha_3 - \alpha_4)(1\ 2) + (-\alpha_2 + 2\alpha_3 - \alpha_4)(1\ 3) + (-\alpha_2 - \alpha_3 + 2\alpha_4)(2\ 3) + (-\alpha_1 + 2\alpha_5 - \alpha_6)(1\ 2\ 3) + (-\alpha_1 - \alpha_5 + 2\alpha_6)(1\ 3\ 2)] \quad (4.4)$$

To prove d($I_{\{e_1\}}$) =2 let u=αg ∈FS_3, α≠0∈F ,since $ue_1 = \alpha e_1 \neq 0$. we have u=αg ∉ $I_{\{e_1\}}$. Now let u=g−h ∈ FS_3 for any distinct g, h∈ S_3. Then ue_1= 0 so u ∈ $I_{\{e_1\}}$ and hence d($I_{\{e_1\}}$) =2.

Similar is the case i=2 thus d($I_{\{e_i\}}$) =2 for i=1,2. Therefore $I_{\{e_i\}}$ is (6,5,2) MDS group code for i=1,2.

(ii) If we take u = I+(1 2 3)+(1 3 2) ∈FS_3 then ue_3 =0 so u∈ $I_{\{e_3\}}$. Hence d($I_{\{e_3\}}$) =3 and dim($I_{\{e_3\}}$) = 2. Therefore ($I_{\{e_3\}}$) is (6,2,3) group code.

(iii), (iv) proofs are similar.

5. Generating idempotents of the group codes in the group algebra FS_4

The Conjugacy classes of S_4 are C_1 ={I}, C_2= $(1\ 2)^G$, $C_3 = (1\ 2\ 3)^G$, $C_4 = (1\ 2\ 3\ 4)^G$, $C_5 = (1\ 2)(3\ 4)^G$.

Notation: If $C_i = \{x_1, x_2, \ldots, x_n\}$ then we denote $\overline{C_i} = \sum_{t=1}^{n} x_t$

Take the partition λ = (4) then k=1 therefore we introduce one independent variable x_1 .

hence $\Delta(x) = 1$ In this case $l_1 = 4$. Identity can be written as product of four 1 cycles, so we have

$$\chi_{(4)}(I) = [\Delta(x)P_1(x)^4]_{(4)} = [x_1{}^4]_{(4)} = 1$$

In the conjugacy class C_2 each element is product of a 1 cycle and 2-cycle, so we have

$$\chi_{(4)}(C_2) = [\Delta(x)P_1(x)^2P_2(x)^1]_{(4)} = [x_1{}^4]_{(4)} = 1$$

In the conjugacy class C_3 each element is product of a 3-cycle and 1 cycle thus, we have

$$\chi_{(4)}(C_3) = [\Delta(x)P_1(x)^1P_3(x)^1]_{(4)} = [x_1{}^4]_{(4)} = 1$$

In the conjugacy class C_4 each element is a 4-cycle, so we have

$$\chi_{(4)}(C_4) = [\Delta(x)P_4(x)^1]_{(4)} = [x_1{}^4]_{(4)} = 1$$

In the conjugacy class C_5 every element is product of two 2-cycle, so we have

$$\chi_{(4)}(C_5) = [\Delta(x)P_2(x)^2]_{(4)} = [x_1{}^4]_{(4)} = 1$$

(ii) Take the partition λ = (3,1) then k =2 therefore we introduce two independent variable x_1, x_2.

hence $\Delta(x) = x_1 - x_2$. Here $l_1 = 4$, $l_2 = 1$

$$\chi_{(3,1)}(I) = [\Delta(x)P_1(x)^4]_{(4,1)} = [(x_1 - x_2)(x_1 + x_2)^4]_{(4,1)} = 3$$

$$\chi_{(3,1)}(C_2) = [(x_1 - x_2)(x_1 + x_2)^2(x_1{}^2 + x_2{}^2)]_{(4,1)} = 1$$

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$\chi_{(3,1)}(C_3) = [(x_1 - x_2)(x_1 + x_2)(x_1{}^3 + x_2{}^3)]_{(4,1)} = 0$

$\chi_{(3,1)}(C_4) = [(x_1 - x_2)(x_1{}^4 + x_2{}^4)]_{(4,1)} = -1$

$\chi_{(3,1)}(C_5) = [(x_1 - x_2)(x_1{}^2 + x_2{}^2)^2]_{(4,1)} = -1$

(iii) Take the partition λ = (2,2) here k=2 therefore we introduce two independent variable x_1, x_2.

Here $\Delta(x) = x_1 - x_2$. Here $l_1 = \lambda_1 + k - 1$ =3, $l_2 = 2$

$\chi_{(2,2)}(I) = [\Delta(x)P_1(x)^4]_{(4,1)} = 2$

similarly

$\chi_{(2,2)}(C_2) = 0$; $\chi_{(2,2)}(C_3) = -1$; $\chi_{(2,2)}(C_4) = 0$; $\chi_{(2,2)}(C_5) = 2$

(iv) Take the partition λ = (2,1,1) here k=3 so we introduce 3 independent variable x_1, x_2, x_3.

$\Delta(x) = x_1{}^2x_2 - x_1x_2{}^2 - x_1{}^2x_3 + x_3x_2{}^2 + x_1x_3{}^2 - x_2x_3{}^2$. In this case $l_1 = 4$, $l_2 = 2$, $l_3 = 1$

$\chi_{(2,1,1)}(I) = 3$; $\chi_{(2,1,1)}(C_2) = -1$; $\chi_{(2,1,1)}(C_3) = 0$

$\chi_{(2,1,1)}(C_4) = 1$; $\chi_{(2,1,1)}(C_5) = -1$

(v) Take the partition λ = (1,1,1,1) here k=4 so we introduce 4 independent variable x_1, x_2, x_3, x_4.

In this case $l_1 = 4$, $l_2 = 3$, $l_3 = 2$, $l_4 = 1$

$\chi_{(1,1,1,1)}(I) = 1$; $\chi_{(1,1,1,1)}(C_2) = -1$; $\chi_{(1,1,1,1)}(C_3) = 1$

$\chi_{(1,1,1,1)}(C_4) = -1$; $\chi_{(1,1,1,1)}(C_5) = 1$

In above characters χ_4 and $\chi_{(1,1,1,1)}$ are two linear and rest three are non linear irreducible characters, corresponding idempotents are given by

$e_1 = \frac{1}{24}\sum_{i=1}^{5}\overline{C_i}$

$e_2 = \frac{1}{24}[\overline{C_1} - \overline{C_2} + \overline{C_3} - \overline{C_4} + \overline{C_5}]$

$e_3 = \frac{1}{12}[2 - \overline{C_3} + 2\overline{C_5}]$

$e_4 = \frac{1}{16}[3 + \overline{C_2} - \overline{C_4} - \overline{C_5}]$

$e_5 = \frac{1}{16}[3 - \overline{C_2} + \overline{C_4} - \overline{C_5}]$

Let F=F_5, The irreducible idempotents in F_5S_4 are given by

$e_1 = 4\sum_{i=1}^{5}\overline{C_i}$

$e_2 = 4[\overline{C_1} + 4\overline{C_2} + \overline{C_3} + 4\overline{C_4} + \overline{C_5}]$

$e_3 = [1 + 2\overline{C_3} + \overline{C_5}]$

$e_4 = [3 + \overline{C_2} + 4\overline{C_4} + 4\overline{C_5}]$

$e_5 = [3 + 4\overline{C_2} + \overline{C_4} + 4\overline{C_5}]$

Proposition 5.1. If I_μ is the group code then $d(I_{\{e_i\}})$ is 2 and $dim(I_{\{e_i\}})$ is 23 for i=1,2.

Proof. Let u = α_1+ α_2 (1 2) + α_3 (1 3) + α_4 (1 4) + α_5 (2 3) + α_6 (2 4)+ α_7 (3 4) + {α_8(1 2 3) + α_9 (1 3 2)+ α_{10}(1 2 4)+ .. . }+ {α_{16} (1 2 3 4)+ α_{17} (1 2 4 3)+ …}+ α_{22} (1 2)(3 4)+ α_{23} (1 3)(2 4)+ α_{24} (1 4)(2 3) be any element of FS_4 then results can be proved easily using the following expressions

$ue_1 = \left(\sum_{i=1}^{24}\alpha_i\right) e_1$ (5.1)

$ue_2 = \left(\alpha_1 - \sum_{i=2}^{7}\alpha_i + \sum_{i=8}^{15}\alpha_i - \sum_{i=16}^{21}\alpha_i + \sum_{i=22}^{24}\alpha_i\right) e_2$ (5.2)

6. Generating idempotents of the group codes in the group algebra FS_5

The Conjugacy classes of S_5 are C_1 ={I}, C_2= (1 2)^G , C_3 = (1 2 3)^G , C_4 = (1 2 3 4)^G , C_5 = (1 2 3 4 5)^G, C_6 = (1 2)(3 4)^G , C_7 = (1 2 3)(4 5)^G .

we take the partition λ = (5) in this case k=1 therefore we introduce one independent variable x. hence $\Delta(x) = 1$ In this case $l_1 = 5$. we have

$\chi_{(5)}(I) = [\Delta(x)P_1(x)^5]_{(5)} = [x^5]_{(5)} = 1$

$\chi_{(5)}(C_2) = [\Delta(x)P_1(x)^3P_2(x)^1]_{(5)} = 1.$

Similarly $\chi_{(5)}(C_i) = 1$ for i= 3,4,5,6,7.

(ii) when λ = (4,1) we have k=2, in this case we introduce two independent variables x_1, x_2.

therefore $\Delta(x) = x_1 - x_2$, and $l_1 = \lambda_1 + k - 1 = 5$, $l_2 = 1$, and the characters are given by

$\chi_{(4,1)}(I) = 4; \chi_{(4,1)}(C_2) = 2; \chi_{(4,1)}(C_3) = 1$

$\chi_{(4,1)}(C_4) = 0; \chi_{(4,1)}(C_5) = -1; \chi_{(4,1)}(C_6) = 0$

$\chi_{(4,1)}(C_7) = -1$

(iii) when λ = (3,2) , $l_1 = 4, l_2 = 2$

$\chi_{(\lambda)}(I) = 5; \chi_{(\lambda)}(C_2) = 1; \chi_{(\lambda)}(C_3) = -1$

$\chi_{(\lambda)}(C_4) = -1; \chi_{(\lambda)}(C_5) = 0; \chi_{(\lambda)}(C_6) = 1$

$\chi_{(\lambda)}(C_7) = 1$

(iv) when λ = (2,2,1) , $l_1 = 4$, $l_2 = 3$, $l_3 = 1$,

$\Delta(x) = x_1{}^2x_2 - x_1x_2{}^2 - x_1{}^2x_3 + x_3x_2{}^2 + x_1x_3{}^2 - x_2x_3{}^2$

$\chi_{(\lambda)}(I) = [(x_1{}^2x_2 - x_1x_2{}^2 - x_1{}^2x_3 + x_3x_2{}^2 + x_1x_3{}^2 - x_2x_3{}^2)(x_1 + x_2)^5]_{(4,3,1)} = 5$

$\chi_{(\lambda)}(C_2) = [(x_1{}^2x_2 - x_1x_2{}^2 - x_1{}^2x_3 + x_3x_2{}^2 + x_1x_3{}^2 - x_2x_3{}^2)(x_1 + x_2 + x_3)^3(x_1{}^2 + x_2{}^2 + x_3{}^2)]_{(4,3,1)} = -1$

$\chi_{(\lambda)}(C_3) = [(x_1{}^2x_2 - x_1x_2{}^2 - x_1{}^2x_3 + x_3x_2{}^2 + x_1x_3{}^2 - x_2x_3{}^2)(x_1 + x_2 + x_3)^2(x_1{}^3 + x_2{}^3 + x_3{}^3)]_{(4,3,1)} = -1$

$\chi_{(\lambda)}(C_4) = [(x_1{}^2x_2 - x_1x_2{}^2 - x_1{}^2x_3 + x_3x_2{}^2 + x_1x_3{}^2 - x_2x_3{}^2)(x_1 + x_2 + x_3)(x_1{}^4 + x_2{}^4 + x_3{}^4)]_{(4,3,1)} = 1$

$\chi_{(\lambda)}(C_5) = [(x_1{}^2x_2 - x_1x_2{}^2 - x_1{}^2x_3 + x_3x_2{}^2 + x_1x_3{}^2 - x_2x_3{}^2)(x_1{}^5 + x_2{}^5 + x_3{}^5)]_{(4,3,1)} = 0$

$\chi_{(\lambda)}(C_6) = [(x_1{}^2x_2 - x_1x_2{}^2 - x_1{}^2x_3 + x_3x_2{}^2 + x_1x_3{}^2 - x_2x_3{}^2)(x_1 + x_2 + x_3)(x_1{}^2 + x_2{}^2 + x_3{}^2)^2]_{(4,3,1)} = 1$

$\chi_{(\lambda)}(C_7) = [(x_1{}^2x_2 - x_1x_2{}^2 - x_1{}^2x_3 + x_3x_2{}^2 + x_1x_3{}^2 - x_2x_3{}^2)(x_1{}^2 + x_2{}^2 + x_3{}^2)(x_1{}^3 + x_2{}^3 + x_3{}^3)]_{(4,3,1)} = -1$

(v) when λ = (3,1,1) , $l_1 = 5, l_2 = 2, l_3 = 1$

$\chi_{(\lambda)}(I) = 6; \chi_{(\lambda)}(C_2) = 0; \chi_{(\lambda)}(C_3) = 0$

$\chi_{(\lambda)}(C_4) = 0; \chi_{(\lambda)}(C_5) = 1; \chi_{(\lambda)}(C_6) = -2$

$\chi_{(\lambda)}(C_7) = 0$.

(vi) when λ = (2,1,1,1) , $l_1 = 5, l_2 = 3, l_3 = 2, l_4 = 1$

$\chi_{(\lambda)}(I) = 4; \chi_{(\lambda)}(C_2) = -2; \chi_{(\lambda)}(C_3) = 1$

$\chi_{(\lambda)}(C_4) = 0; \chi_{(\lambda)}(C_5) = -1; \chi_{(\lambda)}(C_6) = 0$

$\chi_{(\lambda)}(C_7) = 1$

(vii) when λ = (1,1,1,1,1) , $l_1 = 5, l_2 = 4, l_3 = 3, l_4 = 2, l_5 = 1$

$\chi_{(\lambda)}(I) = 1; \chi_{(\lambda)}(C_2) = -1; \chi_{(\lambda)}(C_3) = 1$

$\chi_{(\lambda)}(C_4) = -1; \chi_{(\lambda)}(C_5) = 1; \chi_{(\lambda)}(C_6) = 1$

$\chi_{(\lambda)}(C_7) = -1$

The irreducible idempotents are given by

$e_1 = \frac{1}{120}\sum_{g \in S_5} g$

$e_2 = \frac{1}{120}[\overline{C_1} - \overline{C_2} + \overline{C_3} - \overline{C_4} + \overline{C_5} + \overline{C_6} - \overline{C_7}]$

$e_3 = \frac{1}{75}[4 + 2\overline{C_2} + \overline{C_3} - \overline{C_7} - \overline{C_5}]$

$e_4 = \frac{1}{75} 4 - 2\overline{C_2} + \overline{C_3} - \overline{C_5} + \overline{C_7}]$

$e_5 = \frac{1}{40}[6 - 2\overline{C_6} + \overline{C_5}]$

$e_6 = \frac{1}{96}[5 + \overline{C_2} - \overline{C_3} - \overline{C_4} + \overline{C_6} + \overline{C_7}]$

$e_7 = \frac{1}{96}[5 - \overline{C_2} - \overline{C_3} + \overline{C_4} + \overline{C_6} - \overline{C_7}]$

In particular let F= F_7, then irreducible idempotents in F_7S_5 are given by

$e_1 = \sum_{g \in S_5} g$

$e_2 = [\overline{C_1} + 6\overline{C_2} + \overline{C_3} + 6\overline{C_4} + \overline{C_5} + \overline{C_6} + 6\overline{C_7}]$

$e_3 = 3[4 + 2\overline{C_2} + \overline{C_3} + 6\overline{C_7} + 6\overline{C_5}]$

$e_4 = 3[4 + 5\overline{C_2} + \overline{C_3} + 6\overline{C_5} + \overline{C_7}]$

$e_5 = 3[6 + 5\overline{C_6} + \overline{C_5}]$

$e_6 = 3[5 + \overline{C_2} + 6\overline{C_3} + 6\overline{C_4} + \overline{C_6} + \overline{C_7}]$

$e_7 = 3[5 + 6\overline{C_2} + 6\overline{C_3} + \overline{C_4} + \overline{C_6} + 6\overline{C_7}]$

7. Generating Idempotents of the group codes in the group algebra FS_6

Symmetric group S_6 consist of 11 irreducible characters (two are linear and nine are non linear characters) which can be easily obtained by using the formula 4.1.

Notation: $\overline{C_1} = I$; $\overline{C_2} = \sum(1\ 2)$; $\overline{C_3} = \sum(1\ 2\ 3)$; $\overline{C_4} = \sum(1\ 2\ 3\ 4)$

$\overline{C_5} = \sum(1\ 2)(3\ 4)$; $\overline{C_6} = \sum(1\ 2)(3\ 4\ 5)$; $\overline{C_7} = \sum(1\ 2\ 3\ 4\ 5)$

$\overline{C_8} = \sum(1\ 2)(3\ 4)(5\ 6)$; $\overline{C_9} = \sum(1\ 2\ 3)(4\ 5\ 6)$; $\overline{C_{10}} = \sum(1\ 2\ 3\ 4)(5\ 6)$

$\overline{C_{11}} = \sum(1\ 2\ 3\ 4\ 5\ 6)$

All the irreducible idempotents in FS_6 are given by

$e_1 = \frac{1}{720}\sum_{g \in S_6} g$

$e_2 = \frac{1}{720}[\overline{C_1} - \overline{C_2} + \overline{C_3} - \overline{C_4} + \overline{C_5} - \overline{C_6} + \overline{C_7} - \overline{C_8} + \overline{C_9} + \overline{C_{10}} - \overline{C_{11}}]$

$e_3 = \frac{1}{456}[5 + 3\overline{C_2} + 2\overline{C_3} + \overline{C_4} + \overline{C_5} - \overline{C_8} - \overline{C_9} - \overline{C_{10}} - \overline{C_{11}}]$

$e_4 = \frac{1}{456}[5 - 3\overline{C_2} + 2\overline{C_3} - \overline{C_4} + \overline{C_5} + \overline{C_8} - \overline{C_9} - \overline{C_{10}} + \overline{C_{11}}]$

$e_5 = \frac{1}{396}[10 + 2\overline{C_2} + \overline{C_3} - 2\overline{C_5} - \overline{C_6} - 2\overline{C_8} + \overline{C_9} + \overline{C_{11}}]$

$e_6 = \frac{1}{396}[10 - 2\overline{C_2} + \overline{C_3} - 2\overline{C_5} + \overline{C_6} + 2\overline{C_8} + \overline{C_9} - \overline{C_{11}}]$

$e_7 = \frac{1}{400}[9 + 3\overline{C_2} - \overline{C_4} + \overline{C_5} - \overline{C_7} + 3\overline{C_8} + \overline{C_{10}}]$

$e_8 = \frac{1}{400}[9 - 3\overline{C_2} + \overline{C_4} + \overline{C_5} - \overline{C_7} - 3\overline{C_8} + \overline{C_{10}}]$

$e_9 = \frac{1}{456}[5 + \overline{C_2} - \overline{C_3} - \overline{C_4} + \overline{C_5} + \overline{C_6} - 3\overline{C_8} + 2\overline{C_9} - \overline{C_{10}}]$

$e_{10} = \frac{1}{456}[5 - \overline{C_2} - \overline{C_3} + \overline{C_4} + \overline{C_5} - \overline{C_6} + 3\overline{C_8} + 2\overline{C_9} - \overline{C_{10}}]$

$e_{11} = \frac{1}{225}[16 - 2\overline{C_3} + \overline{C_7} - 2\overline{C_9}]$

The irreducible idempotents in the group algebra F_7S_6 are given by

$e_1 = 6\sum_{g \in S_6} g$

$e_2 = 6[\overline{C_1} + 6\overline{C_2} + \overline{C_3} + 6\overline{C_4} + \overline{C_5} + 6\overline{C_6} + \overline{C_7} + 6\overline{C_8} + \overline{C_9} + \overline{C_{10}} + 6\overline{C_{11}}]$

$e_3 = [5 + 3\overline{C_2} + 2\overline{C_3} + \overline{C_4} + \overline{C_5} + 6\overline{C_8} + 6\overline{C_9} + 6\overline{C_{10}} + 6\overline{C_{11}}]$

$e_4 = [5 + 4\overline{C_2} + 2\overline{C_3} + 6\overline{C_4} + \overline{C_5} + \overline{C_8} + 6\overline{C_9} + 6\overline{C_{10}} + \overline{C_{11}}]$

$e_5 = 2[10 + 2\overline{C_2} + \overline{C_3} + 5\overline{C_5} + 6\overline{C_6} + 5\overline{C_8} + \overline{C_9} + \overline{C_{11}}]$

$e_6 = 2[10 + 5\overline{C_2} + \overline{C_3} + 5\overline{C_5} + \overline{C_6} + 2\overline{C_8} + \overline{C_9} + 6\overline{C_{11}}]$

$e_7 = [9 + 3\overline{C_2} + 6\overline{C_4} + \overline{C_5} + 6\overline{C_7} + 3\overline{C_8} + \overline{C_{10}}]$

$e_8 = [9 + 4\overline{C_2} + \overline{C_4} + \overline{C_5} + 6\overline{C_7} + 4\overline{C_8} + \overline{C_{10}}]$

$e_9 = [5 + \overline{C_2} + 6\overline{C_3} + 6\overline{C_4} + \overline{C_5} + \overline{C_6} + 4\overline{C_8} + 2\overline{C_9} + 6\overline{C_{10}}]$

$e_{10} = [5 + 6\overline{C_2} + 6\overline{C_3} + \overline{C_4} + \overline{C_5} + 6\overline{C_6} + 3\overline{C_8} + 2\overline{C_9} + 6\overline{C_{10}}]$

$e_{11} = [16 + 5\overline{C_3} + \overline{C_7} + 5\overline{C_9}]$

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