



Construction of Fuzzy Mean using Standard Deviation ($\tilde{\bar{X}} - \tilde{s}$) Control Chart with Process Capability

R. Dilipkumar, C. Nanthakumar

Abstract: The Quality has established over a number of points such as inspection, quality control, quality assurance, and total quality control and the effects produced by the above phases are used to check and develop the production/service procedure. Statistical process control (SPC) is a powerful collection of problem solving tools valuable in attaining process steadiness and enlightening capability through the decline of variability. Fuzzy set theory is a utilitarian tool to succeed the uncertainty environmental circumstances and the Fuzzy control limits provide a more accurate and flexible rating than the traditional control charts. The purpose of this research article is to construct the fuzzy mean using standard deviation ($\tilde{\bar{X}} - \tilde{s}$) control chart with the assistance of process capability.

Keywords: Fuzzy control charts, Fuzzy midrange, Statistical Process Control and Trapezoidal fuzzy numbers

I. INTRODUCTION

A fuzzy set is a class of objects with a continuum of grades of membership. Such a set is characterized by a membership (characteristic) function which assigns to each object a grade of membership ranging between zero and one (Zadeh, 1965). The notions of inclusion, union, intersection, complement, relation, convexity, etc., are extended to such sets, and various properties of these notions in the context of fuzzy sets are established. In particular, a separation theorem for convex fuzzy sets is proved without requiring that the fuzzy sets be disjoint. The major contribution of fuzzy set theory is its capability of representing vague data. Fuzzy logic offers a systematic base in dealing with situations, which are ambiguous, or not well defined (Murat Gulbay and Cengiz Kahraman, 2006).

The measures of central tendency in descriptive Statistics are used in variable control charts. These measures can be used to convert fuzzy sets into scalars which are α -level fuzzy midrange, and fuzzy average. There is no theoretical basis to select the appropriate fuzzy measures among these four (Pandurangan and Varadharajan, 2011).

As mentioned, according to the real-world data and information are vague and imprecise expression, this research article with development of Fuzzy control charts, trying to overcome the problem of inaccurate and ambiguous information. Thus, the main objective of this study was to calculate the fuzzy mean using standard deviation ($\tilde{\bar{X}} - \tilde{s}$) control charts with the help of process capability.

II. METHODS AND MATERIALS

Control of the process average or mean quality level is usually done with the control chart for means, or the control chart. Process variability can be monitored with either a control chart for the standard deviation, called the 'S' control chart, or a control chart for the range, called an 'R' control chart. The mean using standard deviation ($\tilde{\bar{X}} - \tilde{s}$) fuzzy control chart is among the most important and useful on-line statistical process monitoring and control techniques.

The Shewhart (1924) control limits for mean based on sample standard deviation ($\bar{X} - s$) are given below:

$$UCL_{\bar{X}-s} = \bar{\bar{X}} + \left[\frac{3}{\sqrt{n}} \frac{\bar{S}}{c_4} \right]$$

$$CL_{\bar{X}-s} = \bar{\bar{X}}$$

$$LCL_{\bar{X}-s} = \bar{\bar{X}} - \left[\frac{3}{\sqrt{n}} \frac{\bar{S}}{c_4} \right]$$

where c_4 is a control chart co-efficient, \bar{S} is the average of S_i that are the standard deviation of samples and n is the size of sample.

$$S_j = \sqrt{\frac{\sum_{i=1}^n (X_{ij} - \bar{X}_j)^2}{n-1}} \text{ and}$$

$$\bar{S} = \frac{\sum_{j=1}^m S_j}{m}, i=1,2,\dots,n; j=1,2,\dots,m$$

The theoretical structure of fuzzy $\tilde{\bar{X}} - \tilde{s}$ control chart and fuzzy $\tilde{\bar{X}} - \tilde{s}$ control chart has been developed by Senturk and Erginel (2009). The fuzzy \tilde{S}_j is the standard deviation of sample j and is calculated as follows

$$S_j = \sqrt{\frac{\sum_{i=1}^n [(X_a, X_b, X_c, X_d)_{ij} - (\bar{X}_a, \bar{X}_b, \bar{X}_c, \bar{X}_d)_j]^2}{n-1}}$$

and the fuzzy average $\tilde{\bar{X}}$ is calculated by using standard deviation represented by the following Trapezoidal fuzzy number

Manuscript published on November 30, 2019.

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The fuzzy $\tilde{X} - S$ control limits using α -cut method for trapezoidal numbers as follows:

$$\tilde{S} = \left\{ \frac{\sum_{i=1}^m S_{aj}}{m}, \frac{\sum_{i=1}^m S_{bj}}{m}, \frac{\sum_{i=1}^m S_{cj}}{m}, \frac{\sum_{i=1}^m S_{dj}}{m} \right\} = (\bar{S}_a, \bar{S}_b, \bar{S}_c, \bar{S}_d)$$

The fuzzy control limits for ($\tilde{X} - S$) are given below:

$$\begin{aligned} U\tilde{C}L_{\tilde{X}-S} &= (\bar{X}_a, \bar{X}_b, \bar{X}_c, \bar{X}_d) + \left[\frac{3}{c_4\sqrt{n}} (\bar{S}_a, \bar{S}_b, \bar{S}_c, \bar{S}_d) \right] \\ &= \left(\bar{X}_a + \frac{3}{c_4\sqrt{n}} \bar{S}_a, \bar{X}_b + \frac{3}{c_4\sqrt{n}} \bar{S}_b, \right. \\ &\quad \left. \bar{X}_c + \frac{3}{c_4\sqrt{n}} \bar{S}_c, \bar{X}_d + \frac{3}{c_4\sqrt{n}} \bar{S}_d \right) \\ &= (U\tilde{C}L_{a.\tilde{X}-S}, U\tilde{C}L_{b.\tilde{X}-S}, U\tilde{C}L_{c.\tilde{X}-S}, U\tilde{C}L_{d.\tilde{X}-S}) \\ \tilde{C}L_{\tilde{X}-S} &= (\bar{X}_{a.\tilde{X}-S}, \bar{X}_{b.\tilde{X}-S}, \bar{X}_{c.\tilde{X}-S}, \bar{X}_{d.\tilde{X}-S}) \\ L\tilde{C}L_{\tilde{X}-S} &= (\bar{X}_a, \bar{X}_b, \bar{X}_c, \bar{X}_d) - \left[\frac{3}{c_4\sqrt{n}} (\bar{S}_a, \bar{S}_b, \bar{S}_c, \bar{S}_d) \right] \\ &= \left(\bar{X}_a - \frac{3}{c_4\sqrt{n}} \bar{S}_a, \bar{X}_b - \frac{3}{c_4\sqrt{n}} \bar{S}_b, \right. \\ &\quad \left. \bar{X}_c - \frac{3}{c_4\sqrt{n}} \bar{S}_c, \bar{X}_d - \frac{3}{c_4\sqrt{n}} \bar{S}_d \right) \\ &= (L\tilde{C}L_{a.\tilde{X}-S}, L\tilde{C}L_{b.\tilde{X}-S}, L\tilde{C}L_{c.\tilde{X}-S}, L\tilde{C}L_{d.\tilde{X}-S}) \end{aligned}$$

The proposed standard deviation

($\tilde{\sigma}_{r.SFC_p}$, $r = a, b, c, d$) for fuzzy $\tilde{X} - S$ control chart with the help of process capability $C_p = \frac{USL_{r.SFC_p} - LSL_{r.SFC_p}}{6\sigma}$, $r = a, b, c, d$ is to calculate by the specified tolerance level from the relation $\frac{\sum_{j=1}^m S_{rj}}{m} / c_4$, $r = a, b, c, d$ and $j = 1, 2, \dots, m$

Therefore the resultant of proposed fuzzy control limits for $\tilde{X} - S$ using process capability is given below:

$$\begin{aligned} U\tilde{C}L_{\tilde{X}-S:C_p} &= \left(\bar{X}_a + \frac{3}{\sqrt{n}} \tilde{\sigma}_{a.SFC_p}, \bar{X}_b + \frac{3}{\sqrt{n}} \tilde{\sigma}_{b.SFC_p}, \right. \\ &\quad \left. \bar{X}_c + \frac{3}{\sqrt{n}} \tilde{\sigma}_{c.SFC_p}, \bar{X}_d + \frac{3}{\sqrt{n}} \tilde{\sigma}_{d.SFC_p} \right) \\ &= \left(U\tilde{C}L_{a.\tilde{X}-S:C_p}, U\tilde{C}L_{b.\tilde{X}-S:C_p}, \right. \\ &\quad \left. U\tilde{C}L_{c.\tilde{X}-S:C_p}, U\tilde{C}L_{d.\tilde{X}-S:C_p} \right) \\ \tilde{C}L_{\tilde{X}-S} &= (\bar{X}_{a.\tilde{X}-S}, \bar{X}_{b.\tilde{X}-S}, \bar{X}_{c.\tilde{X}-S}, \bar{X}_{d.\tilde{X}-S}) \\ L\tilde{C}L_{\tilde{X}-S:C_p} &= \left(\bar{X}_a - \frac{3}{\sqrt{n}} \tilde{\sigma}_{a.SFC_p}, \bar{X}_b - \frac{3}{\sqrt{n}} \tilde{\sigma}_{b.SFC_p}, \right. \\ &\quad \left. \bar{X}_c - \frac{3}{\sqrt{n}} \tilde{\sigma}_{c.SFC_p}, \bar{X}_d - \frac{3}{\sqrt{n}} \tilde{\sigma}_{d.SFC_p} \right) \\ &= \left(L\tilde{C}L_{a.\tilde{X}-S:C_p}, L\tilde{C}L_{b.\tilde{X}-S:C_p}, \right. \\ &\quad \left. L\tilde{C}L_{c.\tilde{X}-S:C_p}, L\tilde{C}L_{d.\tilde{X}-S:C_p} \right) \end{aligned}$$

$$U\tilde{C}L_{\tilde{X}-S}^\alpha = (\bar{X}_a^\alpha, \bar{X}_b^\alpha, \bar{X}_c^\alpha, \bar{X}_d^\alpha)$$

$$+ \left[\frac{3}{c_4\sqrt{n}} (\bar{S}_a^\alpha, \bar{S}_b^\alpha, \bar{S}_c^\alpha, \bar{S}_d^\alpha) \right]$$

$$= \left(\bar{X}_a^\alpha + \frac{3}{c_4\sqrt{n}} \bar{S}_a^\alpha, \bar{X}_b^\alpha + \frac{3}{c_4\sqrt{n}} \bar{S}_b^\alpha, \right. \\ \left. \bar{X}_c^\alpha + \frac{3}{c_4\sqrt{n}} \bar{S}_c^\alpha, \bar{X}_d^\alpha + \frac{3}{c_4\sqrt{n}} \bar{S}_d^\alpha \right)$$

$$= \left(U\tilde{C}L_{a.\tilde{X}-S}^\alpha, U\tilde{C}L_{b.\tilde{X}-S}^\alpha, \right. \\ \left. U\tilde{C}L_{c.\tilde{X}-S}^\alpha, U\tilde{C}L_{d.\tilde{X}-S}^\alpha \right)$$

$$\tilde{C}L_{\tilde{X}-S}^\alpha = (\bar{X}_{a.\tilde{X}-S}^\alpha, \bar{X}_{b.\tilde{X}-S}^\alpha, \bar{X}_{c.\tilde{X}-S}^\alpha, \bar{X}_{d.\tilde{X}-S}^\alpha)$$

$$L\tilde{C}L_{\tilde{X}-S}^\alpha = (\bar{X}_a^\alpha, \bar{X}_b^\alpha, \bar{X}_c^\alpha, \bar{X}_d^\alpha) - \left[\frac{3}{c_4\sqrt{n}} (\bar{S}_a^\alpha, \bar{S}_b^\alpha, \bar{S}_c^\alpha, \bar{S}_d^\alpha) \right]$$

$$= \left(\bar{X}_a^\alpha - \frac{3}{c_4\sqrt{n}} \bar{S}_a^\alpha, \bar{X}_b^\alpha - \frac{3}{c_4\sqrt{n}} \bar{S}_b^\alpha, \right. \\ \left. \bar{X}_c^\alpha - \frac{3}{c_4\sqrt{n}} \bar{S}_c^\alpha, \bar{X}_d^\alpha - \frac{3}{c_4\sqrt{n}} \bar{S}_d^\alpha \right)$$

$$= \left(L\tilde{C}L_{a.\tilde{X}-S}^\alpha, L\tilde{C}L_{b.\tilde{X}-S}^\alpha, \right. \\ \left. L\tilde{C}L_{c.\tilde{X}-S}^\alpha, L\tilde{C}L_{d.\tilde{X}-S}^\alpha \right)$$

where

$$\bar{S}_a^\alpha = \bar{S}_a + \alpha(\bar{S}_b - \bar{S}_a) \text{ and } \bar{S}_d^\alpha = \bar{S}_d + \alpha(\bar{S}_d - \bar{S}_c)$$

The proposed standard deviation

($\tilde{\sigma}_{a.SFC_p}^\alpha$ and $\tilde{\sigma}_{d.SFC_p}^\alpha$) for fuzzy $\tilde{X} - S$ control chart with the help of process capability, Radhakrishnan and Balamurugan (2010) i.e.

$$C_p = \frac{USL_{r.SFC_p}^\alpha - LSL_{r.SFC_p}^\alpha}{6\sigma}, r = a \text{ and } d \text{ using } \alpha\text{-cut method}$$

is to calculate by the specified tolerance level from the relation

$$\frac{\sum_{j=1}^m S_{aj}}{m} + \alpha \left(\frac{\sum_{j=1}^m S_{bj}}{m} - \frac{\sum_{j=1}^m S_{aj}}{m} \right) / c_4 \text{ for } \tilde{\sigma}_{a.SFC_p}^\alpha \text{ and}$$

$$\frac{\sum_{j=1}^m S_{dj}}{m} + \alpha \left(\frac{\sum_{j=1}^m S_{dj}}{m} - \frac{\sum_{j=1}^m S_{cj}}{m} \right) / c_4 \text{ for } \tilde{\sigma}_{d.SFC_p}^\alpha, j = 1, 2, \dots, m.$$

The proposed fuzzy $\tilde{X} - S$ control limits with process capability using α -cut method for trapezoidal numbers as follows:

$$\begin{aligned}
 U\tilde{C}L_{\tilde{X}-S:C_p}^\alpha &= \left(\begin{aligned} &\bar{X}_a^\alpha + \frac{3}{\sqrt{n}} \tilde{\sigma}_{a.SFC_p}^\alpha, \bar{X}_b + \frac{3}{\sqrt{n}} \tilde{\sigma}_{b.SFC_p}^\alpha, \\ &\bar{X}_c + \frac{3}{\sqrt{n}} \tilde{\sigma}_{c.SFC_p}^\alpha, \bar{X}_d^\alpha + \frac{3}{\sqrt{n}} \tilde{\sigma}_{d.SFC_p}^\alpha \end{aligned} \right) \\
 &= \left(\begin{aligned} &U\tilde{C}L_{a.\tilde{X}-S:C_p}^\alpha, U\tilde{C}L_{b.\tilde{X}-S:C_p}^\alpha, \\ &U\tilde{C}L_{c.\tilde{X}-S:C_p}^\alpha, U\tilde{C}L_{d.\tilde{X}-S:C_p}^\alpha \end{aligned} \right) \\
 C\tilde{L}_{\tilde{X}-S}^\alpha &= \left(\bar{X}_{a.\tilde{X}-S}^\alpha, \bar{X}_{b.\tilde{X}-S}^\alpha, \bar{X}_{c.\tilde{X}-S}^\alpha, \bar{X}_{d.\tilde{X}-S}^\alpha \right) \\
 L\tilde{C}L_{\tilde{X}-S:C_p}^\alpha &= \left(\begin{aligned} &\bar{X}_a^\alpha - \frac{3}{\sqrt{n}} \tilde{\sigma}_{a.SFC_p}^\alpha, \bar{X}_b - \frac{3}{\sqrt{n}} \tilde{\sigma}_{b.SFC_p}^\alpha, \\ &\bar{X}_c - \frac{3}{\sqrt{n}} \tilde{\sigma}_{c.SFC_p}^\alpha, \bar{X}_d^\alpha - \frac{3}{\sqrt{n}} \tilde{\sigma}_{d.SFC_p}^\alpha \end{aligned} \right) \\
 &= \left(\begin{aligned} &L\tilde{C}L_{a.\tilde{X}-S:C_p}^\alpha, L\tilde{C}L_{b.\tilde{X}-S:C_p}^\alpha, \\ &L\tilde{C}L_{c.\tilde{X}-S:C_p}^\alpha, L\tilde{C}L_{d.\tilde{X}-S:C_p}^\alpha \end{aligned} \right)
 \end{aligned}$$

$$\left\{ \frac{\sum_{j=1}^m S_{aj}}{m} + \alpha \left(\frac{\sum_{j=1}^m S_{bj}}{m} - \frac{\sum_{j=1}^m S_{aj}}{m} \right) \right\} + \left\{ \frac{\sum_{j=1}^m S_{dj}}{m} + \alpha \left(\frac{\sum_{j=1}^m S_{dj}}{m} - \frac{\sum_{j=1}^m S_{cj}}{m} \right) \right\}$$

for $\tilde{\sigma}_{Mid.SFC_p}^\alpha, j = 1, 2, \dots, m.$

The proposed control limits using process capability of α -level fuzzy midrange for α -cut fuzzy $\tilde{X} - S$ control chart can be obtained as follows:

$$\begin{aligned}
 U\tilde{C}L_{\tilde{X}-S.mid:C_p}^\alpha &= \left(\frac{\bar{X}_a^\alpha + \bar{X}_d^\alpha}{2} \right) + \left[\frac{3}{\sqrt{n}} \tilde{\sigma}_{Mid.SFC_p}^\alpha \right] \\
 C\tilde{L}_{\tilde{X}-S.mid:C_p}^\alpha &= \left(\frac{\bar{X}_a^\alpha + \bar{X}_d^\alpha}{2} \right) \\
 L\tilde{C}L_{\tilde{X}-S.mid:C_p}^\alpha &= \left(\frac{\bar{X}_a^\alpha + \bar{X}_d^\alpha}{2} \right) - \left[\frac{3}{\sqrt{n}} \tilde{\sigma}_{Mid.SFC_p}^\alpha \right]
 \end{aligned}$$

Then, the condition of process control for each sample can be defined as:

$$\text{Process control} = \begin{cases} \text{in control} & ; \\ L\tilde{C}L_{\tilde{X}-S.mid:C_p}^\alpha \leq S_{j:\tilde{X}-S.mid}^\alpha \leq U\tilde{C}L_{\tilde{X}-S.mid:C_p}^\alpha & ; \\ \text{Out-of-control} & ; \text{ Otherwise} \end{cases}$$

III. PERFORMANCE OF THE FUZZY VARIABLE CONTROL CHARTS USING PROCESS CAPABILITY

Consider a process by which coils are manufactured by a company at Ammapet, Salem District. The primary data collected and presented in Table – 1 have been used the samples of size 5 are randomly selected from the process, and the ‘between’ measurements resistance values (in ohms) of the coils are measured for the application. These measurements are then converted into trapezoidal fuzzy numbers (TFN) using computer program and are given in Table – 2.

The control limits of α -level fuzzy midrange for α -cut fuzzy $\tilde{X} - S$ control chart can be obtained as follows:

$$\begin{aligned}
 U\tilde{C}L_{\tilde{X}-S.mid}^\alpha &= \left(\frac{\bar{X}_a^\alpha + \bar{X}_d^\alpha}{2} \right) + \left[\frac{3}{c_4 \sqrt{n}} \left(\frac{\bar{S}_a^\alpha + \bar{S}_d^\alpha}{2} \right) \right] \\
 C\tilde{L}_{\tilde{X}-S.mid}^\alpha &= \left(\frac{\bar{X}_a^\alpha + \bar{X}_d^\alpha}{2} \right) \\
 L\tilde{C}L_{\tilde{X}-S.mid}^\alpha &= \left(\frac{\bar{X}_a^\alpha + \bar{X}_d^\alpha}{2} \right) - \left[\frac{3}{c_4 \sqrt{n}} \left(\frac{\bar{S}_a^\alpha + \bar{S}_d^\alpha}{2} \right) \right]
 \end{aligned}$$

The definition of α -level fuzzy midrange of sample j for fuzzy $\tilde{X} - S$ control chart is

$$S_{j:\tilde{X}-S.mid}^\alpha = \frac{(\bar{X}_{aj} + \bar{X}_{dj}) + \alpha [(\bar{X}_{bj} - \bar{X}_{aj}) - (\bar{X}_{dj} - \bar{X}_{cj})]}{2}$$

Then, the condition of process control for each sample can be defined as:

$$\text{Process control} = \begin{cases} \text{in control} & ; \\ L\tilde{C}L_{\tilde{X}-S.mid}^\alpha \leq S_{j:\tilde{X}-S.mid}^\alpha \leq U\tilde{C}L_{\tilde{X}-S.mid}^\alpha & ; \\ \text{Out-of-control} & ; \text{ Otherwise} \end{cases}$$

The proposed standard deviation ($\tilde{\sigma}_{Mid.SFC_p}^\alpha$) for α -level fuzzy midrange at $\tilde{X} - S$ control chart with the help of process capability $C_p = \frac{USL_{Mid.SFC_p}^\alpha - LSL_{Mid.SFC_p}^\alpha}{6\sigma}$ using α -cut method is to calculate by the specified tolerance level from the relation

Table 1: Resistance values (in ohms) of coils

Sample No.	X ₁		X ₂		X ₃		X ₄		X ₅	
1	0.52	0.55	0.49	0.51	0.56	0.57	0.49	0.50	0.52	0.53
2	0.52	0.53	0.51	0.52	0.52	0.54	0.51	0.53	0.45	0.47
3	0.51	0.52	0.53	0.54	0.54	0.55	0.49	0.55	0.50	0.51
4	0.42	0.45	0.42	0.44	0.46	0.56	0.51	0.54	0.53	0.54
5	0.48	0.50	0.47	0.51	0.50	0.50	0.57	0.58	0.52	0.53
6	0.55	0.56	0.49	0.50	0.50	0.52	0.48	0.49	0.50	0.51
7	0.49	0.53	0.52	0.54	0.49	0.55	0.46	0.48	0.49	0.50
8	0.43	0.46	0.49	0.52	0.49	0.51	0.50	0.53	0.50	0.51

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9	0.54	0.55	0.48	0.53	0.51	0.53	0.47	0.49	0.48	0.50
10	0.50	0.52	0.49	0.51	0.48	0.49	0.48	0.52	0.46	0.47
11	0.47	0.50	0.55	0.57	0.50	0.52	0.49	0.50	0.48	0.50
12	0.50	0.54	0.56	0.57	0.47	0.51	0.48	0.51	0.48	0.50
13	0.46	0.48	0.52	0.54	0.50	0.51	0.53	0.54	0.50	0.51
14	0.50	0.51	0.48	0.51	0.48	0.52	0.51	0.53	0.44	0.45
15	0.49	0.51	0.50	0.51	0.54	0.55	0.48	0.52	0.49	0.50

Table 2: Trapezoidal Fuzzy measurement resistance values of coils

Sample No.	X_1				X_2				X_3			
1	0.45	0.52	0.55	0.58	0.48	0.49	0.51	0.53	0.55	0.56	0.57	0.58
2	0.50	0.52	0.53	0.55	0.45	0.51	0.52	0.55	0.50	0.52	0.54	0.56
3	0.47	0.51	0.52	0.56	0.52	0.53	0.54	0.58	0.53	0.54	0.55	0.56
4	0.40	0.42	0.45	0.48	0.41	0.42	0.44	0.49	0.41	0.46	0.56	0.57
5	0.45	0.48	0.50	0.53	0.45	0.47	0.51	0.54	0.46	0.50	0.50	0.52
6	0.52	0.55	0.56	0.57	0.47	0.49	0.50	0.53	0.47	0.50	0.52	0.53
7	0.46	0.49	0.53	0.56	0.51	0.52	0.54	0.55	0.45	0.49	0.55	0.56
8	0.40	0.43	0.46	0.47	0.44	0.49	0.52	0.55	0.46	0.49	0.51	0.54
9	0.53	0.54	0.55	0.57	0.41	0.48	0.53	0.56	0.46	0.51	0.53	0.55
10	0.49	0.50	0.52	0.54	0.44	0.49	0.51	0.54	0.45	0.48	0.49	0.52
11	0.41	0.47	0.50	0.53	0.54	0.55	0.57	0.58	0.46	0.50	0.52	0.55
12	0.48	0.50	0.54	0.56	0.54	0.56	0.57	0.59	0.42	0.47	0.51	0.53
13	0.44	0.46	0.48	0.50	0.51	0.52	0.54	0.55	0.48	0.50	0.51	0.56
14	0.47	0.50	0.51	0.53	0.45	0.48	0.51	0.57	0.46	0.48	0.52	0.58
15	0.46	0.49	0.51	0.55	0.48	0.50	0.51	0.56	0.53	0.54	0.55	0.56

Sample No.	X_4				X_5			
1	0.47	0.49	0.50	0.52	0.49	0.52	0.53	0.54
2	0.49	0.51	0.53	0.55	0.42	0.45	0.47	0.49
3	0.48	0.49	0.55	0.58	0.48	0.50	0.51	0.52
4	0.46	0.51	0.54	0.57	0.50	0.53	0.54	0.55
5	0.56	0.57	0.58	0.59	0.49	0.52	0.53	0.54
6	0.46	0.48	0.49	0.50	0.48	0.50	0.51	0.52
7	0.44	0.46	0.48	0.51	0.47	0.49	0.50	0.55
8	0.48	0.50	0.53	0.55	0.49	0.50	0.51	0.53
9	0.46	0.47	0.49	0.54	0.45	0.48	0.50	0.52
10	0.45	0.48	0.52	0.56	0.42	0.46	0.47	0.48
11	0.41	0.49	0.50	0.54	0.45	0.48	0.50	0.52
12	0.45	0.48	0.51	0.55	0.47	0.48	0.50	0.52
13	0.48	0.53	0.54	0.55	0.48	0.50	0.51	0.53
14	0.50	0.51	0.53	0.54	0.42	0.44	0.45	0.48
15	0.46	0.48	0.52	0.55	0.41	0.49	0.50	0.51

2.2 Construction of fuzzy control chart for mean using standard deviation ($\tilde{X} - S$) with process capability

The fuzzy standard deviation is calculated as follows:

$$S_{a1} = \sqrt{\frac{\sum_{i=1}^n [(X_a, X_b, X_c, X_d)_{i1} - (\bar{X}_a, \bar{X}_b, \bar{X}_c, \bar{X}_d)_1]}{m-1}}$$

= 0.038

$$S_{a2} = \sqrt{\frac{\sum_{i=1}^n [(X_a, X_b, X_c, X_d)_{i2} - (\bar{X}_a, \bar{X}_b, \bar{X}_c, \bar{X}_d)_1]}{m-1}}$$

= 0.036

and so on...

The fuzzy average \tilde{S} is calculated by using standard deviation represented by the following Trapezoidal fuzzy number

$$\bar{S}_a = \frac{S_{a1} + S_{a2} + \dots + S_{a15}}{m} = \frac{0.038 + 0.036 + \dots + 0.043}{15}$$

= 0.0360

$$\bar{S}_b = \frac{S_{b1} + S_{b2} + \dots + S_{b15}}{m} = \frac{0.029 + 0.029 + \dots + 0.023}{15}$$

= 0.0290

$$\bar{S}_c = \frac{S_{c1} + S_{c2} + \dots + S_{c15}}{m} = \frac{0.029 + 0.028 + \dots + 0.019}{15}$$

= 0.0286

$$\bar{S}_d = \frac{S_{d1} + S_{d2} + \dots + S_{d15}}{m} = \frac{0.028 + 0.028 + \dots + 0.021}{15}$$

= 0.0277

$$\begin{aligned} \tilde{S} &= (\bar{S}_a, \bar{S}_b, \bar{S}_c, \bar{S}_d) \\ &= (0.0360, 0.0290, 0.0286, 0.0277) \end{aligned}$$

The fuzzy control limits for $(\tilde{X} - S)$ are given below:

$$\begin{aligned} U\tilde{C}L_{\tilde{X}-S} &= (0.4679, 0.4961, 0.5173, 0.5416) \\ &+ \left[\frac{3}{0.94\sqrt{5}} (0.0360, 0.0290, 0.0286, 0.0277) \right] \\ &= \left(0.4679 + \frac{3}{0.94\sqrt{5}} \cdot 0.0360, 0.4961 + \frac{3}{0.94\sqrt{5}} \cdot 0.0290, \right. \\ &\quad \left. 0.5173 + \frac{3}{0.94\sqrt{5}} \cdot 0.0286, 0.5416 + \frac{3}{0.94\sqrt{5}} \cdot 0.0277 \right) \\ &= (0.5192, 0.5376, 0.5581, 0.5811) \\ C\tilde{L}_{\tilde{X}-S} &= (0.4679, 0.4961, 0.5173, 0.5416) \\ L\tilde{C}L_{\tilde{X}-S} &= (0.4679, 0.4961, 0.5173, 0.5416) \\ &- \left[\frac{3}{0.94\sqrt{5}} (0.0360, 0.0290, 0.0286, 0.0277) \right] \\ &= \left(0.4679 - \frac{3}{0.94\sqrt{5}} \cdot 0.0360, 0.4961 - \frac{3}{0.94\sqrt{5}} \cdot 0.0290, \right. \\ &\quad \left. 0.5173 - \frac{3}{0.94\sqrt{5}} \cdot 0.0286, 0.5416 - \frac{3}{0.94\sqrt{5}} \cdot 0.0277 \right) \\ &= (0.4165, 0.4547, 0.4765, 0.5021) \end{aligned}$$

The proposed standard deviation

$$\begin{aligned} USL_{a.SFC_p} - LSL_{a.SFC_p} &= 0.05659 - 0.02495 \\ \Rightarrow \tilde{\sigma}_{a.SFC_p} &= 0.00264 \\ USL_{b.SFC_p} - LSL_{b.SFC_p} &= 0.05393 - 0.01578 \\ \Rightarrow \tilde{\sigma}_{b.SFC_p} &= 0.00318 \\ USL_{c.SFC_p} - LSL_{c.SFC_p} &= 0.05999 - 0.01933 \\ \Rightarrow \tilde{\sigma}_{c.SFC_p} &= 0.00339 \\ USL_{d.SFC_p} - LSL_{d.SFC_p} &= 0.04612 - 0.02025 \\ \Rightarrow \tilde{\sigma}_{d.SFC_p} &= 0.00216 \end{aligned}$$

Therefore the resultant of proposed fuzzy control limits for $\tilde{X} - S$ using process capability is given below:

$$\begin{aligned} U\tilde{C}L_{\tilde{X}-S:C_p} &= \left(0.4679 + \frac{3}{\sqrt{5}} \cdot 0.00264, 0.4961 + \frac{3}{\sqrt{5}} \cdot 0.00318, \right. \\ &\quad \left. 0.5173 + \frac{3}{\sqrt{5}} \cdot 0.00339, 0.5416 + \frac{3}{\sqrt{5}} \cdot 0.00216 \right) \\ &= (0.4714, 0.5004, 0.5219, 0.5445) \\ C\tilde{L}_{\tilde{X}-S} &= (0.4679, 0.4961, 0.5173, 0.5416) \\ L\tilde{C}L_{\tilde{X}-S:C_p} &= \left(0.4679 - \frac{3}{\sqrt{5}} \cdot 0.00264, 0.4961 - \frac{3}{\sqrt{5}} \cdot 0.00318, \right. \\ &\quad \left. 0.5173 - \frac{3}{\sqrt{5}} \cdot 0.00339, 0.5416 - \frac{3}{\sqrt{5}} \cdot 0.00216 \right) \\ &= (0.4643, 0.4919, 0.5128, 0.5387) \end{aligned}$$

The fuzzy $\tilde{X} - S$ control limits using α -cut method for trapezoidal numbers as follows:

$$\bar{S}_a^\alpha = 0.00264 + 0.65(0.00318 - 0.00264) = 0.00299$$

and

$$\bar{S}_d^\alpha = 0.00216 + 0.65(0.00216 - 0.00339) = 0.00136$$

$$\begin{aligned} U\tilde{C}L_{\tilde{X}-S}^\alpha &= \left(\bar{X}_a^\alpha, \bar{X}_b^\alpha, \bar{X}_c^\alpha, \bar{X}_d^\alpha \right) + \left[\frac{3}{c_4\sqrt{n}} (\bar{S}_a^\alpha, \bar{S}_b^\alpha, \bar{S}_c^\alpha, \bar{S}_d^\alpha) \right] \\ &= \left(0.4862 + \frac{3}{0.94\sqrt{5}} \cdot 0.00299, 0.4961 + \frac{3}{0.94\sqrt{5}} \cdot 0.00318, \right. \\ &\quad \left. 0.5173 + \frac{3}{0.94\sqrt{5}} \cdot 0.00339, 0.5258 + \frac{3}{0.94\sqrt{5}} \cdot 0.00136 \right) \\ &= (0.5311, 0.5376, 0.5581, 0.5646) \\ C\tilde{L}_{\tilde{X}-S}^\alpha &= \left(\bar{X}_a^\alpha, \bar{X}_b^\alpha, \bar{X}_c^\alpha, \bar{X}_d^\alpha \right) = (0.4862, 0.4961, 0.5173, 0.5258) \end{aligned}$$

$$\begin{aligned} L\tilde{C}L_{\tilde{X}-S}^\alpha &= \left(\bar{X}_a^\alpha, \bar{X}_b^\alpha, \bar{X}_c^\alpha, \bar{X}_d^\alpha \right) - \left[\frac{3}{c_4\sqrt{n}} (\bar{S}_a^\alpha, \bar{S}_b^\alpha, \bar{S}_c^\alpha, \bar{S}_d^\alpha) \right] \\ &= \left(0.4862 - \frac{3}{0.94\sqrt{5}} \cdot 0.00299, 0.4961 - \frac{3}{0.94\sqrt{5}} \cdot 0.00318, \right. \\ &\quad \left. 0.5173 - \frac{3}{0.94\sqrt{5}} \cdot 0.00339, 0.5258 - \frac{3}{0.94\sqrt{5}} \cdot 0.00136 \right) \\ &= (0.4413, 0.4547, 0.4765, 0.4871) \end{aligned}$$

The proposed standard deviation

$$\begin{aligned} USL_{a.SFC_p}^\alpha - LSL_{a.SFC_p}^\alpha &= 0.050983 - 0.019749 \\ \Rightarrow \tilde{\sigma}_{a.SFC_p}^\alpha &= 0.002603 \\ USL_{d.SFC_p}^\alpha - LSL_{d.SFC_p}^\alpha &= 0.047460 - 0.016239 \\ \Rightarrow \tilde{\sigma}_{d.SFC_p}^\alpha &= 0.002602 \end{aligned}$$

The proposed fuzzy $\tilde{X} - S$ control limits with process capability using α -cut method for trapezoidal numbers as follows:

$$\begin{aligned} U\tilde{C}L_{\tilde{X}-S:C_p}^\alpha &= \left(\bar{X}_a^\alpha + \frac{3}{\sqrt{n}} \tilde{\sigma}_{a.SFC_p}^\alpha, \bar{X}_b^\alpha + \frac{3}{\sqrt{n}} \tilde{\sigma}_{b.SFC_p}^\alpha, \right. \\ &\quad \left. \bar{X}_c^\alpha + \frac{3}{\sqrt{n}} \tilde{\sigma}_{c.SFC_p}^\alpha, \bar{X}_d^\alpha + \frac{3}{\sqrt{n}} \tilde{\sigma}_{d.SFC_p}^\alpha \right) \\ &= \left(0.4862 + \frac{3}{\sqrt{5}} \cdot 0.002603, 0.4961 + \frac{3}{\sqrt{5}} \cdot 0.003179, \right. \\ &\quad \left. 0.5173 + \frac{3}{\sqrt{5}} \cdot 0.003389, 0.5258 + \frac{3}{\sqrt{5}} \cdot 0.002602 \right) \\ &= (0.4897, 0.5004, 0.5219, 0.5445) \\ C\tilde{L}_{\tilde{X}-S}^\alpha &= \left(\bar{X}_{a.\tilde{X}-S}^\alpha, \bar{X}_{b.\tilde{X}-S}^\alpha, \bar{X}_{c.\tilde{X}-S}^\alpha, \bar{X}_{d.\tilde{X}-S}^\alpha \right) \\ &= (0.4862, 0.4961, 0.5173, 0.5258) \\ L\tilde{C}L_{\tilde{X}-S:C_p}^\alpha &= \left(\bar{X}_a^\alpha - \frac{3}{\sqrt{n}} \tilde{\sigma}_{a.SFC_p}^\alpha, \bar{X}_b^\alpha - \frac{3}{\sqrt{n}} \tilde{\sigma}_{b.SFC_p}^\alpha, \right. \\ &\quad \left. \bar{X}_c^\alpha - \frac{3}{\sqrt{n}} \tilde{\sigma}_{c.SFC_p}^\alpha, \bar{X}_d^\alpha - \frac{3}{\sqrt{n}} \tilde{\sigma}_{d.SFC_p}^\alpha \right) \\ &= \left(0.4862 - \frac{3}{\sqrt{5}} \cdot 0.002603, 0.4961 - \frac{3}{\sqrt{5}} \cdot 0.003179, \right. \\ &\quad \left. 0.5173 - \frac{3}{\sqrt{5}} \cdot 0.003389, 0.5258 - \frac{3}{\sqrt{5}} \cdot 0.002602 \right) \\ &= (0.4827, 0.4919, 0.5128, 0.5223) \end{aligned}$$

The control limits of α -level fuzzy midrange for α -cut fuzzy $\tilde{X} - S$ control chart can be obtained as follows:

Construction of Fuzzy Mean using Standard Deviation ($\tilde{X} - S$) Control Chart with Process Capability

$$\begin{aligned}
 U\tilde{C}L_{\tilde{X}-S.mid}^{\alpha} &= \left(\frac{\bar{X}_a^{\alpha} + \bar{X}_d^{\alpha}}{2} \right) + \left[\frac{3}{c_4\sqrt{n}} \left(\frac{\bar{S}_a^{\alpha} + \bar{S}_d^{\alpha}}{2} \right) \right] \\
 &= \left(\frac{0.4862 + 0.5258}{2} \right) \\
 &\quad + \left[\frac{3}{0.94\sqrt{5}} \left(\frac{0.0315 + 0.0271}{2} \right) \right] = 0.5478 \\
 C\tilde{L}_{\tilde{X}-S.mid}^{\alpha} &= \left(\frac{\bar{X}_a^{\alpha} + \bar{X}_d^{\alpha}}{2} \right) = \left(\frac{0.4862 + 0.5258}{2} \right) = 0.5060 \\
 L\tilde{C}L_{\tilde{X}-S.mid}^{\alpha} &= \left(\frac{\bar{X}_a^{\alpha} + \bar{X}_d^{\alpha}}{2} \right) - \left[\frac{3}{c_4\sqrt{n}} \left(\frac{\bar{S}_a^{\alpha} + \bar{S}_d^{\alpha}}{2} \right) \right] \\
 &= \left(\frac{0.4862 + 0.5258}{2} \right) \\
 &\quad - \left[\frac{3}{0.94\sqrt{5}} \left(\frac{0.0315 + 0.0271}{2} \right) \right] = 0.4642
 \end{aligned}$$

The definition of α -level fuzzy midrange of sample j for fuzzy $\tilde{X} - S$ control chart is

$$S_{j:\tilde{X}-S.mid}^{\alpha} = \frac{(\bar{X}_{aj} + \bar{X}_{dj}) + \alpha [(\bar{X}_{bj} - \bar{X}_{aj}) - (\bar{X}_{dj} - \bar{X}_{cj})]}{2}$$

Then, the condition of process control for each sample can be defined as:

$$\text{Process control} = \begin{cases} \text{in control} & ; \\ L\tilde{C}L_{\tilde{X}-S.mid}^{\alpha} \leq S_{j:\tilde{X}-S.mid}^{\alpha} & \\ \leq U\tilde{C}L_{\tilde{X}-S.mid}^{\alpha} & \\ \text{Out-of-control} & ; \text{ Otherwise} \end{cases}$$

The proposed standard deviation of α -level fuzzy midrange for α -cut fuzzy $\tilde{X} - S$ control chart is as follows:

$$\begin{aligned}
 USL_{Mid.SFC_p}^{\alpha} - LSL_{Mid.SFC_p}^{\alpha} &= 0.04445 - 0.020484 \\
 \Rightarrow \tilde{\sigma}_{Mid.SFC_p}^{\alpha} &= 0.001997
 \end{aligned}$$

The proposed control limits using process capability of α -level fuzzy midrange for α -cut fuzzy $\tilde{X} - S$ control chart can be obtained as follows:

$$\begin{aligned}
 U\tilde{C}L_{\tilde{X}-S.mid:C_p}^{\alpha} &= \left(\frac{\bar{X}_a^{\alpha} + \bar{X}_d^{\alpha}}{2} \right) + \left[\frac{3}{\sqrt{n}} \tilde{\sigma}_{Mid.SFC_p}^{\alpha} \right] \\
 &= \left(\frac{0.4862 + 0.5258}{2} \right) + \left[\frac{3}{\sqrt{5}} 0.001997 \right] = 0.5089 \\
 C\tilde{L}_{\tilde{X}-S.mid:C_p}^{\alpha} &= \left(\frac{\bar{X}_a^{\alpha} + \bar{X}_d^{\alpha}}{2} \right) \\
 &= \left(\frac{0.4862 + 0.5258}{2} \right) = 0.5060 \\
 L\tilde{C}L_{\tilde{X}-S.mid:C_p}^{\alpha} &= \left(\frac{\bar{X}_a^{\alpha} + \bar{X}_d^{\alpha}}{2} \right) - \left[\frac{3}{\sqrt{n}} \tilde{\sigma}_{Mid.SFC_p}^{\alpha} \right] \\
 &= \left(\frac{0.4862 + 0.5258}{2} \right) - \left[\frac{3}{\sqrt{5}} 0.001997 \right] = 0.5031
 \end{aligned}$$

Table 3: Decision process for α -level fuzzy midrange of fuzzy $\tilde{X} - S$ control chart

Sample No.	$S_{j:\tilde{X}-S.mid}^{\alpha}$	$0.4642 \leq S_{j:\tilde{X}-S.mid}^{\alpha} \leq 0.5478$
1	0.5223	In control
2	0.5086	In control

3	0.5254	In control
4	0.4860	In control
5	0.5150	In control
6	0.5083	In control
7	0.5054	In control
8	0.4930	In control
9	0.5070	In control
10	0.4910	In control
11	0.5049	In control
12	0.5117	In control
13	0.5087	In control
14	0.4955	In control
15	0.5083	In control

Then, the condition of process control for each sample can be defined as:

$$\text{Process control} = \begin{cases} \text{in control} & ; \\ L\tilde{C}L_{\tilde{X}-S.mid:C_p}^{\alpha} \leq S_{j:\tilde{X}-S.mid}^{\alpha} & \\ \leq U\tilde{C}L_{\tilde{X}-S.mid:C_p}^{\alpha} & \\ \text{Out-of-control} & ; \text{ Otherwise} \end{cases}$$

From the resulting Figure 1, the process is out of control in Fuzzy $\tilde{X} - S$ using process capability compared with Fuzzy $\tilde{X} - S$ control chart since the sample numbers 1, 3, 5 and 12 placed above the upper control limit and the sample numbers 4, 8, 10 and 14 placed below the lower control limit.

Table 4: Decision process for α -level fuzzy midrange of fuzzy $\tilde{X} - S$ process capability control chart

Sample No.	$S_{j:\tilde{X}-S.mid}^{\alpha}$	$0.5031 \leq S_{j:\tilde{X}-S.mid}^{\alpha} \leq 0.5089$
1	0.5223	Out-of-control
2	0.5086	In control
3	0.5254	Out-of-control
4	0.4860	Out-of-control
5	0.5150	Out-of-control
6	0.5083	In control
7	0.5054	In control
8	0.4930	Out-of-control
9	0.5070	In control
10	0.4910	Out-of-control
11	0.5049	In control
12	0.5117	Out-of-control
13	0.5087	In control
14	0.4955	Out-of-control
15	0.5083	In control

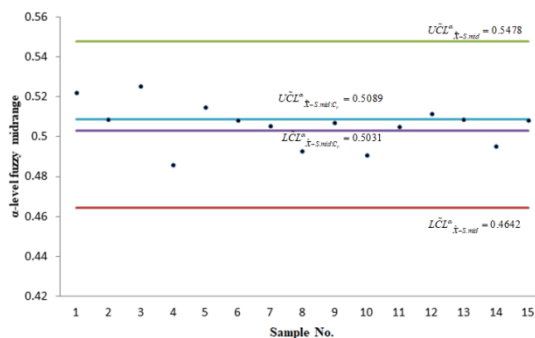


Figure 1: Comparison of Fuzzy $\tilde{X} - s$ control chart with $\tilde{X} - s$ using process capability

In general the expected number of samples needed to detect a shift of 'multiple of σ ' under the control chart for fuzzy $\tilde{X} - s$ using process capability is more agile than the existing fuzzy $\tilde{X} - s$ control limits.

Table 5: Efficiency results through average run length (ARL) for $\tilde{X} - s$ control charts

multiple of σ	Fuzzy $\tilde{X} - s$ control chart	Control chart for Fuzzy $\tilde{X} - s$ using process capability
0.001	356.2671	59.8895
0.002	331.4839	6.3780
0.003	296.4213	1.8363
0.004	257.1979	1.1224
0.005	218.5715	1.0094
0.006	183.3430	1.0003
0.007	152.6915	1.0000
0.008	126.7727	1.0000
0.009	105.2187	1.0000
0.010	87.4576	1.0000

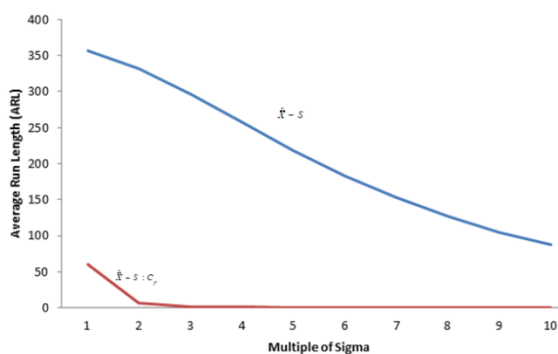


Figure 2: Average run length (ARL) for $\tilde{X} - s$ control charts

IV. CONCLUSION

The constructed control chart for fuzzy mean using standard deviation with the help of process capability, procedures adopted and discussed in the research article by taking the process capability (C_p) as the base only. In this article offers the possibility of using fuzzy mean using standard deviation control chart using process

capability, which rules out the weaknesses compared to the existing control charts. Specifically, it presents one of the fuzzy mean using standard deviation control chart using process capability and on the real-life data illustrates the simplicity of its usage in practice. The procedures can also be constructed by changing the suitable process capability (C_p) used in this article and the performance with efficiency of the control charts is tested through average run length.

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