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Replacement Srategy for Depriciating Stock Considering Stockouts and Time Dependent Demand

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***Abstract:** Researchers, till now, have faced only 2 variants in demand - time variations i.e. linear and exponential variations. A linear variation shows constant change in rate of demand per unit of time for a product, which is a virtual assumption and will rarely be found in any real industry. To exemplify, the demand for the latest technology spare parts, chips, computers etc increases rapidly contrary to obsolete technology and routine gadgets. Some researchers have also termed these variations in exponential category as increasing or decreasing function with respect to per unit of time. From a researcher view an exponential rate of variation is very high to get actually satisfied be demand in actual market. Hence, a realistic approach matching with the researched theories and the actual market conditions is neither linear nor exponential but a quadratic variation in demand and time showing growth in both up and down directions of demand. The problem is solved and formulated in a model form and elaborated using sensitivity analysis.*

***Key words:** Linear, exponential, inventory, demand, quadratic function.*

I. INTRODUCTION

In major research problems a lot has been discussed about inventory models with fixed demand. Donaldson [35] firstly suggested an exact replacement policy for a stock type having demand increasing with linear trend in finite time duration. Following this many like Silver and Meal [131], Ritchie[115], Goyal et al. [56], Silver [132] etc. suggest much simplified models to achieve result equivalent to the optimum values in replacement policies. Deb and Chaudhary [33] restudied and formulated with shortages in the model given by Silver[132].

The limitation of study in above discussed models was lack of consideration of depreciation of stocked material. Dave and Patel [30] discussed an inventory model for depreciating items and items having rate of demand dependent on time. Bahari and Kashani [5] gave a heuristic methodology for calculating quantity of order for a stock with constant depreciation and time dependent rate of demand.

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Taking this forward,Goswami and Chaudhary [47] studied the concept assuming fixed time for planning, constant rate of depreciation, linear rate of demand and finite cost of shortages. Hariga [64] studied the latter model for rectifications and gave corresponding corrections. Recently, Lin, Tan and Lee [89], Chakraborty and Chaudhuri [42], Chang and Dye [19] also discussed EOQ models with shortages and demand dependent on time for deteriorating stock units. In another category of Inventory models, researchers usually discuss demand and time varying exponentially.Hariga and Benkherouf [66] used this pattern to give a customized model for depreciating stock. Kishan and Misra [82] also developed a model having constant rate of depreciation and shortages, using the same demand – time pattern.

Considering the literature review, we find that researchers, till now, have faced only 2 variants in demand - time variations i.e linear and exponential variations. A linear variation shows constant change in rate of demand per unit of time for a product, which is a virtual assumption and will rarely be found in any real industry. To exemplify, the demand for the latest technology spare parts, chips, computers etc increases rapidly contrary to obsolete technology and routine gadgets. Some researchers have also termed these variations in exponential category as increasing or decreasing function with respect to per unit of time. From a researcher view an exponential rate of variation is very high to get actually satisfied be demand in actual market. Hence, a realistic approach matching with the researched theories and the actual market conditions is neither linear nor exponential but a quadratic variation in demand and time showing growth in both up and down directions of demand.

II. PROBLEM STATEMENT

In this study, the author have developed, a replacement strategy for a product, Assuming fixed rate of depreciation fixed stock outs and demand time variation as a quadratic function in inventory. The problem is solved and formulated in a model form and elaborated using sensitivity analysis by taking b, c > 0 and different cases are exemplified like b, c < 0; b, c > 0; b > 0 and c < 0; b < 0 and c > 0.

NOTATIONS

1) H: Planning Horizon
2) F(t): Rate of Demand at any time 't'
3) θ: Constant Fraction
4) c_1: holding cost
5) c_2: ordering cost

6) c_3: Purchasing Cost
7) c_4: Shortage cost

ASSUMPTIONS

1) The inventory system has finite H.
2) $F(t) = a+bt+ct^2$ where $a>0$; $b, c \neq 0$
3) Instant Replacement
4) Nil Lead time value.
5) Allowed Shortages.
6) θ decreases with time
7) c_1, c_2, c_3, c_4 per unit time are known and constant during horizon timeline.

III. MODEL FORMULATION

The stock depreciation with time due to either demand satisfaction or inventory depletion nature, and finally tends to stock outs, at any time $t_1 < T$, where T is the cycle time and (t_1, T) is the time interval for stock out.

Consider $I_1(t)$ to be level of inventory at any time 't', then from $t > 0$ to $t < t_1$, the differential equation used is stated as $\frac{dI_1(t)}{dt} + \theta I_1(t) = -(a+bt+ct^2)$ where $0 < t < t_1$

(1.1)

With boundary conditions $I_1(t) = 0$

(1.2)

(1.1) w.r.t (1.2) can be solved to give

$$I_1(t) = \left[\frac{a+bt_1+ct_1^2}{\theta} - \frac{b+2ct_1}{\theta^2} + \frac{2c}{\theta^3}\right] e^{\theta(t_1-t)} - \frac{a+bt+ct^2}{\theta} + \frac{b+2ct}{\theta^2} - \frac{2c}{\theta^3} \quad (1.3)$$

Hence, $I_1(t)$ can be called size of replacement, say, S_1 at time $t = 0$

Therefore, $S_1 = \left[\frac{a+bt_1+ct_1^2}{\theta} - \frac{b+2ct_1}{\theta^2} + \frac{2c}{\theta^3}\right] e^{\theta t_1} - \frac{a}{\theta} + \frac{b}{\theta^2} - \frac{2c}{\theta^3}$

(1.4)

The size of inventory that has been depreciated in $(0, t_1)$ time interval is stated as,

$$D_1 = S_1 - \int_0^{t_1} (a + bt + ct^2)dt$$

(1.5)

Equation (1.4) and (1.5) can be stated in combined form as,

$$D_1 = \left[\frac{a+bt_1+ct_1^2}{\theta} - \frac{b+2ct_1}{\theta^2} + \frac{2c}{\theta^3}\right] e^{\theta t_1} - \frac{a}{\theta} + \frac{b}{\theta^2} - \frac{2c}{\theta^3} - \frac{ct_1^3}{3} - \frac{bt_1^2}{2} - at_1 \quad (1.6)$$

Assuming c_h to be the holding cost of inventory. Then, c_h in $(0, t_1)$ is,

$$C_h = \int_0^t I_1(t)dt$$

$$= c_1 \left[\left(\frac{ct_1^2+bt_1+a}{\theta^2} - \frac{ct_1+b}{\theta^3} + \frac{2c}{\theta^4}\right)\left(e^{t_1\theta} - 1\right) - \frac{2ct_1^3+3bt_1^2+6at_1}{6\theta} + ct12+bt1\theta 2-2ct1\theta 3 \quad (1.7)$$

To interpret the instantaneous level of inventory $I_2(t)$, in time interval of shortage (t_1, T), the differential equationconsidered is,

$$\frac{dI_2(t)}{dt} = a + bt + ct^2 \qquad t_1 \le t \le T$$

(1.8)

With boundary conditions $I_2(t_1) = 0$

(1.9)

Using condition (1.9), equation (1.8) can be solved to get,

$$I_2(t) = \left[\frac{ct^3}{3} + \frac{bt^2}{2} + at\right] - \left[\frac{ct_1{}^3}{3} + \frac{bt_1{}^2}{2} + at_1\right], \ t_1 \le t \le T \quad (1.10)$$

At this level, let S_2 be the level of shortage given by,

$$S_2 = c_4 \int_{t_1}^{T} I_2(t)dt$$

(1.11)

Solving (1.10) and (1.11) by integration, we get

$$S_2 = \left[\frac{a}{2}(T^2 + t_1^2) + \frac{b}{6}(T^3 + 2t_1^3) + \frac{c}{12}(T^4 + t3_1^4) - aTt_1 - bTt122 - cTt133 \quad (1.12)$$

Therefore, mean cost of system $C_1(T, t_1)$ w. r. t time is given by,

$$C(T,t_1) = \left[\frac{S_2+c_2+c_3D_2+C_h}{T}\right]$$

(1.13)

Assuming $t_1 = KT$ where $0 < K < 1$

(1.14)

Substitute (1.14) in (1.6) to get,

$$D_1 = \left[\frac{a+bKT+cK^2T^2}{\theta} - \frac{b+2cKT}{\theta^2} + \frac{2c}{\theta^3}\right] e^{\theta KT} - \frac{a}{\theta} + \frac{b}{\theta^2} - \frac{2c}{\theta^3} - aKT - \frac{bT^2K^2}{2} - \frac{cK^3T^3}{3} \quad (1.15)$$

Also, substituting (1.14) in(1.7) again, we get,

$$c_h = c_1 \left[\left(\frac{a+bKT+cK^2T^2}{\theta^2} - \frac{b+2cKT}{\theta^3} + \frac{2cKT}{\theta^4}\right)\left(e^{\theta KT} - 1\right)\right] - c_1 \left[\frac{6aKT+3bK^2T^2+2cK^3T^3}{6\theta} - \frac{bKT+cK^2T^2}{\theta^2} + \frac{2cKT}{\theta^3}\right]$$

(1.16)

Again substituting (1.14) in (1.12), it gives

$$S_2 = c_4 \left[\frac{aT^2}{2}(1 + K^2 - 2K) + \frac{bT^3}{6}(1 - 3K^2 + 2K^3) + cT4121-4K3+3K4 \quad (1.17)$$

Substituting (1.15), (1.16), (1.17) in (1.13), gives mean cost of system $C_1(K, T)$ as

$$C\ (K,T) = \frac{c_1}{T}\left[\left(\frac{a+bKT+cK^2T^2}{\theta^2} - \frac{b+2cKT}{\theta^3} + \frac{2cKT}{\theta^4}\right)\left(e^{\theta KT} - 1-c1T6aKT+3bK2T2+2cK3T36\theta-bKT+cK2T2\theta 2+2cKT\theta 3+c2T+c3a\theta T+bK\theta+cK2T\theta-bT\theta 2-2cK\theta 2+2cT\theta 3e\theta KT-\right.$$

$$c_3\left[\frac{a}{\theta T} - \frac{b}{\theta T^2} + \frac{2c}{\theta T^3} + aK + \frac{bTK^2}{2} + \frac{cK^3T^2}{3}\right] + c_4\left[\frac{aT}{2}(1 + K2-2K+bT261-3K2+2K3+cT3121-4K3+3K4=0$$

(1.18)

Minimizing Condition for eqn (1.18) is,

$$\frac{\partial C(K,T)}{\partial T} = 0$$

(1.19)

$$\frac{\partial C(K,T)}{\partial K} = 0$$

(1.20)

Using (1.19), it gives

$$e^{\theta KT}K\left(\frac{a}{T} + bK + cK^2T - \frac{b}{T\theta} - \frac{2cK}{\theta} + \frac{2c}{T\theta^2}\right) - \left(\frac{e^{\theta KT}}{\theta} - c1\theta c1+\theta c3aT2-cK2-bT2\theta+2cT2\theta 2-\left(\frac{3bK^2+4cTK^3}{6}\right) - \frac{\theta c_2}{T^2(c_1+\theta c_3)} + \frac{cc_1K^2}{\theta(c_1+\theta c_3)} + \left(\frac{c_3}{T^2(c_1+\theta c_3)}\right)\left(a - b\theta+2c\theta 2+\theta c4(c1+\theta c3)a21+K2-2K+bT31-3K+2K3+cT24(1-4K2+3K4=0\right.$$

(1.21)

Equation (1.20) gives,

$$e^{\theta KT}\left(a+bKT+cK^2T^2-\frac{b}{\theta}-\frac{2cKT}{\theta}+\frac{2c}{\theta^2}\right)+\left(b+2cKT-\right.$$
$$2c\theta e\theta KT\theta- \quad c1\theta(c1+\theta c3)+c1b+2cKT\theta c1+\theta c3-$$
$$\frac{2cc_1}{\theta^2(c_1+\theta c_3)}-(a+bKT+cK^2T^2)+\left(\frac{\theta c_4}{c_1+\theta c_3}\right)[(K-$$
$$1aT+bT2K+cT3K2=0$$

(1.22)

The Probable feasble solution of the problem given by (1.21) and (1.22), is the minimum value of C(K, T) given by K^* and T^*, if we assume minC(K, T) = C^*, say, (K^*, T^*) satisfy the necessary conditions as,

$$\frac{\partial^2 C(K,T)}{\partial K^2}\cdot\frac{\partial^2 C(K,T)}{\partial T^2}-\left(\frac{\partial^2 C(K,T)}{\partial K\partial T}\right)^2>0$$

(1.23(a))

And,

$$\frac{\partial^2 C(K,T)}{\partial K^2}>0,\frac{\partial^2 C(K,T)}{\partial T^2}>0$$

(1.23(b))

Since equation (1.21) and (1.22) are not linear in parameters K and T. Therefore, some numerical illustration has to be used through computer algorithm using a given set of values to parameters. This solution is further checked on sufficient conditions stated in equation (1.23(a) and (b))

In case the output from equation (1.23(a) and (b)) and (1.22) do not comply the sufficient condition in (1.23(a) and (b)), then, it is concluded that an optimal solution does not exist for the illustrated set of values for parameters. Hence, indicating inconsistent values and error in estimation.

IV. ILLUSTRATION ANALYSIS

Let us take a = 100, θ= 0.05, c_1 = 4, c_2 = 150, c_4 = 6 (in respective units)

Solving (1.21) and (1.22) through computerized algorithm for variation in "b" and "c".

Table 1.1 shows the results obtained. Then viability of (1.23(a) and (b)) is checked for this illustration.

Table 1.1

b	c	K^*	T^*	C^*
2.5	0.25	0.2765	0.3695	402.3795
-2.5	0.25	0.2764	0.3766	398.5143
2.5	-0.25	0.2765	0.3699	402.2053
-1	-0.1	0.2764	0.3747	399.5601

SENSTIVITY ANALYSIS

On the basis of above stated illustration, the senstivity analysis of parameters b and c (b, c > 0) is done by increasing and decreasing anyone parameter by 10% and 25% at a time. The other parameters are kept at their actual values. Table 1.2 shows the followed results along with the below mentioned interpreted points.

Table 1.2

Change in parameter	By (%)	K^*	T^*	C^*
b	-10%	0.2765	0.3702	401.9867
	-25%	0.2765	0.3712	401.4135
	10%	0.2765	0.3688	402.7430
	25%	0.2765	0.3678	403.3179
c	-10%	0.2765	0.3695	402.3557
	-25%	0.2765	0.3696	402.3349
	10%	0.2765	0.3694	402.3837
	25%	0.2765	0.3694	402.4161
a	-10%	0.2762	0.4111	360.7645
	-25%	0.2755	0.5118	287.5247
	10%	0.2767	0.3383	440.1178
	25%	0.2770	0.3035	491.3379
c_1	-10%	0.2987	0.3888	381.8870
	-25%	0.3396	0.4343	340.8550
	10%	0.2574	0.3550	419.1892
	25%	0.2333	0.3389	439.5549
c_2	-10%	0.2768	0.3311	359.5638
	-25%	0.2772	0.2621	283.7896
	10%	0.2763	0.4041	441.1517
	25%	0.2760	0.4507	893.7918
c_3	-10%	0.2804	0.3727	398.8358
	-25%	0.2865	0.3778	393.2914
	10%	0.2727	0.3665	405.7726
	25%	0.2672	0.3622	410.6567
c_4	-10%	0.2489	0.3897	381.6505
	-25%	0.1915	0.4448	334.6773
	10%	0.2987	0.3553	418.2651
	25%	0.3247	0.3406	436.1415
θ	-10%	0.2809	0.3732	398.5358
	-25%	0.2878	0.3793	392.4990
	10%	0.2722	0.3660	406.0570
	25%	0.2660	0.3610	411.3436

FINDINGS

From table 1.1 and 1.2 we note the following observations:-

(i) The model is not much sensitive for variation in parameter b > 0 and c > 0.

(ii) The values of T^* changes inversely with respect tp value of C^*. The change in T^* is also inversely proportional to change in a and c_4.

(iii) The direction of changes in K^* and T^* is opposite to direction of changes in C^* as well as direction of changes in parameter c_1, c_3 and θ.

(iv) The direction of change in K^* is opposite to direction of changes in T^* and C^* as well as direction of changes in parameter c_2

V. CONCLUSION

In this paper, an inventory model for level of order where rate of demand is dependent on time, is discussed.the rate of demand assumed is taken in a quadratic function form for variation. The model is solved asumming the shortages.

The outcome is achieved by solving non linear equations in 2 variables with Newton – Raphson method. Also, optimum system cost is proved to justify the optimisation criteria.Traditionally linear or exponential variation in demand with respect to time was studied by researchers. The linear function for rate of demand with respect to time is $D(t) = a + bt$, $b \neq 0$, $a \geq 0$, reflects a slow and constant change in demand. Similarly, the exponential function for rate of demand with respect to time is $D(t) = ae^{bt}$, $b\neq0$, $a>0$, reflects hange in demand exponentially. The exponential change is to high in number which is not in sync with demand patern in real life market for any product.The change in demand also depends on type and nature of product. Hence, to justify the real life market pattern, it is suggested that quadratic function gives results that are neither constant like linear function nor extremely high like exponential. Hence, the Quadratic function for rate of demand with respect to time studied id $D(t) = a + bt + ct^2$ is more realistic. Also using the concept of absolute maxima , it can be justified that D(t) possess maximum values at $t = -b/2c$. This also concludes that the rate of demand increases gradually to the highest and the start decreasing.

VI. FUTURE SCOPE

In future, this model has a scope to be further developed as a computer software for practcal industry application in Cosmetics, woolen, computer Machine aviation industry etc.with $b > 0$, $c > 0$, for which this model can be further studied in future.

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Dr. Shradha Goyal has over 10 years of rich experience in Academics. Dr. Goyal completed her doctorate from Jagannath University and is Graduate and Post Graduate in Mathematics honors from Panjab University Campus, Chandigarh She has participated and presented Papers in various seminar and conferences of National & International repute. She has published several research papers in refereed and reputed journals indexed in UGC and SCOPUS. She has also authored a book titled "Basics of Linear Programming". Dr. Shradha Goyal is associated with Jagannath International Management School, Kalkaji, New Delhi. She is committed towards her responsibilities and gives a mentorship support in Progress of students.

Dr. Rudresh Pandey has over 16 years of experience in corporate, management education and entrepreneurship. He has diverse experience in the areas of general management, marketing, entrepreneurship, B2B marketing, research and advertising. Dr. Pandey's corporaste work experience has been in the field of education rating, women entrepreneurship and education technologies including MNCs and Top Indian companies like - Reed Business Information, India Today (Living Media Group) in Mumbai. ONICRA Credit Rating Agency of India Ltd and Impact Marketing Services Pvt Ltd in Delhi/NCR.

Entrepreneurship has been his area of special interest and his Phd topic had been Women Entrepreneurship. He had been instrumental in setting up Entrepreneurship cell in various colleges where he served as a Professor