

Results on Homomorphism of a Vague Γ -Nearing



S Ragamayi, Y Bhargavi, N Konda Reddy

Abstract: In this article, we originate and inspect the idea of homomorphism of a Vague Γ - NR and prove that the vague pre-Homomorphic-image and Homomorphic-image of a $V\Gamma - NR$ of M is also a Vague Γ - NR of M . Also we introduce Vague characteristic-set. We established one-one correspondence between vague characteristic Γ - Nearing of M and crisp characteristic $\Gamma - NR$ of M .

Keywords: Vague characteristic set, Vague set, Vague Γ -NR, Homomorphism of a vague Γ -NR.

I. INTRODUCTION

The theory of fuzzy Γ -NRs has been introduced and developed by Bh.Satyanarayana[1] and G.L.Booth. Later D.J.Buehrer [20] introduced vague sets as generalization of fuzzy sets. Further S.Ragamayi ([12],[17]) has introduced and studied the notion of Normal Vague Γ - NR and Normal Vague Ideals of a Γ - NR. Also she proved some interesting results on it. Earlier, Ranjit Biswas[4] had introduced the notion of vague homomorphism in groups. Now we study the notation of homomorphism in vague Γ -NR. We show that the vague pre-Homomorphic-image and Homomorphic-image of a $V\Gamma - NR$ of M is a $V\Gamma$ - NR of M . Later, we show that pre-Homomorphic -image and it's image of a $V\Gamma$ - NR possessing Supremum property is a $V\Gamma$ - NR possessing Supremum property. Furthermore, we show that if P is a $VC\tilde{A} - NR$ of M then it's (\acute{o}, \grave{e}) - cut, $P_{(\acute{o}, \grave{e})}$ is also a $CS\tilde{A} - NR$ of M and vice versa.

Notations: Throughout this article, we use the following notations.

- 1) $\tilde{A} - NR$ stands for Γ -NR.
- 2) $V\tilde{A} - NR$ stands for Vague Γ -NR.
- 3) M stands for Zero-Symmetric Γ -NR.
- 4) $S\tilde{A} - NR$ stands for Sub Γ -NR.
- 5) $VC\tilde{A} - NR$ stands for Vague characteristic Γ -NR.
- 6) $CS\tilde{A} - NR$ stands for Characteristic Sub Γ -NR
- 7) **H-image** stands for Homomorphic-image.
- 8) **Aut**(M) is the set of all automorphisms of M

II. PRELIMINARIES

Definition 2.1: A triple $(M, +, \Gamma)$ is called Zero-Symmetric $\tilde{A} - NR$, if

- (i) M is a group with '+'.
 - (ii) $(M, +, \gamma)$ is a nearing $\forall \gamma \in \Gamma$, where Γ is a non-empty set of binary operators on M
 - (iii) $m \gamma_1 (n \gamma_2 p) = (m \gamma_1 n) \gamma_2 p$, for all $m, n, p \in M$ and $\gamma_1, \gamma_2 \in \Gamma$.
 - (iv) $m \gamma_1 0 = 0$ for every $m \in M, \gamma_1 \in \Gamma$

Definition 2.2: A nonempty subset 'E' of M is said to be $S\tilde{A} - NR$ if

- i) $m - n \in E$
- ii) $m \alpha n \in E$, for each $\alpha \in \Gamma$ and $m, n \in M$.

Definition 2.3: A Fuzzy subset μ of M is a Fuzzy $S\tilde{A} - NR$ of M if

- 1) $\mu(m - n) \geq \min\{\mu(m), \mu(n)\}$
- 2) $\mu(m \alpha n) \geq \min\{\mu(m), \mu(n)\}$ for all $m, n \in M, \alpha \in \Gamma$.

Definition 2.4: A vague set P in the universe of discourse U is a pair (t_p, f_p) , where

$t_p, f_p: U \rightarrow [0, 1]$ are mappings in such a way that $t_p(a) + f_p(a) \leq 1, \forall a \in U$. The functions

t_p and f_p are called true and false membership functions.

Definition 2.5: The interval $[t_p(a), 1 - f_p(a)]$ is said to be the vague value of 'a' in P is defined as

$$V_P(a) = [t_p(a), 1 - f_p(a)], \forall a \in U.$$

Definition 2.6: For given vague sets P and $Q, P \subseteq Q$ if and only if $V_P(a) \subseteq V_Q(a)$ i.e., $t_p(a) \leq t_q(a)$ and $1 - f_p(a) \leq 1 - f_q(a), \forall a \in U$.

Definition 2.7: We can say that two vague sets P and Q are equal, $P = Q$, if and only if $P \subseteq Q$ and $Q \subseteq P$ i.e., $V_P(a) \subseteq V_Q(a)$ and $V_Q(a) \subseteq V_P(a), \forall a \in U$.

Definition 2.8: The union of vague sets P and Q is a vague set R , is represented by

$R = P \cup Q$ then it's truth membership function is $t_R = \max\{t_p, t_q\}$ and false membership function $1 - f_R = \max\{1 - f_p, 1 - f_q\} = 1 - \min\{f_p, f_q\}$.

Definition 2.9: The intersection of two vague sets P and Q is a vague set $R = P \cap Q$, whose truth membership function is $t_R = \min\{t_p, t_q\}$ and false membership function is $1 - f_R = \min\{1 - f_p, 1 - f_q\} = 1 - \max\{f_p, f_q\}$.

Definition 2.10: The intersection and union of a family $\{P_i / i \in \Delta\}$ of vague sets of U are defined by

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$$V_{\cup_{i \in \Delta} P_i}(r) = \sup_{i \in \Delta} V_{P_i}(r), \forall r \in U$$

$$V_{\cap_{i \in \Delta} P_i}(r) = \inf_{i \in \Delta} V_{P_i}(r), \forall r \in U$$

Definition 2.11: The vaguecut or (\acute{o}, \grave{e}) - cut of a vagueset P of a universeset U is defined for every $\acute{o}, \grave{e} \in [0,1]$ with $\acute{o} \leq \grave{e}$ as $P_{(\acute{o}, \grave{e})} = \{r \in U / V_P(r) \geq [\acute{o}, \grave{e}]\}$ where $P_{(\acute{o}, \acute{o})} = \{r \in U / t_P(r) \geq \acute{o}, f_P(r) \leq \acute{o}\}$ is the crisp subset of U.

Definition 2.12: The \acute{o} -cut of a vagueset P of a universe U is defined by $P_\sigma = \{x \in U / t_P(x) \geq \acute{o}\}$.

Definition 2.13: Consider $g : X \rightarrow Y$ be homomorphism. Let P be a vagueset in X whose vaguevalue V_P . Then the H-image of P, $g(P)$ is the vagueset on Y described as

$$V_{g(P)}(n) = \begin{cases} \sup_{r \in g^{-1}(n)} V_P(r) & \text{if } g^{-1}(n) \neq \emptyset \\ [0,0] & \text{Otherwise} \end{cases}$$

For all $n \in Y$, where $g^{-1}(n) = \{l/g(x) = n\}$.

If Q is a vagueset in Y then the pre-H-image of Q, $g^{-1}(Q)$ is the vagueset on X by

$$V(l) = V_{g^{-1}(Q)}(l) = V_{(Q)}(g(l)), \forall l \in X.$$

Definition 2.14: A vagueset P of possessing the Supr. property if $S \subseteq X$,

$$\exists \alpha_0 \in S \ni V_P(\alpha_0) = \sup_{\acute{a} \in S} V_P(\acute{a})$$

Definition 2.15: If g is homomorphism from $X \rightarrow Y$ then

$$g(m\gamma_1 n) = g(m) \gamma_1 g(n), \forall m, n \in X; \gamma_1 \in \Gamma_1.$$

Definition 2.16: A vagueset $P = (t_P, f_P)$ of M is said to be a $V\check{A} - NR$ if for every $r, m \in M$ and $\alpha \in \Gamma$,

$$1) V_P(t-m) \geq \min\{V_P(t), V_P(m)\} \text{ and } 2) V_P(t\alpha m) \geq \min\{V_P(t), V_P(m)\}$$

i.e., (i). $t_P(t-m) \geq \min\{t_P(t), t_P(m)\}$, $1-f_P(t-m) \geq \min\{1-f_P(t), 1-f_P(m)\}$ and

(ii). $t_P(t\alpha m) \geq \min\{t_P(t), t_P(m)\}$, $1-f_P(t\alpha m) \geq \min\{1-f_P(t), 1-f_P(m)\}$.

III. VAGUE HOMOMORPHISM OF A Γ -NR

In the following section, we invented the notion of vague homomorphism of a $\check{A} - NR$. We shown that the pre-H-image and H-image of a $V\check{A} - NR$ of M is a $V\check{A} - NR$ of M. Later, we prove that the pre-H-image and H-image of a $V\check{A} - NR$ having Supremum property is a $V\check{A} - NR$ possess Supremum property.

Definition 3.1: If $g : M \rightarrow N$ be homomorphism and P be a vagueset on M whose vaguevalue V_P then the image of P, $g(P)$ is the vagueset on N defined as

$$V_{g(P)}(n) = \begin{cases} \sup_{r \in g^{-1}(n)} V_P(r) & \text{if } g^{-1}(n) \neq \emptyset \\ [0,0] & \text{Otherwise} \end{cases}$$

$\forall n \in N, g^{-1}(n) = \{m/g(m) = n\}$.

Let Q be a vagueset on N. Then the pre-H-image of Q, $g^{-1}(Q)$ is the vagueset on M by

$$V_{g^{-1}(Q)}(m) = V_Q(g(m)), \forall m \in M.$$

Definition 3.2: If $\exists m_0 \in S$ in such a way that $V_P(m_0) = \sup_{m \in S} V_P(m)$,

then we say that the vagueset P of M is having Supr. Property, $\forall S \in M$.

Definition 3.3: Let P and Q be two $\check{A} - NR$ s and if 'g' is homomorphism from P into Q then

$$g(m\gamma_1 n) = g(m) \gamma_1 g(n), \forall m, n \in P; \gamma_1 \in \Gamma_1.$$

Theorem 3.4: If 'g' is homomorphism from $\check{A} - NR, M$ into $\check{A} - NR, N$ and if Q is a $V\check{A} - NR$ of N, then it's pre-H-image of Q, $g^{-1}(Q)$ is a $V\check{A} - NR$ of M.

Proof. : $\forall t, r \in M, \beta \in \Gamma_1$,

$$1) V_{g^{-1}(Q)}(t-r) = V_Q(g(t-r)) = V_Q(g(t)-g(r)) \geq \min\{V_Q(g(t)), V_Q(g(r))\} = \min\{V_{g^{-1}(Q)}(t), V_{g^{-1}(Q)}(r)\}$$

$$2) V_{g^{-1}(Q)}(t\beta r) = V_Q(g(t\beta r)) = V_Q(g(t)\beta g(r)) \geq \min\{V_Q(g(t)), V_Q(g(r))\} = \min\{V_{g^{-1}(Q)}(t), V_{g^{-1}(Q)}(r)\}.$$

hence, $g^{-1}(Q)$ is a $V\check{A} - NR$ of M.

Theorem 3.5: If 'g' is a homomorphism of $\check{A} - NR, M$ onto $\check{A} - NR, N$. If 'P' is a $V\check{A} - NR$ of M possess Supr. property, then the H-image of P, $g(P)$ is a $V\check{A} - NR$ of N.

Proof: Let $m, n \in N; \beta \in \Gamma$.

If either $g^{-1}(m) = \emptyset$ or $g^{-1}(n) = \emptyset$ then the proof is obviously true.

Let's have a Supposition, i.e., neither $g^{-1}(m) \neq \emptyset$ nor $g^{-1}(n) \neq \emptyset$.

Let $m_0 \in g^{-1}(m)$ and $n_0 \in g^{-1}(n)$ be $\exists V_P(m_0) = \sup V_P(a)$

where $a \in g^{-1}(m)$

and $V_P(n_0) = \sup V_P(b)$ where $b \in g^{-1}(n)$

$$1) V_{g(P)}(m-n) = \sup_{r \in g^{-1}(m-n)} V_P(r) \geq V_P(r), r \in g^{-1}(m-n)$$

$$= V_P(m_0 - n_0) \geq \min\{V_P(m_0), V_P(n_0)\} = \min\{V_{g(P)}(m), V_{g(P)}(n)\}$$

$$2) V_{g(P)}(m\gamma_2 n) = \sup_{r \in g^{-1}(m\gamma_2 n)} V_P(r) \geq V_P(r), r \in g^{-1}(m\gamma_2 n)$$

$$= V_P(m_0\gamma_1 n_0), \gamma_1 \in \Gamma_1 \geq$$

$$\min\{V_P(m_0), V_P(n_0)\}$$

$$= \min\{V_{g(P)}(m), V_{g(P)}(n)\}.$$

Hence we have shown that $g(P)$ is a $V\check{A} - NR$ of N.

Theorem 3.6: Let g be an epimorphism from a $\check{A} - NR, M$ onto a $\check{A} - NR, N$. Let P be

$V\check{A} - NR$ of M possessing Supr. property. Then the H-image $g(P)$ of a vagueset P, is a $V\check{A} - NR$ of N possessing Supr. property.

Proof. : From theorem: 3.5, $g(P)$ is a $V\check{A} - NR$ of N.

Now, we prove that $g(P)$ possess sup. property.

Let $S \subseteq M_2$.

Since g is epimorphism $g^{-1}(S) \neq \emptyset$.

According to definition, $V_{g(P)}(s) = \sup\{V_P(m) / m \in g^{-1}(s)\}$.

Since P possess sup. property and $g^{-1}(S) \subset M, \exists m_0 \in g^{-1}(S)$

$$\exists V_P(m_0) = \sup\{V_P(m) / m \in g^{-1}(S)\} \dots \dots \dots (1).$$

Also,



$$\sup\{V_{g(P)}(s)/s \in S\} = \sup\{\sup\{V_P(m)/m \in g^{-1}(s)\}/s \in g^{-1}(S)\}$$

$$= \sup\{V_P(m)/m \in g^{-1}(S)\} \dots \dots \dots (2).$$

From (1) and (2), $V_P(m_0) = \sup\{V_{g(P)}(s)/s \in S\}$.
 Let $s_0 \in S$.
 Since g is epimorphism, we have $m_0 \in M_1 \ni s_0 = g(m_0)$.
 Now, we have $V_{g(P)}(s_0) = \sup\{V_P(m)/m \in g^{-1}(s_0)\}$.
 But $m_0 \in g^{-1}(s_0) \subseteq g^{-1}(S)$.
 So, $\sup\{V_P(m)/m \in g^{-1}(S)\} \geq \sup\{V_P(m)/m \in g^{-1}(s_0)\}$.
 Thus $V_P(m_0) = \sup\{V_P(m)/m \in g^{-1}(S)\} \geq \sup\{V_P(m)/m \in g^{-1}(s_0)\} \geq V_P(m_0)$ is not possible.
 Therefore $V_P(m_0) = \sup\{V_P(m)/m \in g^{-1}(s_0)\}$.
 Now, $V_{g(P)}(s_0) = \sup\{V_P(m)/m \in g^{-1}(s_0)\} = V_P(m_0) = \sup\{V_{g(P)}(s)/s \in S\}$.
 i.e., $V_{g(P)}(s_0) = \sup\{V_{g(P)}(s)/s \in S\}$.
 Hence $g(P)$ possess Sup. property.

Theorem 3.7: If $g: M \rightarrow N$ is a homomorphism onto and Q be a $V\check{A} - NR$ of N possessing Supr. property. Then the pre-H-image $g^{-1}(Q)$ of Q , is a $V\Gamma - NR$ of M possess Supr. property.

Proof. : From theorem: 3.4, $g^{-1}(Q)$ is a $V\Gamma - NR$ of M . we will prove that $g^{-1}(Q)$ possess Supremum- property. Let $T \subseteq M_1$. By definition, $V_{g^{-1}(Q)}(m) = V_Q(g(m))$, $\forall m \in T$. Since Q possessing sup. property, $\exists l_0 \in g(T) \ni V_Q(l_0) = \sup\{V_Q(l)/l \in g(T)\} \dots \dots \dots (1)$. Also, $\sup\{V_{g^{-1}(Q)}(l)/l \in T\} = \sup\{V_Q(g(l))/l \in T\} = \sup\{V_Q(l)/l \in g(T)\} \dots \dots \dots (2)$. From (1) and (2), $V_Q(l_0) = \sup\{V_{g^{-1}(Q)}(l)/l \in T\}$. Now, $l_0 \in g(T)$ which implies $\exists l_0 \in T$ in such a way that $l_0 = g(m_0)$. Thus $V_{g^{-1}(Q)}(m_0) = V_Q(g(m_0)) = V_Q(l_0) = \sup\{V_{g^{-1}(Q)}(m)/m \in T\}$.

Hence proved that $g^{-1}(Q)$ possessed Sup. property.
Definition 3.8: If $g(P) = P$, $\forall g \in \text{Aut}(M)$ then a $S\check{A} - NR$, P of M is said to be $CS\check{A} - NR$.

Definition 3.9: If $V_P(g(l)) = V_P(l)$, $\forall l \in M$, $g \in \text{Aut}(M)$ then a $V\check{A} - NR$, P of M is said to be $VC\check{A} - NR$.

Lemma 3.10: Let $P = (t_p, f_p)$ be a $V\check{A} - NR$ of M and let $m \in M$. Then $V_P(m) = [\alpha, \beta]$ if and only if $m \in P_{(\alpha, \beta)}$ and $m \notin P_{(\gamma, \delta)}$, $\forall [\gamma, \eta] > [\alpha, \beta]$, where $\gamma, \eta \in [0, 1]$.

Proof. : The proof of lemma is clear.
Theorem 3.11: Let $P = (t_p, f_p)$ be a $V\check{A} - NR$ of M . Then P is $VC\check{A} - NR$ of M if and only if it's vaguecut of P or (\acute{o}, \grave{e}) -cut, $P_{(\acute{o}, \grave{e})}$, where $\acute{o}, \grave{e} \in [0, 1]$ is a $CS\check{A} - NR$ of M .

Proof. : Let P be a $VC\check{A} - NR$ of M .
 $\rightarrow P_{(\acute{o}, \grave{e})}$ is a $VC\check{A} - NR$ of M .
 Let $g \in \text{Aut}(M)$.
 We have to prove that $g(P_{(\acute{o}, \grave{e})}) = P_{(\acute{o}, \grave{e})}$.
 Let $n \in g(P_{(\acute{o}, \grave{e})})$
 $\rightarrow n = g(m)$, for some $m \in P_{(\acute{o}, \grave{e})}$.
 Now, $V_P(n) = V_P(g(m)) = V_P(m) = [\acute{o}, \grave{e}] \Rightarrow n \in P_{(\acute{o}, \grave{e})}$.
 So, $g(P_{(\acute{o}, \grave{e})}) \subseteq P_{(\acute{o}, \grave{e})}$.
 Let $n \in P_{(\acute{o}, \grave{e})}$.
 because g is automorphism on M , we have $n = g(m)$, for some $p \in M$.
 Now, $[\acute{o}, \grave{e}] \leq V_P(n) = V_P(g(m)) = V_P(m)$.
 $\Rightarrow m \in P_{(\acute{o}, \grave{e})} \Rightarrow n \in g(P_{(\acute{o}, \grave{e})})$
 So, $P_{(\acute{o}, \grave{e})} \subseteq g(P_{(\acute{o}, \grave{e})})$.
 $\rightarrow P_{(\acute{o}, \grave{e})} = g(P_{(\acute{o}, \grave{e})})$.
 Thus, $P_{(\acute{o}, \grave{e})}$ is a $CS\check{A} - NR$ of M .

By Converse, we have a supposition that $P_{(\acute{o}, \grave{e})}$ is a $CS\check{A} - NR$ of M .
 Consider $g \in (M)$, $m \in M$.
 Let $V_P(m) = [\acute{o}, \grave{e}]$.
 This implies $m \in P_{(\acute{o}, \grave{e})}$, $m \notin P_{(\acute{o}, \grave{e})} \forall [\acute{o}, \grave{e}] > [\acute{o}, \grave{e}]$.
 Let $V_P(g(m)) = [\acute{o}, \grave{e}]$
 if it is possible $[\acute{o}, \grave{e}] > [\acute{o}, \grave{e}]$.
 Then $g(m) \in P_{(\acute{o}, \grave{e})} = g(P_{(\acute{o}, \grave{e})}) \Rightarrow m \in P_{(\acute{o}, \grave{e})}$ is contradiction.
 $\rightarrow P_{(\acute{o}, \grave{e})} = [\acute{o}, \grave{e}]$.
 So, $V_P(g(m)) = V_P(m)$.
 Thus P is a $VC\check{A} - NR$ of M .

Definition 3.12: Let $P = (t_p, f_p)$ and $Q = (t_q, f_q)$ be $V\check{A} - NR$ s of M . If $\exists \vartheta \in \text{Aut}(M) \ni V_P(m) = V_Q(\vartheta(m))$, $\forall m \in M$. i.e., $t_p(m) = t_q(\vartheta(m))$ and $f_p(m) = f_q(\vartheta(m))$, then P and Q are said to be homologous $V\check{A} - NR$ s of M .

If P and Q are homologous then Q, P are also homologous.
Theorem 3.13: Consider $Q = (t_q, f_q)$ be a $V\check{A} - NR$ of M and $\vartheta \in \text{Aut}(M)$. If $P = (t_p, f_p)$ is a vague set of $M \ni V_P(m) = V_Q(\vartheta(m))$, $\forall m \in M$, then P and Q are homologous $V\check{A} - NR$ s of M .

Proof. : Let $t, r \in M$; $\gamma_1 \in \Gamma$.
 1. $V_P(t-r) = V_Q(\vartheta(t-r))$
 $= V_Q(\vartheta(t) - \vartheta(r))$
 $\geq \min\{V_Q(\vartheta(t)), V_Q(\vartheta(r))\}$
 $= \min\{V_P(t), V_P(r)\}$
 2. $V_P(t\gamma_1 r) = V_Q(\vartheta(t\gamma_1 r))$
 $= V_Q(\vartheta(t)\gamma_1 \vartheta(r))$
 $\geq \min\{V_Q(\vartheta(t)), V_Q(\vartheta(r))\}$

Therefore P is a $V\check{A} - NR$ of M .
 Thus P and Q are homologous $V\check{A} - NR$ s of M .
Theorem 3.14: Let $P = (t_p, f_p)$ be a $V\check{A} - NR$ of M and let g be an onto homomorphism from M into itself. Then the vague set $P^f = (t_p^f, f_p^f)$ defined by $V_{P^f} g(m) = V_P(g(m))$, $\forall m \in M$ is a $V\check{A} - NR$ of M .

Proof. : Let $m, n \in M$; $\gamma_1 \in \Gamma$.
 1. $V_{P^f}(t-r) = V_P(g(t-r))$
 $= V_P(g(t) - g(r))$
 $\geq \min\{V_P(g(t)), V_P(g(r))\}$
 $= \min\{V_{P^f}(t), V_{P^f}(r)\}$.
 2. $V_{P^f}(t\gamma_1 r) = V_P(g(t\gamma_1 r))$
 $= V_P(g(t)\gamma_1 g(r))$
 $\geq \min\{V_P(g(t)), V_P(g(r))\}$
 $= \min\{V_{P^f}(t), V_{P^f}(r)\}$.

Hence P is a $V\check{A} - NR$ of M .

IV. CONCLUSION

In this research article, the idea of vague homomorphism of a $\check{A} - NR$ has been introduced and proved some interesting results on it. We proved that the homomorphic Pre- image and image of a $V\check{A} - NR$ of M is a $V\check{A} - NR$ of M . Also, we proved that pre-H-image and image of a $\check{A} - NR$ possessing Sup. property is a $V\check{A} - NR$ possessing Sup. property. Moreover, we proved that if P is a $VC\check{A} - NR$ of M then it's $(\acute{a}, \acute{\alpha})$ -cut, $P_{(\alpha, \beta)}$ is also a $CS\check{A} - NR$ of M and vice versa.



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