

# Modelling & Position Control for Electro-Hydraulic System



Meera C S, Mukul Kumar Gupta

**Abstract:** Electro-hydraulic systems (EHS) are widely used in industrial applications due to the high-power density and accuracy. However, EHS are highly nonlinear which makes its modelling and control aspects a complex process. In this paper, we present the modelling and position control for an electro-hydraulic system (EHS). The mathematical modelling is carried out considering the non-linearities like friction, discharge coefficient and load mass present in the system. A back-stepping control scheme is developed for maintaining the accuracy in the position control. The closed-loop stability of the proposed control system is analyzed with Lyapunov's theory. The performance of the control system under the effect of bounded external uncertainties is validated with simulation study. The study indicates that the proposed controller gives an effective motion control in presence of the system uncertainties.

**Keywords:** Backstepping control, electro-hydraulic system, modelling, position control

## I. INTRODUCTION

Electro-hydraulic systems are an integral part of earth-moving machineries like loaders and excavator backhoes that have become essential in the natural disaster management, rescue operations, agriculture, mining and construction [1]. The electro hydraulic systems consisting of hydraulic cylinders, servo valves and pumps. An autonomous operation with these machineries would require modelling and accurate motion control of the EHS. However, the EHS systems are highly non-linear and modelling them is a complex process [2-5]. The presence of friction, leakages, valve overlapping, and external load makes the development of functions that describe them a complicated task. In order to ensure accurate motion control in presence of the model uncertainties, development of robust non-linear controllers is crucial. Various studies have been performed on the development of nonlinear controllers for position control [6-7], parameter-identification [8-9] and estimation of states [10]. Moreover, studies showing the application of adaptive control and feedback linearization for the control of electro

hydraulic systems are reported in [11-13]. Various studies relating to the development and implementation of disturbance-observer assisted non-linear tracking controllers showed good tracking performance by estimating and compensating the effect of disturbances [14-16]. Some of the previous studies also shows the application of backstepping based control techniques to compensate for unmodeled disturbances like unknown load mass, friction etc. Unknown load disturbance is compensated using a combination of backstepping control and a disturbance observer in [17]. The position control system was developed by Qing et.al with extended-state observer and backstepping controller. In this study, it was shown that even under the uncertain servo-valve dynamics, the error converged to steady state value [18].

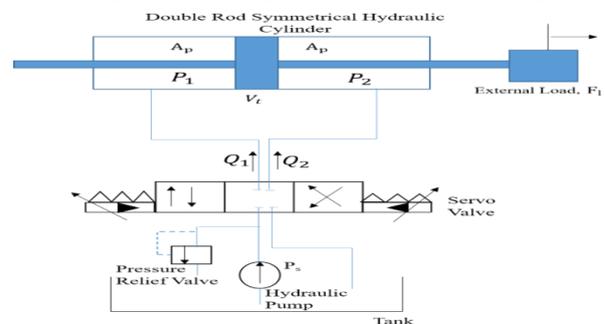
The paper proposes the development of a backstepping controller for the motion control of an EHS. The stability analysis of the proposed motion control approach is verified with the method of Lyapunov direct analysis which parallels similar works presented by [19]. The paper organization is as follows: The system modelling is demonstrated in section 2. The development of the controller and its stability analysis is presented in section 3. In section 4, the simulation results with the proposed controller and its comparison to a conventional PI controller is discussed, followed by conclusion in section 5.

## II. DYNAMIC MODELLING OF EHS.

The EHS is modelled as a fourth order system. A hydraulic actuator shown in figure 1 is symmetric. The servo-valve is modelled as a 1st order linear dynamic equation given by,

$$T_s \dot{X}_v + X_v = K_s u \tag{1}$$

where,  $K_s$  is the servo valve's gain constant and  $T_s$  is the time constant.  $\dot{X}_v$  and  $X_v$  are the spool position and valve velocity respectively.  $u$  represents the control voltage.



**Fig.1. Hydraulic System Model Consisting Of A Symmetric Hydraulic Cylinder, Servo Valve, Pressure Relief Valve And A Hydraulic Pump**

Manuscript published on November 30, 2019.

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$Q_l$  is the load flow-rate in the servo valve is and is given by ,

$$Q_l = C_d w X_v \sqrt{\frac{1}{\rho} (P_s - \text{sgn}(X_v) P_l)} \quad (2)$$

where,  $C_d$ ,  $w$  and  $\rho$  represents the discharge co-efficient, area-gradient of the servo-valve spool and hydraulic fluid density respectively.  $P_1$  denotes the supply-pressure in the hydraulic pump and  $P_2$  is the load-pressure of the hydraulic cylinder given by,

$$P_l = P_1 - P_2 \quad (3)$$

The law of continuity is applied to both chambers of the hydraulic cylinder and the load flow-rate can be given as,

$$\dot{Q}_l = A_p \dot{y} + C_{tl} P_l + \frac{V_t}{4\beta_e} \dot{P}_l \quad (4)$$

In equation 4,  $y$  and  $A_p$  represents the displacement and area of the cylinder respectively,  $V_t$  denotes the complete cylinder volume. The coefficient of leakage is  $C_{tl}$  and  $\beta_e$  denotes the bulk-modulus of the system. The piston dynamics can be given with Newton's second law as,

$$m\ddot{y} = -ky - b\dot{y} + P_l A_p - F_l \quad (5)$$

Here, the mass of the piston is given by  $m$ , the spring constant by  $k$ , the external load acting on the cylinder by  $F_l$ . and the co-efficient of viscous damping by  $b$ . The state variables can be defined as,

$$[x_1, x_2, x_3, x_4]^T = [y, \dot{y}, P_l, X_v]^T \quad (6)$$

The state-space model of the electro- hydraulic system can be represented as in equation 7 as,

$$\begin{aligned} \dot{x}_1 &= x_2 & (7) \\ \dot{x}_2 &= \frac{1}{m} (-kx_1 - bx_2 + A_p x_3 - F_l) \\ \dot{x}_3 &= \frac{4\beta_e A_p}{V_t} x_2 - \frac{4\beta_e C_{tl}}{V_t} x_3 + \frac{4\beta_e C_d w}{V_t \sqrt{\rho}} \sqrt{P_s - \tanh(kx_4)} x_3 x_4 \\ \dot{x}_4 &= -\frac{1}{T_s} x_4 + \frac{K_s}{T_s} u \end{aligned}$$

For the derivation of the proposed backstepping control, the function  $\text{sgn}(\cdot)$  is smoothed and replaced by  $\tanh(\cdot)$  function [20].

$$\tanh(kX_v) = \frac{e^{kX_v} - e^{-kX_v}}{e^{kX_v} + e^{-kX_v}}, k \gg 0 \quad (8)$$

### III. DESIGN OF BACKSTEPPING CONTROL

For the position tracking of the EHS, the tracking error between the desired output and the actual output  $e$ , can be defined as,

$$e_i = x_i - x_{id} \quad (9)$$

where  $x_{id}$  is the desired position and  $x_i$  is the actual position. The Lyapunov function is chosen as,

$$V_1 = \frac{1}{2} e_1^2 \quad (10)$$

The derivative of  $V_1$  w.r.t to time is given by,

$$\begin{aligned} \dot{V}_1 &= e_1 \cdot \dot{e}_1 = e_1 (x_{2d} - \dot{x}_{id}) \\ x_{2d} &= \dot{x}_{id} - k_1 e_1 \\ \dot{V}_1 &= -k_1 e_1^2 \leq 0 \end{aligned} \quad (11)$$

For the next step, the Lyapunov function  $V_2$  is chosen as,

$$V_2 = V_1 + \frac{1}{2} e_2^2. \quad (12)$$

$$\begin{aligned} \dot{V}_2 &= \dot{V}_1 + e_2 \cdot \dot{e}_2 = \dot{V}_1 + (\dot{x}_2 - \dot{x}_{2d}) e_2 \\ &= \dot{V}_1 + \left( \frac{kx_1}{m} - \frac{bx_2}{m} + \frac{A_p x_{3d}}{m} \frac{F_l}{m} - \dot{x}_{2d} \right) e_2 \end{aligned} \quad (13)$$

By considering  $x_{3d}$  as,

$$x_{3d} = \left( \frac{kx_1}{m} - \frac{bx_2}{m} + \frac{F_l}{m} + \dot{x}_{2d} - k_2 e_2 \right) \frac{m}{A_p} \quad (14)$$

And substituting  $x_{3d}$  in equation 13  $\dot{V}_2$  is simplified to,

$$\dot{V}_2 = -k_1 e_1^2 - k_2 e_2^2 \leq 0 \quad (15)$$

By considering  $x_{4d}$  as,

$$x_{4d} = \frac{1}{g_3 \sqrt{P_s - \tanh(kx_4)}} (-g_1 x_2 - g_2 x_3 + \dot{x}_{3d} - k_3 e_3) \quad (16)$$

where,  $g_1 = \frac{4\beta_e A_p}{V_t}$ ,  $g_2 = \frac{4\beta_e C_{tl}}{V_t}$ ,  $g_3 = \frac{4\beta_e C_d w}{V_t \sqrt{\rho}}$ , the Lyapunov function  $V_3$  can be written as,

$$V_3 = V_2 + \frac{1}{2} e_3^2, \quad (17)$$

$$\dot{V}_3 = \dot{V}_2 + e_3 \cdot \dot{e}_3$$

$$= \dot{V}_2 + e_3 (x_{4d} - \dot{x}_{3d}) \quad (18)$$

Substituting  $\dot{V}_2$  (from equation 15) in equation 18,

$$\dot{V}_3 = -k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2 \leq 0 \quad (19)$$

The Lyapunov function  $V_4$  is defined as,

$$V_4 = V_3 + \frac{1}{2} e_4^2. \quad (20)$$

$$\begin{aligned} \dot{V}_4 &= \dot{V}_3 + e_4 \cdot \dot{e}_4 \\ &= \dot{V}_3 + e_4 \cdot (\dot{x}_4 - \dot{x}_{4d}) \end{aligned} \quad (21)$$

The controller  $u$  is chosen as,

$$u = \frac{T_s}{K_s} \left( \frac{x_4}{T_s} + \dot{x}_{4d} - k_4 e_4 \right) \quad (22)$$

$$\dot{V}_4 = -k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2 - k_4 e_4^2 \leq 0 \quad (23)$$

As  $\dot{V}_4$  is negative the stability is proved with Lyapunov's direct method. The stability analysis in the above equations is proved such that the external disturbances are considered to be bounded.

IV. RESULTS AND DISCUSSION

Fig.2. Simulink model of the EHS used in the study.

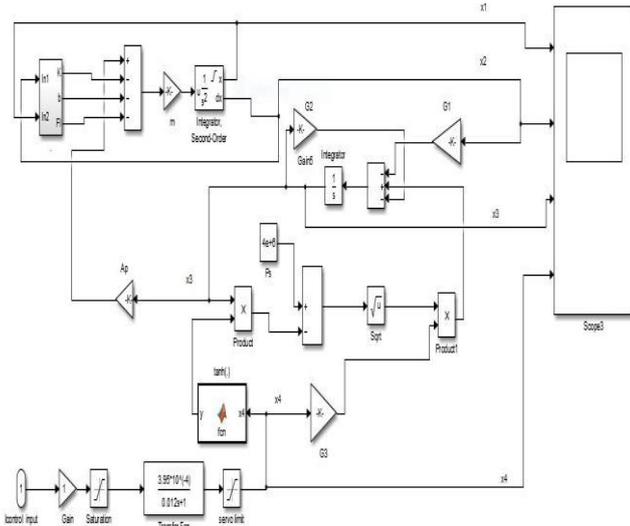


Fig.3.b Performance of the PI controller in tracking the input signal 0.5 Hz.

To verify the performance efficiency of the proposed control system a simulation study carried out in MATLAB/SIMULINK by varying input signal frequency. The input signal was taken as sine wave at 0.5 Hz and 0.1 Hz and the output was observed. The system modelling presented is similar to other studies as [21]. The Simulink model developed for EHS is shown in figure 2. In addition to this, the proposed controller performance was compared to a conventional PI controller for both the frequencies of input sine wave.

The PI controller was given as,

$$u = K_p e + K_i \int e dt \quad (24)$$

Figure 3a and 3b shows the tracking performance of the proposed controller and the PI controller for input frequency at 0.5Hz respectively. The gain values  $K_p$  and  $K_i$  for PI controller are chosen as 100 and 15 respectively.

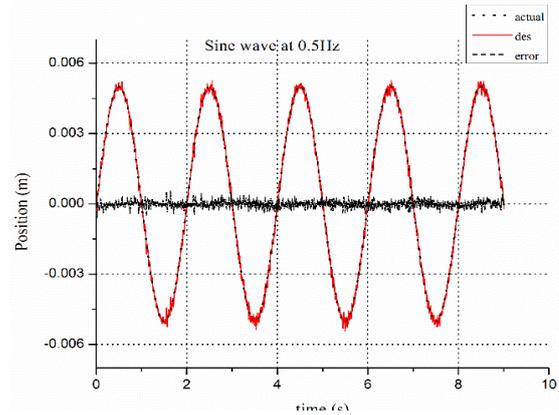
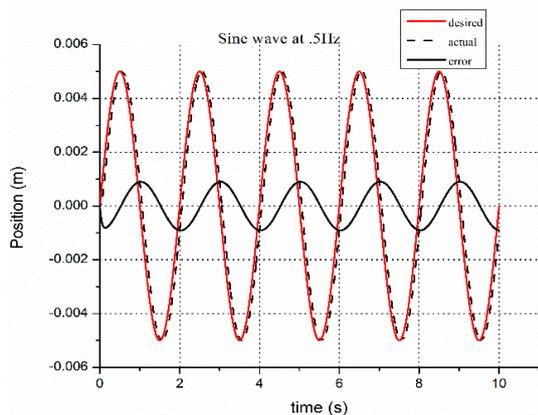


Fig.3.a Performance of the proposed controller in tracking the input signal 0.5 Hz. The root mean square error (RMSE) for the proposed controller is obtained as 0.00012, whereas the RMSE of the PI controller was 0.00121 for the input signal tracking. The error signal is shown in black line for both the controllers. The system non-linearities like friction is considered for motion control study. From the result it can be seen that the error for proposed controller is almost zero, whereas error with PI controller is higher. The proposed controller shows a superior performance than the PI controller.

Figure 4a and 4b shows the performance of the proposed controller and the PI controller in tracking a reference signal at 1 Hz.

The RMSE for the proposed backstepping controller is 0.0024 and the conventional PI controller is found to be 0.2313.

From the results it can be seen that the error is more for the PI controller at 1 Hz. The steady-state error does not converge and shows an oscillatory nature for the PI controller. The steady-state error of the developed controller converges to zero, which asserts the superiority of the proposed motion control system.

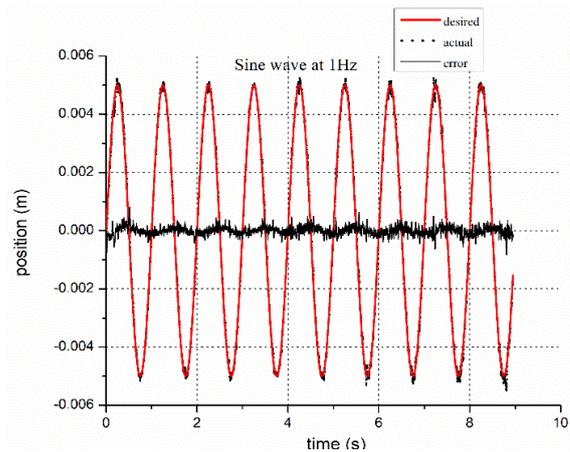
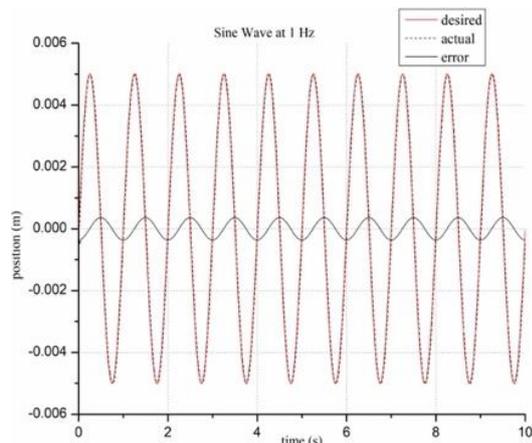


Fig. 4.a. Performance of the proposed controller in tracking an input reference signal of 1 Hz



**Fig.4.b Performance of the PI controller in tracking an input reference signal of 1 Hz.**

The parameters of the system used for the study are given in table 1.

**Table 1: System parameters used in the study**

Actuator Load	50 kg	Discharge coefficient, Cd	0.62
Supply Pressure, Ps	$4 \times 10^6$ Pa	Area gradient servo valve, w	0.024m
Actuator area, Ap	$2.04 \times 10^{-4}$ m <sup>2</sup>	Bulk modulus, $\beta_e$	$2.2 \times 10^8$ Pa
Spring Constant, K	1000 N/m	Servo gain, Ks	$3.95 \times 10^4$ m/V
Total leakage coefficient, Ctl	$2.5 \times 10^{11}$ m <sup>3</sup> /(sPa)	Time constant, Ts	0.012
k1	10	k3	500
k2	200	k4	10

## V. CONCLUSIONS

Electro hydraulic systems with its high-power density offers great benefits in earthmoving and construction industry. An autonomous operation with EHS would require an accurate motion control. This paper presents modelling and nonlinear motion control of an EHS using a state-feedback backstepping controller. EHS is modelled as a fourth order system and the simulation study was conducted to verify the control system performance. The tracking control of the proposed control system was analyzed in presence of non-linearities like friction and the performance was compared against a conventional PI controller. The results showed the proposed controller track the reference input signal accurately and the state system error quickly converges to steady-state value.

## ACKNOWLEDGEMENTS

The authors would like to thank Dept of Electrical and Electronics Engineering and Bosch Centre of Excellence, UPES and for the support and encouragement.

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