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L – Fuzzy Ordered ‘Γ’- Semi rings

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Abstract— *‘Γ’- ring concept was introduced by Nobusawa which is the generalization of a ring. In this paper we studied the concept of L – Fuzzy Ordered ‘Γ’- Semi ring along with non-membership and membership functions whose values are taken from a complete lattice and some properties.*

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Key words : ordered ‘Γ’-semi ring, complete lattice, L-fuzzy sets, L-fuzzy ordered ‘Γ’-semi ring

I. INTRODUCTION

The approach of Fuzzy sets was invented by Zadeh, L.A[1] in the year 1965. Fuzzy sets which are using in different fields such as linguistics and clustering are special cases of L-relations. Goguen J.A [2] introduced L-fuzzy subset from a no empty subset to a complete lattice, L which is the generalization of fuzzy subset. The concept of generalization of a ring in ‘Γ’- ring was invented by Nobusawa [3] in 1961. M.K.Rao [4] studied the nation of a ‘Γ’- semi ring , ternary semiring and semiring. Majority of the researchers deduced many important results on ‘Γ’- Semi rings. Several authors studied fuzzy sets on ordered ‘Γ’- Semi rings, ideals, prime ideals and many more. Bhargavi, Y. et.al, [5] studied fuzzy ideals on ‘Γ’- Semi rings and deduced some important properties related to ‘Γ’- Semi rings. Bhargavi, Y. and Eswarlal, T, Nageswara Rao, B., Ramakrishana, N [6-9&15], studied the translates applications of vague sets in carier decision making and vague set in medical diagnosis. Vasavi, C. H et.al[10,11&14] deliberated fuzzy dynamic equations on time scales under second type hukuhara delta derivative. Bind et.al[12] discussed about the approach of Complete left ideals in ternary semi groups. Nayagam, V. L. G et.al [13] studied fuzzy multi-criteria decision-making. Srinuvasa Rao, B et.al [16] introduced some properties and generalization on α, β - rational contractions in ordered sb-metric spaces with applications. Bhargavi, Y [17] introduced Vague filters on ‘Γ’-semiring and some properties. Nayagam et.al [18] discussed about the application of trapezoidal intuitionistic fuzzy numbers in decision making problems. Seethe, M. P [19] studied some important results on Ideal theory in a-ternary semi groups. In the present paper we studied the perception of L – Fuzzy Ordered ‘Γ’- Semi rings of a ordered ‘Γ’- Semiring and we deduced there is a one to one mapping between L – Fuzzy Ordered ‘Γ’- Semi rings to the crisp Ordered Ordered’- Semi rings

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II. PRELIMINARIES

In the present part we discussed some basic concepts and definitions related to this article.

2.1 Definition:

For two additive commutative semi groups R and ‘Γ’, R is called ‘Γ’-semi ring if there exists a mapping RX’Γ’XR →R defined by p ∝ q for p, q ∈ R and ∝∈’Γ’ satisfying the below four conditions

1. p α (q + r) = p α q + q α r
2. (p + q) α r = p α r + q α r
3. p $(\alpha + \beta)$ r = p α r + q β r
4. p α (q β r) = (p α q) β r, ∀ p,q,r ∈ R, α, β ∈ ‘Γ’.

2.2 Definition:

For a ‘Γ’-semi ring, R, which is named as an ordered ‘Γ’-semi ring if it satisfying the relation ≤ i.e., ≤ is a partial ordering on R satisfies the below three conditions. If p ≤ q and r ≤ s then

(i)p+r ≤ q+s
(ii) p αr ≤ q α s
(iii) r α p ≤ s α q, ∀ p,q,r ∈ R, α ∈ ‘Γ’.

2.3 Definition:

Suppose a non-empty sub set S, of an ordered ‘Γ’-semi ring, R then S is called an ordered sub ‘Γ’-semi ring of R if (S,+) is sub semi ring of (R,+) and p α q ∈ S, α ∈ ‘Γ’.
Definition:
A partially ordered set is a complete lattice in which all sub sets have both supremum (join) and infimum (meet).

2.4 Definition:

A mapping: M → [0, 1] is called a fuzzy subset of a universe of discourse M.

2.5 Definition:

Suppose M is a non-empty set. A mapping μ : M → L is called a L-fuzzy subset of M.
2.6 Definition: An L-fuzzy subset μ of a set M and let t ∈ L. The set μ_t = { g ∈ M / μ (g) ≥ t } is called a level subset of μ. The set of all level subsets of μ is denoted by F_μ and F_μ = { μ_t / t ∈ im μ }.

2.7 Definition:

Two L-fuzzy subsets μ and σ

of a set M, then μ is said to be continued in, denoted by μ is subset of σ. If $\mu\,(g) \leq \sigma(g), \forall$ g∈ M. If $\mu\,(g) = \sigma(g), \forall$ g∈ M, then μ and σ are said to be equal.

2.8 Definition:

Let S be a subset of ordered ‘Γ’-semi ring R. The L-fuzzy set is the characteristic function of S taking values in L which is a given by

$$\delta_S(g) = \begin{cases} 1_L \text{ if } g \in \text{M} \\ 0_L \text{ if } g \text{ is not in M} \end{cases}$$

Then δ_S is a ‘L’- fuzzy characteristic function of Sin L.

III. L-FUZZY ORDERED ‘Γ’-SEMI RINGS & RESULTS

In this section we study the concept of L-fuzzy ordered ‘Γ’-semi ring along with non-membership and membership functions taking values in a complete lattice. Also we deduce that there a one-to-one correspondence between L-fuzzy ordered ‘Γ’-semi rings to the crisp ordered ‘Γ’-semi rings. In this section R stands for an ordered ‘Γ’-semi ring.

3.1 Definition:

A fuzzy set μ is said to be L-fuzzy ordered ‘Γ’-semi ring of R if for all g, h ∈ R , γ ∈ ‘Γ’.

(i) μ (g+h) ≥ μ(g) ∧ μ (h)

(ii) μ (gγh) ≥ μ(g) ∧ μ (h)

(iii) g ≤ h → μ(g) ≥ μ (h)

3.2 Example:

Suppose that R is the set of whole numbers (W) and let ‘Γ’ = {0,1 }. The mapping R, from M‘Γ’M R → R by p α q usual product of p, α, q $\forall\, p$,q ∈ R, α ∈ ‘Γ’. Then R is ordered ‘Γ’-semi ring.

Consider L = [0, 1] and define the mapping μ : R → L by

$$\mu\,(x) = \begin{cases} 0.7 \text{ if } g = 0 \\ 0.6 \text{ if } g \text{ is even} \\ 0.4 \text{ if } g \text{ is odd} \end{cases}$$

By this definition we can observe clearly μ is L-fuzzy ordered ‘Γ’-semi ring.

3.3 Theorem

An L-fuzzy subset μ of R is a L-fuzzy ordered ‘Γ’-semi ring if and only if its level set μ_t, t ∈ L is an ordered sub ‘Γ’-semi ring of R.

Proof:

Let us suppose that μ is an L-fuzzy ordered ‘Γ’-semi ring.

Let g, h ∈ μ_t and γ ∈ ‘Γ’.

⇒ μ (g) ≥ t, μ (h) ≥ t

⇒ μ(g) ∧ μ (h) ≥ t

⇒ μ (g+h) ≥ t and μ (gγh) ≥ t

⇒ g+h ∈ μ_t , gγh ∈ μ_t.

Then μ_t is ordered sub ‘Γ’-semi ring of R.

Conversely, μ_t is an ordered sub ‘Γ’-semi ring of R.

Now we show that μ is L-fuzzy ordered ‘Γ’-semi ring of R.

Let g, h ∈ R and γ ∈ ‘Γ’.

Suppose μ(g) = p and μ(h) = q.

Put t = a ∧ b.

Then μ (g) ≥ t, μ (h) ≥ t

⇒ g . h ∈ μ_t

⇒ g+ h ≥ μ_t and gγh ≥ μ_t.

⇒ μ (g+ h) ≥ t and μ (gγh)≥ t.

⇒ μ (g+ h) ≥ μ(g) ∧ μ (h) and μ (gγh)≥ μ(g) ∧ μ (h).

Let g ≤ h.

If possible, suppose that μ(g) < μ (h).

Then there exists t_1 ∈ L such that μ(g) < t_1 < μ (h).

Then h ∈ μ_{t_1} and x does not belongs to μ_{t_1}.

Which is a contradiction?

The contradiction arises our supposition is wrong.

Therefore μ (g) ≥ μ (h).

Thus μ is L-fuzzy ordered ‘Γ’-semi ring of R.

3.4 Theorem

Let S is a non- empty subset of an ordered ‘Γ’-semi ring R. Then δ_S is an L-fuzzy ordered ‘Γ’-semi ring of R if and only if S is an ordered sub ‘Γ’-semi ring of R.

Proof:

Suppose that δ_S is an L-fuzzy ordered ‘Γ’-semi ring of R.

Let g, h ∈ R and γ ∈ ‘Γ’

By the definition δ_S(g+h) ≥ $\delta_S(g) \wedge \delta_S(h)$ = 1_L

⇒ g . h ∈ S.

Also δ_S(gγh) ≥ $\delta_S(g) \wedge \delta_S(h)$ = 1_L

⇒ gγh ∈ S.

Then S is an ordered sub ‘Γ’-semi ring of R.

Conversely suppose that S is an ordered sub ‘Γ’-semi ring of R.

Let g, h ∈ R and γ ∈ ‘Γ’

g . h ∈ R and γ ∈ ‘Γ’.

If g . h ∈ S ⇒ g+h ∈S and gγh ∈ S.

⇒ δ_S(g+h) = 1_L = min { $\delta_S(g)$, $\delta_S(h)$ }and $\delta_S(g\gamma h)$ = 0_L = δ_S.(h)

If one of g or h are not in S then g+h and gγh are not in S.

⇒ δ_S(g+h) = 0_L = min { $\delta_S(g)$, $\delta_S(h)$ } and δ_S(gγh) = 0_L = $\delta_S(h)$.

Suppose that g ≤ h then we have μ (g) = μ (h).

Therefore μ (g) ≥ μ (h).

Thus δ_S L-fuzzy ordered ‘Γ’-semi ring of R.

3.5 Theorem

Let μ and σ be L-fuzzy ordered ‘Γ’-semi ring of R. Then $\mu \cap \sigma$ is a L-fuzzy ordered ‘Γ’-semi ring of R.

Proof:

Let g, h ∈ R and γ ∈ ‘Γ’.

($\mu \cap \sigma$) (g+h) = μ(g+h) ∧ σ (g+h).

≥ (μ(g) ∧ μ (h)) ∧ (σ(g) ∧ σ (h))

= (μ(g) ∧ σ (g)) ∧ (μ (h) ∧ σ (h))

=($\mu \cap \sigma$) (g) ∧ ($\mu \cap \sigma$) (g)

Also we have

($\mu \cap \sigma$) (gγh) = μ(gγh) ∧ σ (gγh).

≥ (μ(g) ∧ μ (h)) ∧ (σ(g) ∧ σ (h))

= (μ(g) ∧ σ (g)) ∧ (μ (h) ∧ σ (h))

=($\mu \cap \sigma$) (h) ∧ ($\mu \cap \sigma$) (h)

Moreover, let suppose that g ≤ h.

($\mu \cap \sigma$) = μ(g) ∧ σ (g) ≥ μ(h) ∧ σ (h).

Thus $\mu \cap \sigma$ is an L-fuzzy ordered ‘Γ’-semi ring of R.

3.6 Theorem

Two L-fuzzy ordered ‘Γ’-semi rings μ and θ of Rsuch that card Im $\mu < \infty$ and card Im $\theta < \infty$ are equal if and only if Im μ = Im θ and $F_\mu = F_\theta$.

Proof:

Suppose μ and θ are equal.

Let t ∈Im μ

↔ μ(g) = t.

↔ θ(g) = t. (because μ = θ).

↔ t ∈Im θ.

Therefore Im μ = Im θ.

Now

F_μ = { μ_t / t ∈Im μ }

= { θ_t / t ∈Im θ } = F_θ.

Therefore $F_\mu = F_\theta$.

Conversely suppose that Im μ = Im θ and $F_\mu = F_\theta$.

Let t ∈Im μ

t ∈Im θ

Then μ(g) = t and θ(g) = t.

i.e., μ(g) = t = θ(g) , ∀ g ∈ R.

⇒ $\mu = \theta$.

IV. CONCLUSIONS

In the present work we studied the concept of L – Fuzzy Ordered 'Γ'- Semi ring with membership and non-membership functions whose values are taken from a complete lattice and some important properties. Also we discussed L-fuzzy ordered 'Γ'-semi ring with suitable example.

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