Results on Homomorphism of a Vague Γ-Nearing

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Abstract: In this article, we originate and inspect the idea of homomorphism of a Vague Γ-NR and prove that the vague pre-Homomorphic-image and Homomorphic-image of a VΓ − NR of M is also a Vague Γ-NR of M. Also we introduce Vague characteristic-set. We established one-to-one correspondence between vague characteristic Γ-Nearing of M and crisp characteristic Γ − NR of M.

Keywords: Vague characteristic set, Vague set, Vague Γ-NR, Homomorphism of a vague Γ-NR.

I. INTRODUCTION

The theory of fuzzy Γ-NRs has been introduced and developed by Bh.Satyanarayana[1] and G.L.Booth. Later D.J.Buehrer [20] introduced vague sets as generalization of fuzzy sets. Further S.Ragamayi ([12],[17]) has introduced and studied the notion of Normal Vague Γ-NR and Normal Vague Ideals of a Γ-NR. Also she proved some interesting results on it. Earlier, Ranjit Biswas[4] had introduced the notion of vague homomorphism in groups. Now we study the notion of homomorphism in vague Γ-NR. We show that the vague pre-Homomorphic-image and Homomorphic-image of a VΓ − NR of M is also a VΓ − NR of M. Later, we show that pre-Homomorphic -image and it’s image of a VT- NR possessing Supremum property is a VT- NR possessing Supremum property. Furthermore, we show that if P is a VΓ − NR of M then it’s (6,è) - cut, P(6,è)is also a CSĂ − NR of M and vice versa.

Notations: Throughout this article, we use the following notations.

1) A − NR stands for Γ-NR.
2) VĂ − NR stands for Vague Γ-NR.
3) M stands for Zero-Symmetric Γ-NR.
4) SĂ − NR stands for Sub Γ-NR.
5) VCĂ − NR stands for Vague characteristic Γ-NR.
6) CSĂ − NR stands for Characteristic Sub Γ-NR.
7) H-image stands for Homomorphic-image.
8) Aut(M) is the set of all automorphisms of M

II. PRELIMINARIES

Definition 2.1: A triple (M, +, Γ) is called Zero-Symmetric A − NR, if:
(i) M is a group with ‘+’.
(ii) (M, +, γ) is a nearring ∀ γ ∈ Γ, where Γ is a non-empty set of binary operators on M
(iii) m γ1 (n γ2 p) = (m γ1 n) γ2 p, for all m, n, p ∈ M and γ1, γ2 ∈ Γ.
(iv) m γ1 0 = 0 for every m ∈ M, γ1 ∈ Γ

Definition 2.2: A nonempty subset ‘E’ of M is said to be SĂ − NR if:
i) m − n ∈ E
ii) m α n ∈ E, for each α ∈ Γ and m, n ∈ M.

Definition 2.3: A Fuzzy subset μ of M is a Fuzzy SĂ − NR of M if:
1) μ(m − n) ≥ min{μ(m), μ(n)}
2) μ(m α n) ≥ min{μ(m), μ(n)}

Definition 2.4: A vagueset P in the universe of discourse U is a pair (fP, fP), where fP : U → [0, 1] are mappings in such a way that fP(a) + fP(a) ≤ 1, ∀ a ∈ U. The functions fP and fP are called true and false membership functions.

Definition 2.5: The interval [fP(a), 1-fP(a)] is said to be the vague value of ‘a’ in P is defined as:

Definition 2.6: For given vague sets P and Q, P ⊆ Q if and only if fP(a) ≤ fQ(a) for all a ∈ M.

Definition 2.7: We can say that two vague sets P and Q are equal, P = Q, if and only if P ⊆ Q and Q ⊆ P i.e., fP(a) ≤ fQ(a) and fQ(a) ≤ fP(a), ∀ a ∈ M.

Definition 2.8: The union of vague sets P and Q is a vague set, R, is represented by:
R = P ∪ Q then it’s truth membership function is tR = max{tP, tQ} and false membership function 1-fR = max{1-fP, 1-fQ} = 1-min{fP, fQ}.

Definition 2.9: The intersection of two vague sets P and Q is a vagueset R = P ∩ Q, whose truth membership function is tR = min{tP, tQ} and false membership function is 1-fR = min{1-fP, 1-fQ} = 1-max{fP, fQ}.

Definition 2.10: The intersection and union of a family {P/ i ∈ Δ} of vague sets of U are defined by:

VU∩ΔPi(r)=Supi∈ΔVPi(r), ∀r ∈ U

VU∪ΔPi(r)=Infi∈ΔVPi(r), ∀r ∈ U
Definition 2.11: The vaguecut or $(\delta, \epsilon)$-cut of a vagueset $P$ of a universe set $U$ is defined for every $\delta, \epsilon \in [0, 1]$ with $\delta \leq \epsilon$ as $P(\delta, \epsilon) = \{ r \in U/V(r) \geq \delta, \epsilon \}$ where $P(\delta, \epsilon) = \{ r \in U/t(r) \geq \delta, \epsilon \}$ is the crisp subset of $U$.

Definition 2.12: The $\delta$-cut of a vagueset $P$ of a universe $U$ is defined by $P_{\delta} = \{ r \in U/t(r) \geq \delta \}$.

Definition 2.13: Consider $g : X \rightarrow Y$ be homomorphism. Let $P$ be a vagueset in $X$ whose vaguevalue $V_P$. Then the H-image of $P$, $g(P)$ is the vagueset on $Y$ described as

$$V_{g(P)}(n) = \begin{cases} \sup \{ \epsilon V(r) \mid g^{-1}(n) \neq 0 \} & \text{if } g^{-1}(n) \neq 0 \\ [0,0] & \text{if } g^{-1}(n) = 0 \end{cases}$$

For all $n \in Y$, where $g^{-1}(n) = \{ x \in X \mid g(x) = n \}$. If $Q$ is a vagueset in $Y$ then the pre-H-image of $Q$, $g^{-1}(Q)$ is the vagueset on $X$ by

$$V(l) = V_{g^{-1}(Q)}(l) = V_Q(g(l)), \forall l \in X.$$

Definition 2.14: A vagueset $P$ of possessing the Supr. property if $S \subseteq X$,

$$\exists a_0 \in S \ni V_P(a_0) = \sup_{a \in S} V_P(\alpha).$$

Definition 2.15: If $g$ is homomorphism from $X \rightarrow Y$ then $g(m \wedge n) = g(m) \wedge g(n), \forall m, n \in X; \gamma_1 \in \Gamma_1$.

Definition 2.16: A vagueset $P = (t_0, t_1)$ of $M$ is said to be a $V \subset NR$ if for every $r, m \in M$ and $\alpha \in \Gamma_1$,

1) $V_P(t_0) \supseteq \min \{ V_P(t_1), V_P(m) \}$ and $V_P(t_1) \supseteq \min \{ V_P(t_0), V_P(m) \}$

i.e., (i). $t_0(t_0 - t) \supseteq \min \{ t_0(t_1), t_0(m) \}$, $t_0(t_1 - t) \supseteq \min \{ t_1(t_0), t_1(m) \}$ and

(ii). $t_1(t_0 - t) \supseteq \min \{ t_0(t_1), t_0(m) \}$, $t_1(t_1 - t) \supseteq \min \{ t_1(t_0), t_1(m) \}.$

III. VAGUE HOMOMORPHISM OF A Γ-NR

In the following section, we invented the notion of vague homomorphism of a $\tilde{A} \subset NR$. We shown that the pre-H-image and H-image of a $\tilde{V} \subset NR$ of $M$ is a $\tilde{V} \subset NR$ of $M$. Later, we prove that the pre-H-image and H-image of a $\tilde{V} \subset NR$ having Supr. property is a $\tilde{V} \subset NR$ possess Supr. property.

Definition 3.1: If $g : M \rightarrow N$ be homomorphism and $P$ be a vagueset on $M$ whose vaguevalue $V_P$ then the image of $P$, $g(P)$ is the vagueset on $N$ defined as

$$V_{g(P)}(n) = \begin{cases} \sup \{ \epsilon V(r) \mid g^{-1}(n) \neq 0 \} & \text{if } g^{-1}(n) \neq 0 \\ [0,0] & \text{if } g^{-1}(n) = 0 \end{cases}$$

$\forall n \in N; \epsilon V_{g^{-1}}(n) = \{ m \mid g(m) = n \}$.

Let $Q$ be a vagueset on $N$. Then the pre-H-image of $Q$, $g^{-1}(Q)$ is the vagueset on $M$ by

$$V_{g^{-1}(Q)}(m) = V_Q(g(m)), \forall m \in M.$$

Definition 3.2: If $\exists m_0 \in S$ in such a way that $V_P(m_0) = \sup_{m \in S} V_P(m)$, we say that $V_P$ is having Supr. Property, $\forall S \subseteq M$.

Definition 3.3: Let $P$ and $Q$ be two $\tilde{A} \subset NRs$ and if $g$ is homomorphism from $P$ into $Q$ then $g(m \wedge n) = g(m) \wedge g(n), \forall m, n \in P; \gamma_1 \in \Gamma_1$.

Theorem 3.4: If $g$ is homomorphism from $\tilde{A} \subset NRs$, $M$ into $\tilde{A} \subset NR$, $N$ and if $Q$ is a $\tilde{V} \subset NR$ of $N$, then it’s pre-H-image of $Q$, $g^{-1}(Q)$ is a $\tilde{V} \subset NR$ of $M$.

Proof: : $\forall \in M, \beta \in \Gamma$,

1. $V_{g^{-1}(Q)}(t_0) = V_Q(g(t_0)) = V_Q(g(t_0) - g(t_0)) \geq \min \{ V_Q(g(t_0)), V_Q(g(t_0)) \} = \min \{ V_Q(g(t_0)), V_{g^{-1}(Q)}(t_0) \}$

2. $V_{g^{-1}(Q)}(t_1) = V_Q(g(t_1)) = V_Q(g(t_1) - g(t_1)) \geq \min \{ V_Q(g(t_1)), V_Q(g(t_1)) \} = \min \{ V_Q(g(t_1)), V_{g^{-1}(Q)}(t_1) \}$

hence, $g^{-1}(Q)$ is a $\tilde{V} \subset NR$ of $M$.

Theorem 3.5: If ‘$g$’ is a homomorphism of $\tilde{A} \subset NR$, $M$ onto $\tilde{A} \subset NR$, $N$. If ‘$P$’ is a $\tilde{V} \subset NR$ of $M$ possess Supr. property, then the H-image of $P$, $g(P)$ is a $\tilde{V} \subset NR$ of $N$.

Proof: : Let $m, n \in N; \beta \in \Gamma$,

If either $g^{-1}(m) = 0$ or $g^{-1}(n) = 0$ then the proof is obviously true.

Let’s have a Supposition, i.e., neither $g^{-1}(m) \neq 0$ nor $g^{-1}(n) \neq 0$.

Let $m_0 \in g^{-1}(m)$ and $n_0 \in g^{-1}(n)$. Then $\exists \beta \in (m_0, n_0) = \sup_{g(P)}(a)$ where $a \in g^{-1}(m)$ and $V_P(n_0) = V_P(b)$ where $b \in g^{-1}(n)$

1. $V_{g(P)}(m-n) = \sup \{ V_P(r) \mid g^{-1}(m-n) \} = \sup \{ V_P(r) \mid g^{-1}(m-n) \} = \sup \{ V_P(r) \mid g^{-1}(m-n) \} = \sup \{ V_P(r) \mid g^{-1}(m-n) \}

2. $V_{g(P)}(m+n) = \sup \{ V_P(r) \mid g^{-1}(m+n) \} = \sup \{ V_P(r) \mid g^{-1}(m+n) \} = \sup \{ V_P(r) \mid g^{-1}(m+n) \}$

Hence we have shown that $g(P)$ is a $\tilde{V} \subset NR$ of $N$.

Theorem 3.6: Let $g$ be an epimorphism from a $\tilde{A} \subset NR$, $M$ onto a $\tilde{A} \subset NR$, $N$. Let $P$ be a $\tilde{V} \subset NR$ of $M$ possessing Supr. property. Then the H-image of a vagueset of $P$, $\tilde{V} \subset NR$ of $N$ possessing Supr. property.

Proof: : From theorem 3.5. $g(P)$ is a $\tilde{V} \subset NR$ of $N$.

Now, we prove that $g(P)$ possess sup. property.

Let $S \subseteq M$.

Since $g$ is epimorphism $g^{-1}(S) \neq \phi$.

According to definition, $V_{g(P)}(s) = \sup \{ V_P(m) \mid m \in g^{-1}(s) \}$.

Since $P$ possess sup. property and $g^{-1}(S) \subseteq M$, $\exists m_0 \in g^{-1}(S)$

$V_P(m_0) = \sup \{ V_P(m) \mid m \in g^{-1}(S) \} \sup \{ V_P(m) \mid m \in g^{-1}(S) \} \sup \{ V_P(m) \mid m \in g^{-1}(S) \}$

$\sup \{ V_P(m) \mid m \in g^{-1}(S) \}$
Let $V \rightarrow \mathbb{S}$ be a $\mathbb{N}$ possessing Supr. property. Then the pre-H-image $g^{-1}(Q)$ of $Q$ is a $\mathbb{V} \rightarrow \mathbb{N}$ of M possessing Supr. property. 

Proof. : From theorem 3.4, $g^{-1}(Q)$ is a $\mathbb{V} \rightarrow \mathbb{N}$ of M. We will prove that $g^{-1}(Q)$ possess Supremum- property. 

Let $T \subseteq M$. By definition, $V^{-1}_{\mathbb{N}}(m) = V_{\mathbb{G}}(m)$, $\forall m \in T$. Since $Q$ possess sup property, $\exists l_0 \in g(T) \ni V_{\mathbb{G}}(l_0) = sup\{V_{\mathbb{G}}(l) \mid l \in g(T)\}$. 

Also, $sup\{V_{\mathbb{G}}(l) \mid l \in T\} = sup\{V_{\mathbb{G}}(l) \mid l \in g(T)\}$. 

From (1) and (2), $V_{\mathbb{G}}(l_0) = sup\{V_{\mathbb{G}}(l) \mid l \in T\}$.

Hence proved that $g^{-1}(Q)$ possess Supr. property.

Definition 3.8: If $g(P) = P$, $P \in Aut(M)$ then a $\mathbb{S} \rightarrow \mathbb{N}$, P of M is said to be $CSA \rightarrow \mathbb{N}$.

Definition 3.9: If $g(P)(l) = V_{\mathbb{G}}(l)$, $\forall l \in M$, $P \in Aut(M)$ then a $\mathbb{V} \rightarrow \mathbb{N}$, P of M is said to be $VC\mathbb{A} \rightarrow \mathbb{N}$.

Lemma 3.10: Let P = $(t_0, t_0)$ be a $\mathbb{V} \rightarrow \mathbb{N}$ M and let $m \in M$. Then $V_{\mathbb{G}}(m) = [\alpha, \beta]$ if and only if $m \in P_{(\alpha, \beta)}$ and $\forall P_{(\alpha, \beta)} \ni \gamma \in [\alpha, \beta]$, $\gamma \in [0, 1]$. 

Proof. : The proof of lemma is clear.

Theorem 3.11: Let P = $(t_0, f_0)$ be a $\mathbb{V} \rightarrow \mathbb{N}$ M. Then P is $VC\mathbb{A} \rightarrow \mathbb{N}$ M if and only if its vaguecut of P or $(\delta, \epsilon)$ - cut, $P_{(\delta, \epsilon)}$, where $\delta, \epsilon \in [0, 1]$ is a $CSA \rightarrow \mathbb{N}$ M. 

Proof. : Let a $\mathbb{V} \rightarrow \mathbb{N}$ M. 

$P_{(\delta, \epsilon)}$ is a $VC\mathbb{A} \rightarrow \mathbb{N}$ M. 

We have to prove that $g(P_{(\delta, \epsilon)}) = P_{(\delta, \epsilon)}$. 

Let n $\in g(P_{(\delta, \epsilon)})$ 

$\rightarrow n = g(m)$, for some $m \in P_{(\delta, \epsilon)}$.

Now, $V_{\mathbb{G}}(n) = V_{\mathbb{G}}(m) = [\delta, \epsilon] \rightarrow n \in P_{(\delta, \epsilon)}$.

So, $g(P_{(\delta, \epsilon)}) \subseteq P_{(\delta, \epsilon)}$.

Let $n \in P_{(\delta, \epsilon)}$ because g is automorphism on M, we have n = g(m), for some $m \in P_{(\delta, \epsilon)}$. 

$\rightarrow [\delta, \epsilon] \subseteq P_{(\delta, \epsilon)}$.

We consider $g(P_{(\delta, \epsilon)}) = P_{(\delta, \epsilon)}$. 

Thus, $P_{(\delta, \epsilon)}$ is a $CSA \rightarrow \mathbb{N}$ M. 

By Converse, we have a supposition that $P_{(\delta, \epsilon)}$ is a $CSA \rightarrow \mathbb{N}$ M. 

Consider g $\in M$, $m \in M$. 

Let $V_{\mathbb{G}}(m) = [\delta, \epsilon]$. 

This implies $m \in P_{(\delta, \epsilon)}$.

Let $V_{\mathbb{G}}(m) = [\delta, \epsilon]$ if it is possible $[\delta, \epsilon] > [0, \delta]$.

Then $g(m) \in P_{(\delta, \epsilon)}$ be $g(P_{(\delta, \epsilon)})$ implies $m \in P_{(\delta, \epsilon)}$ is a $CSA \rightarrow \mathbb{N}$ M.

IV. CONCLUSION

In this research article, the idea of vague homomorphism of a $\mathbb{A} \rightarrow \mathbb{N}$ has been introduced and proved some interesting results on it. We proved that the homomorphic Pre- image and image of a $\mathbb{V} \rightarrow \mathbb{N}$ M is a $\mathbb{V} \rightarrow \mathbb{N}$ M. Also, we proved that pre-H-image and image of a $\mathbb{L} \rightarrow \mathbb{N}$ possessing Sup. property is a $\mathbb{V} \rightarrow \mathbb{N}$ M possessing Sup. property. Moreover, we proved that if P is a $VC\mathbb{A} \rightarrow \mathbb{N}$ M then it’s $(\delta, \alpha)$ - cut, $P_{(\alpha, \delta)}$ is also a $CSA \rightarrow \mathbb{N}$ M and vice versa.

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