L – Fuzzy Ordered ‘Γ’- Semi rings


Abstract— ‘Γ’- ring concept was introduced by Nobusawa which is the generalization of a ring. In this paper we studied the concept of L – Fuzzy Ordered ‘Γ’. Semi ring along with non-membership and membership functions whose values are taken from a complete lattice and some properties.

AMS Mathematics Subject Classification: 03B52

Key words : ordered ‘Γ’-semi ring, complete lattice, L-fuzzy sets, L-fuzzy ordered ‘Γ’-semi ring

I. INTRODUCTION


Revised Manuscript Received on November 19, 2019

T. Srinivasa Rao, “Koneru Lakshmaiah Education Foundation, vaddeswaram”, Guntur, A. P, India. (Email: tsr_2505@klniversity.in)
B. Srinivasa Kumar, “Koneru Lakshmaiah Education Foundation, vaddeswaram”, Guntur, AP, India. (Email: sk_bhavuresetty@klniversity.in)
S. Hanumantha Rao, “Vignan’s Foundation for Science, Technology and Research, Guntur, AP, India. (Email: sama.hanumantharao@gmail.com)
T. Nageswara Rao, “Koneru Lakshmaiah Education Foundation, vaddeswaram”, Guntur, A. P, India. (Email: tna@klniversity.in)
2.8 Definition: Let S be a subset of ordered 'Γ'-semi ring R. The L-fuzzy set is the characteristic function of S taking values in L which is a given by

\[ \delta_S(g) = \begin{cases} 1_L & \text{if } g \in M \\ 0_L & \text{if } g \text{ is not } \in M \end{cases} \]

Then \( \delta_S \) is a 'L'-fuzzy characteristic function of Sin L.

III. L-FUZZY ORDERED 'Γ'-SEMI RINGS & RESULTS

In this section we study the concept of L-fuzzy ordered 'Γ'-semi ring along with non-membership and membership functions taking values in a complete lattice. Also we deduce that there is a one-to-one correspondence between L-fuzzy ordered 'Γ'-semi rings to the crisp ordered 'Γ'-semi rings.

3.1 Definition: A fuzzy set \( \mu \) is said to be L-fuzzy ordered 'Γ'-semi ring of R if for all g, h \( \in R \), \( \gamma \in \Gamma \).

(i) \( \mu(g+h) \geq \mu(g) \land \mu(h) \)

(ii) \( \mu(gh) \geq \mu(g) \land \mu(h) \)

(iii) \( g \leq h \Rightarrow \mu(g) \geq \mu(h) \)

3.2 Example: Suppose that R is the set of whole numbers (W) and let \( \Gamma = \{0,1\} \). The mapping R, from \( \Gamma \)-\( \Gamma \)-R \( \rightarrow \) R by \( p \alpha q \) usual product of p, \( \alpha \), q \( \forall \) p,q \( \in \Gamma \). Then R is ordered 'Γ'-semi ring.

Consider L = [0,1] and define the mapping \( \mu : R \rightarrow L \) by

\[ \mu(x) = \begin{cases} 0.7 & \text{if } g = 0 \\ 0.6 & \text{if } g \text{ is even} \\ 0.4 & \text{if } g \text{ is odd} \end{cases} \]

By this definition we can observe clearly \( \mu \) is a L-fuzzy ordered 'Γ'-semi ring.

3.3 Theorem

An L-fuzzy subset \( \mu \) of R is a L-fuzzy ordered 'Γ'-semi ring if and only if its level set \( \mu_t \), t \( \in L \) is an ordered sub 'Γ'-semi ring of R.

Proof:

Let us suppose that \( \mu \) is an L-fuzzy ordered 'Γ'-semi ring. Let g, h \( \in \mu \) and \( \gamma \in \Gamma \).

\( \Rightarrow \mu(g) \geq t, \quad \mu(h) \geq t \)

\( \Rightarrow \mu(g) \land \mu(h) \geq t \)

\( \Rightarrow (g)(h) \geq t \quad \text{and} \quad \mu(gh) \geq t \)

\( \Rightarrow g+h \in \mu_t, \quad gh \in \mu_t \).

Then \( \mu_t \) is ordered sub 'Γ'-semi ring of R.

Conversely, \( \mu_t \) is an ordered sub 'Γ'-semi ring of R.

Now we show that \( \mu_t \) is L-fuzzy ordered 'Γ'-semi ring of R.

Let g, h \( \in R \) and \( \gamma \in \Gamma \).

Suppose \( \mu(g) = p \) and \( \mu(h) = q \).

Put \( t = a \land b \).

Then \( \mu(g) \geq t, \quad \mu(h) \geq t \)

\( \Rightarrow g+h \in \mu_t \)

\( \Rightarrow g+h \geq \mu_t \) and \( gh \geq \mu_t \)

\( \Rightarrow \mu(g+h) \geq t \quad \text{and} \quad \mu(gh) \geq t \)

\( \Rightarrow \mu(g+h) \geq \mu(g) \land \mu(h) \) and \( \mu(gh) \geq \mu(g) \land \mu(h) \).

Let \( g \leq h \).

If possible, suppose that \( \mu(g) < \mu(h) \).

Then there exists \( t_1 \in L \) such that \( \mu(g) < t_1 < \mu(h) \).

Then \( h \in \mu_{t_1} \) and \( x \) does not belong to \( \mu_{t_1} \).

Which is a contradiction?

The contradiction arises our supposition is wrong.

Therefore \( \mu(g) \geq \mu(h) \).

Thus \( \mu \) is L-fuzzy ordered 'Γ'-semi ring of R.

3.4 Theorem

Let S is a non-empty subset of an ordered 'Γ'-semi ring R. Then \( \delta_S \) is an L-fuzzy ordered 'Γ'-semi ring of R if and only if S is an ordered sub 'Γ'-semi ring of R.

Proof:

Suppose that \( \delta_S \) is an L-fuzzy ordered 'Γ'-semi ring of R.

Let g, h \( \in R \) and \( \gamma \in \Gamma \).

By the definition \( \delta_S(g+h) = \delta_S(g) \land \delta_S(h) \).

\( \Rightarrow g \cdot h \in S \Rightarrow g \cdot h \in S \) and \( gh \in S \).

\( \Rightarrow \delta_S(gh) = 0_L = \delta_S(h) \).

If there exists \( t \) \( \in \Gamma \).

Thus \( \delta_S(g+h) = \delta_S(g) \land \delta_S(h) \).

Which is a contradiction?

Therefore \( \mu(g) \geq \mu(h) \).

Thus \( \mu \) is L-fuzzy ordered 'Γ'-semi ring of R.

3.5 Theorem

Let \( \mu \) and \( \sigma \) be L-fuzzy ordered 'Γ'-semi ring of R. Then \( \mu \cap \sigma \) is a L-fuzzy ordered 'Γ'-semi ring of R.

Proof:

Let g, h \( \in R \) and \( \gamma \in \Gamma \).

\( (\mu \cap \sigma)(g+h) = (\mu(g)+h) \land (\sigma(g)+h) \geq (\mu(g) \land \mu(h)) \land (\sigma(g) \land \sigma(h)) \geq (\mu \cap \sigma)(g) \land (\mu \cap \sigma)(h) \).

Also we have

\( (\mu \cap \sigma)(gh) = (\mu(gh)) \land (\sigma(gh)) \geq (\mu(gh) \land \mu(h)) \land (\sigma(gh) \land \sigma(h)) \geq (\mu \cap \sigma)(h) \land (\mu \cap \sigma)(h) \).

Moreover, let suppose that \( g \leq h \).

\( (\mu \cap \sigma) = (\mu(g) \land \sigma(g)) \geq (\mu(h) \land \sigma(h)) \).

Thus \( \mu \cap \sigma \) is an L-fuzzy ordered 'Γ'-semi ring of R.

3.6 Theorem

Two L-fuzzy ordered 'Γ'-semi rings \( \mu \) and \( \theta \) of R such that card Im \( \mu < \infty \) and card Im \( \theta < \infty \) are equal if and only if Im \( \mu = \theta = F_{\mu} = F_{\theta} \).

Proof:

Suppose \( \mu \) and \( \theta \) are equal.
Let $t \in \text{Im } \mu$

$\leftrightarrow \mu(g) = t$.

$\leftrightarrow \theta(g) = t$ (because $\mu = \theta$).

$\leftrightarrow t \in \text{Im } \theta$.

Therefore $\text{Im } \mu = \text{Im } \theta$.

Now

$F_{\mu} = \{ \mu_t / t \in \text{Im } \mu \} = \{ \theta_t / t \in \text{Im } \theta \} = F_{\theta}$.

Therefore $F_{\mu} = F_{\theta}$.

Conversely suppose that $\text{Im } \mu = \text{Im } \theta$ and $F_{\mu} = F_{\theta}$.

Let $t \in \text{Im } \mu$

$t \in \text{Im } \theta$

Then $\mu(g) = t$ and $\theta(g) = t$.

i.e., $\mu(g) = t = \theta(g)$, $\forall g \in \mathbb{R}$.

$\Rightarrow \mu = \theta$.

IV. CONCLUSIONS

In the present work we studied the concept of $L – \text{Fuzzy Ordered } '\Gamma' – \text{Semi ring with membership}$ and non-membership functions whose values are taken from a complete lattice and some important properties. Also we discussed $L$-fuzzy ordered $'\Gamma'$-semi ring with suitable example.

REFERENCES