International Journal of Recent Technology and Engineering (IJRTE)
ISSN: 2277-3878, Volume-8 Issue-4, November 2019

Couple Stress Impacton S-Wavesin an Anisotropic Pre-Stressed Medium under Gravity and Magnetic Field

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Abstract— The gravity fieldand magnetic field impact with the inclusion of couple stress r_s onshear waves in a transversely isotropic elastic solid medium is studied. Thedispersion equations for the particular problem have been obtained. Severalparticular cases are also examined. The numerical calculations are carriedout for specific rigidity parameter A, density parameter C, anisotropic factor $\overline{N}$, initial stress parameter $\overline{P}$, gravity parameter G, magnetic fieldparameter $\overline{H}$, couple stress parameter $\Re$ as well as wave number κ and theoutcomes are visually represented by graph.

Keywords: incompressible, anisotropic, gravity field, pre-stressed, couplestress, magnetic field..

I. INTRODUCTION

In 2004, A. M. Abd-Alla., et.al. [1] examined the Rayleigh waves in anelasto-magnetic pre-stressed orthotropic material of half space under gravityfield. Abd-Alla, A.M., et.al. [2] discussed the gravity field impact on S-wavesin a non-homogeneous, incompressible and initially stressed anisotropic solidmedium. In 1965, Biot [3] observed that the initial stresses in the solidmedium have eminence effect on the reproduction of elastic waves and acomparative interpretation of a stratified medium by an extended structurewith anisotropic features. Das., et.al. [6] have investigated the effect ofinitial stresses in the propagation of edge waves in homogeneous isotropicplates. In his study, compound structures of thinly stratified materials weredeliberated and used in the study of twisting and oscillations. The spreading of "Edge Waves" in an anisotropic pre-stressed plate of limited thicknessand absolute length was investigated and the dispersion equations for theedge waves were derived by Dey, S., P. K. De.,[7]. The propagation natureof the elastic waves in a layered stratified medium under both initial andcouple stresses was discussed by Pal Roy [8]. Also P. Pal Roy examinedthe dynamics of a stratified medium of Maxwell type solids and weaknessof the surface of a pre-stressed laminated material. Pijush Pal Roy., andLokenathDebnath., [9] considered about the development of edge waves ina stratified pre-stressed medium with stress couples. Mainly the work wasbased on the comparative interpretation of a stratified medium by an equivalent pre-stressed anisotropic continuation with couple stresses and concludedthat for a particular compression, the couple stress increases the speed ofwave propagation with the increment in wave numbers. But in general theincrement in wave numbers diminishes the speed of the wave. Selim [11]inspected the thermal stress impact on development of transverse waves anddebilitation of seismic body waves in pre-stressed dissipative medium undercouple stresses.

In this work, an attempt has been made to find the velocities of propagation, damping of S-waves in a non-homogeneous, anisotropic, incompressible and initially stressed medium under the presence of magneto-gravity field with couple stress r_s. Also the dispersion equations for this particularproblem have been obtained to determine the S-wave velocity using linearinhomogeneities and outcomes are visually represented by graph

II. FORMULATION OF THE PROBLEM

A stratified medium of n thin layers and of total thickness H underthe common initial compressive stress P on the $x-$ direction is considered.Suppose that the $y-$ axis is normal to the plane of the laminations, therelationship between stress-strain in the composite medium are given by [3]

$$s_{11} = 2Ne_{xx} + s \qquad (2.1a)$$
$$s_{22} = 2Ne_{yy} + s \qquad (2.1b)$$
$$s_{12} = 2Qe_{xy} \qquad (2.1c)$$

where s_{11} *and* s_{22} are the components of principal stress along x and y directions respectively,

s_{12} is the shear stress component in the xy -plane and

$$s_{33} = s_{13} = s_{23} = 0 \text{ and } s = \frac{1}{2}(s_{11} + s_{22}) \qquad (2.1d)$$

The bending rigidity of the medium taken into accounts by introducingcouple stresses r_s [4], the total bending moment Dof the medium is

Manuscript published on November 30, 2019.
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$$D = r_s \frac{\partial^2 u_2}{\partial x^2} \quad (2.1e)$$

. where $\vec{H}$ is the magnetic field intensity, $\vec{B}$ is the magnetic induction, $\vec{E}$ isthe electric field intensity, $\vec{J}$ is the current density vector, $\vec{D}$ is thedisplacement current vector, c is the speed of the light in vacuum, μ_e is themagnetic permeability and ε_e is the permittivity respectively.

Limiting the analysis to the plane strain only. The modified dynamical equations of equilibrium in $x-y$ plane of the composite anisotropic mediumunder the impact of couple stress are given by [5] and [9]for the present problem

$$\frac{\partial s_{11}}{\partial x} + \frac{\partial s_{12}}{\partial y} - P\frac{\partial \omega}{\partial y} - \rho g \frac{\partial u_2}{\partial x} = \rho \frac{\partial^2 u_1}{\partial t^2} \quad (2.2a)$$

$$\frac{\partial s_{12}}{\partial x} + \frac{\partial s_{22}}{\partial y} - P\frac{\partial \omega}{\partial x} + \rho g \frac{\partial u_1}{\partial x} + \mu_e H_0^2 \left(\frac{\partial^2 u_2}{\partial x^2} - \frac{\partial^2 u_1}{\partial x \partial y}\right) = \rho \frac{\partial^2 u_2}{\partial t^2} + r_s \frac{\partial^4 u_2}{\partial x^4} \quad (2.2b)$$

where ρ is the density, g is the acceleration due to gravityand ω is the component of the local

rotation perpendicular to $x-y$ plane

$$\omega = \frac{1}{2}\left(\frac{\partial u_2}{\partial x} - \frac{\partial u_1}{\partial y}\right) \quad (2.3)$$

The components of incremental strain are given by [1]

$$e_{xx} = \frac{\partial u_1}{\partial x},\ e_{yy} = \frac{\partial u_2}{\partial y},\ e_{xy} = \frac{1}{2}\left(\frac{\partial u_2}{\partial x} + \frac{\partial u_1}{\partial y}\right) \quad (2.4)$$

The condition of incompressibility $e_{xx} + e_{yy} = 0$ is satisfied by

$$u_1 = -\frac{\partial \varsigma}{\partial y} \ and \ u_2 = \frac{\partial \varsigma}{\partial x} \quad (2.5)$$

where $\varsigma = \varsigma(x, y, t)$.

III. SOLUTION OF THE PROBLEM

Applying the equations from(2.1a) to (2.1d) and (2.3) to (2.5) in (2.2a) and (2.2b), we obtain

$$\frac{\partial s}{\partial x} - 2N\frac{\partial^3 \varsigma}{\partial x^2 \partial y} + \frac{\partial}{\partial y}\left[Q\left(\frac{\partial^2 \varsigma}{\partial x^2} - \frac{\partial^2 \varsigma}{\partial y^2}\right)\right] - \frac{P}{2}\left(\frac{\partial^3 \varsigma}{\partial x^2 \partial y} + \frac{\partial^3 \varsigma}{\partial y^3}\right) = \rho\left(g\frac{\partial^2 \varsigma}{\partial x^2} - \frac{\partial^3 \varsigma}{\partial t^2 \partial y}\right) \quad (3.1)$$

$$\frac{\partial s}{\partial y} + \left(Q - \frac{P}{2} + \mu_e H_0^2\right)\left(\frac{\partial^3 \varsigma}{\partial x^3} + \frac{\partial^3 \varsigma}{\partial x \partial y^2}\right) - 2(Q-N)\frac{\partial^3 \varsigma}{\partial x \partial y^2} - \rho g \frac{\partial^2 \varsigma}{\partial x \partial y} = \rho \frac{\partial^3 \varsigma}{\partial t^2 \partial x} + r_s \frac{\partial^5 \varsigma}{\partial x^5} \quad (3.2)$$

Assuming the inhomogeneities

$$Q = Q_{11}(1+ay),\ N = N_{11}(1+by),\ \rho = \rho_{11}(1+cy) \quad (3.3)$$

where Q_{11} and N_{11} are rigidities and ρ_{11} is the density in homogeneous isotropic generalized medium.

Applying the equation(3.3) in (3.1) and (3.2)

$$\left\{\begin{aligned}&\left[Q_{11}(1+ay) + \frac{P}{2}\right]\frac{\partial^4 \varsigma}{\partial y^4} - \left[Q_{11}(1+ay) - \frac{P}{2} + \mu_e H_0^2\right]\frac{\partial^4 \varsigma}{\partial x^4} + \\ &\left[4N_{11}(1+by) - 2Q_{11}(1+ay) + \mu_e H_0^2\right]\frac{\partial^4 \varsigma}{\partial x^2 \partial y^2} + \left[2bN_{11} - aQ_{11}\right]\frac{\partial^3 \varsigma}{\partial x^2 \partial y} \\ &+ aQ_{11}\frac{\partial^3 \varsigma}{\partial y^3} - \rho_{11}(1+cy)\left[\frac{\partial^4 \varsigma}{\partial t^2 \partial x^2} + \frac{\partial^4 \varsigma}{\partial t^2 \partial y^2}\right] + \rho_{11} c\left(g\frac{\partial^2 \varsigma}{\partial x^2} - \frac{\partial^3 \varsigma}{\partial t^2 \partial y}\right) - r_s \frac{\partial^6 \varsigma}{\partial x^6}\end{aligned}\right\} = 0 \quad (3.4)$$

The solution of equation (3.4) for propagation of sinusoidal waves at anydirectionis

$$\varsigma(x, y, t) = M_1 e^{ik(x\cos\phi + y\sin\phi - v_1 t)} \quad (3.5)$$

where ϕ is the angle of propagation with the $x-$ axis, v_1 is the S-wavevelocity and k is the wave number.

Applying equation(3.5) in (3.4)

$$\left(\frac{v_1}{\beta}\right)^2 = \frac{1}{1+cy}\left\{\begin{aligned}&\left[(1+ay) + \frac{P}{2Q_{11}}\right]\sin^4\phi + \left[(1+ay) - \frac{P}{2Q_{11}} + \mu_e H_0^2\right]\cos^4\phi \\ &+\left[\frac{4N_{11}}{Q_{11}}(1+by) - 2(1+ay) + \mu_e H_0^2\right]\sin^2\phi\cos^2\phi - \frac{gc}{k^2\beta^2}\cos^2\phi + \frac{r_s k^2}{Q_{11}}\cos^6\phi\end{aligned}\right\} \quad (3.6)$$

$$\left(\frac{v_1}{\beta}\right)^2 = \left[\frac{2bN_{11}}{cQ_{11}} - \frac{a}{c}\right]\cos^2\phi + \frac{a}{c}\sin^2\phi \quad (3.7)$$

where β is the S-wave velocity in generalized homogeneous isotropic medium.

The equation (3.6) represents the square of the phase velocity V_R and equation (3.7)represents the square of the damping velocity V_I of the shear waves.

IV INSPECTION OF PARTICULAR PROBLEM IN GENERAL HOMOGENEOUS MEDIUM:

(i)Investigation of equation (3.6):

Case A: When $a \to 0$, Q is homogeneous i.e.,Constant rigidity along vertical direction.

$$\left(\frac{v_1}{\beta}\right)^2 = \frac{1}{1+cy}\left\{\begin{aligned}&\left[1 + \frac{P}{2Q_{11}}\right]\sin^4\phi + \left[1 - \frac{P}{2Q_{11}} + \mu_e H_0^2\right]\cos^4\phi \\ &+\left[\frac{4N_{11}}{Q_{11}}(1+by) - 2 + \mu_e H_0^2\right]\sin^2\phi\cos^2\phi - \frac{gc}{k^2\beta^2}\cos^2\phi + \frac{r_s k^2}{Q_{11}}\cos^6\phi\end{aligned}\right\} \quad (4.1)$$

When $\cos\phi = 1, \sin\phi = 0$, the velocity along (horizontal) $x-direction$ is

$$v_{11}^2 = \frac{\beta^2}{1+cy}\left[1 - \frac{P}{2Q_{11}} + \mu_e H_0^2 - \frac{gc}{k^2\beta^2} + \frac{r_s k^2}{Q_{11}}\right] \quad (4.2)$$

Equation (4.2) depends on the couple stress coefficient.

When $\cos\phi = 0, \sin\phi = 1$, the velocity along (vertical) $y-direction$ is

$$v_{22}^2 = \frac{\beta^2}{1+cy}\left[1 + \frac{P}{2Q_{11}}\right] \quad (4.3)$$

Case B: When $b \to 0$, N is homogeneous i.e.,Constant rigidity along horizontal direction.

$$\left(\frac{v_1}{\beta}\right)^2 = \frac{1}{1+cy}\left\{\begin{aligned}&\left[(1+ay) + \frac{P}{2Q_{11}}\right]\sin^4\phi + \left[(1+ay) - \frac{P}{2Q_{11}} + \mu_e H_0^2\right]\cos^4\phi \\ &+\left[\frac{4N_{11}}{Q_{11}} - 2(1+ay) + \mu_e H_0^2\right]\sin^2\phi\cos^2\phi - \frac{gc}{k^2\beta^2}\cos^2\phi + \frac{r_s k^2}{Q_{11}}\cos^6\phi\end{aligned}\right\} \quad (4.4)$$

When $\cos\phi = 1, \sin\phi = 0$, the velocity along (horizontal) $x-direction$ is

$$v_{11}^2 = \frac{\beta^2}{1+cy}\left[(1+ay) - \frac{P}{2Q_{11}} + \mu_e H_0^2 - \frac{gc}{k^2\beta^2} + \frac{r_s k}{Q_1}\right.$$

Retrieval Number: D5332118419/2019©BEIESP
DOI:10.35940/ijrte.D5332.118419
Journal Website: www.ijrte.org

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Blue Eyes Intelligence Engineering
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Equation (4.5) depends on the average couple stress coefficient.

When $\cos\phi = 0, \sin\phi = 1$, the velocity of propagation along (vertical) $y - direction$ i

$$v_{22}^2 = \frac{\beta^2}{1+cy}\left[(1+ay) + \frac{P}{2Q_{11}}\right] \tag{4.6}$$

$$v_{22}^2 = \frac{\beta^2}{1+cy}\left[(1+ay) + \frac{P}{2Q_{11}}\right] \tag{4.6}$$

The velocity along vertical direction increases for $P > 0$ and the waves are dispersive.

Case C: WhenN($b \to 0$), Q($a \to 0$) and ρ ($c \to 0$) are homogeneous

$$\left(\frac{v_1}{\beta}\right)^2 = \begin{Bmatrix} \left[1 + \frac{P}{2Q_{11}}\right]\sin^4\phi + \left[1 - \frac{P}{2Q_{11}} + \mu_e H_0^2\right]\cos^4\phi \\ + \left[\frac{4N_{11}}{Q_{11}} - 2 + \mu_e H_0^2\right]\sin^2\phi\cos^2\phi + \frac{r_s k^2}{Q_{11}}\cos^6\phi \end{Bmatrix} \tag{4.7}$$

The velocity equation when the initial stress is absent becomes

$$\left(\frac{v_{11}}{\beta}\right)^2 = \begin{Bmatrix} \sin^4\phi + \left[1 + \mu_e H_0^2\right]\cos^4\phi \\ + \left[\frac{4N_{11}}{Q_{11}} - 2 + \mu_e H_0^2\right]\sin^2\phi\cos^2\phi + \frac{r_s k^2}{Q_{11}}\cos^6\phi \end{Bmatrix} \tag{4.8}$$

When $\cos\phi = 1, \sin\phi = 0$, the velocity along (horizontal) $x - direction$ is

$$v_{11}^2 = \beta^2\left[1 + \mu_e H_0^2 + \frac{r_s k^2}{Q_{11}}\right] \tag{4.9}$$

When $\cos\phi = 0, \sin\phi = 1$, the velocity along (vertical) $y - direction$ is

$$v_{22}^2 = \beta^2$$

When $\cos\phi = 0, \sin\phi = 1$, the velocity along (vertical) $y - direction$ is

$$v_{22}^2 = \beta^2$$

In this case, the anisotropy does not affect the velocity.

(ii)Analysis of Equation (3.7):

Case A: When $a \to 0$, Q is homogeneous

i.e., Constant rigidity along vertical direction.

$$\left(\frac{v_1}{\beta}\right)^2 = \frac{2bN_{11}}{cQ_{11}}\cos^2\phi \tag{4.13}$$

i.e., the S-wave velocity is damped

When $\cos\phi = 1, \sin\phi = 0$, the velocity along (horizontal) $x - direction$ is

$$\left(\frac{v_{22}}{\beta}\right)^2 = 0 \tag{4.15}$$

Case B: When $b \to 0$, N is homogeneous

i.e., Constant rigidity along the horizontal direction.

$$\left(\frac{v_1}{\beta}\right)^2 = -\frac{a}{c}\cos^2\phi + \frac{a}{c}\sin^2\phi \tag{4.16}$$

When $\cos\phi = 1, \sin\phi = 0$, the velocity along (horizontal) $x - direction$ is

$$\left(\frac{v_{11}}{\beta}\right)^2 = -\frac{a}{c} \tag{4.17}$$

The velocity in horizontal direction is damped by $-\frac{a}{c}$.

When $\cos\phi = 0, \sin\phi = 1$, the velocity along (vertical) $y - direction$ is

$$\left(\frac{v_{22}}{\beta}\right)^2 = \frac{a}{c} \tag{4.18}$$

Case C: When N($b \to 0$), Q($a \to 0$) both are homogeneous

$$\left(\frac{v_1}{\beta}\right)^2 = 0 \tag{4.19}$$

V MATHEMATICAL ANALYSIS AND DISCUSSION RESULTS

Consider the non-dimensional parameters to get numerical data on the phase velocity V_R of shear waves in the non-homogeneous pre-stressed medium:

$A = \frac{a}{b}$ (rigidityparameter), $C = \frac{c}{b}$ (density parameter), $\overline{N} = \frac{N_{11}}{Q_{11}}$ (anisotropy factor), $B = by$ (depth), $V_R = \frac{v_1}{\beta}$ (Shear wave velocity), $\overline{P} = \frac{P}{2Q_{11}}$ (initial stress factor), $\overline{H} = \frac{\mu_e {H_0}^2}{Q_{11}}$ (magnetic field parameter), $G = \frac{gb}{k^2\beta^2}$ (gravity parameter), $\Re = \frac{r_s}{Q_{11}}$ (couple stress parameter).

Using these parameters in the equation (3.8)

$$V_R^2 = \frac{1}{1+BC}\begin{Bmatrix} \left[(1+AB) + \overline{P}\right]\sin^4\phi + \left[1 + AB - \overline{P} + \overline{H}\right]\cos^4\phi \\ + \left[4\overline{N}(1+B) - 2(1+AB) + \overline{H}\right]\cos^2\phi\sin^2\phi - CG\cos^2\phi + \Re k^2\cos^6\phi \end{Bmatrix}$$

The outcomes are plotted for the parametersA=4; B=0 to 6;C=0.8; $\overline{P}$ =0.5; $\overline{N}$ =0.5; $\Re$ =0.08; k=1; $\overline{H} = 0.5$; G=0.2given by [11] and [9].

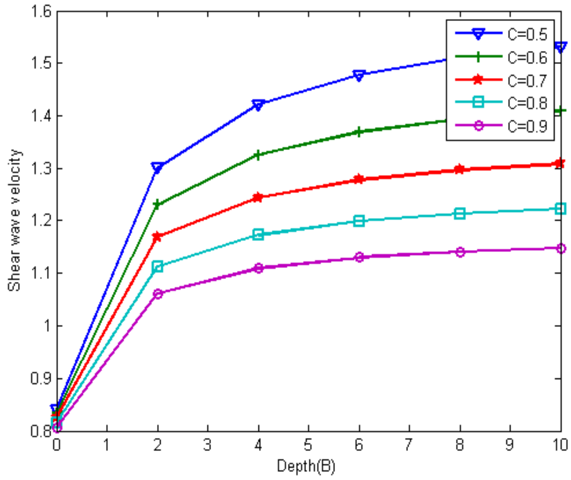

Figure 1: Changes in wave velocity at $\phi = 30^\circ$ with depth B and distinctdensity parameters $C = 0.5, 0.6, 0.7, 0.8, 0.9$.

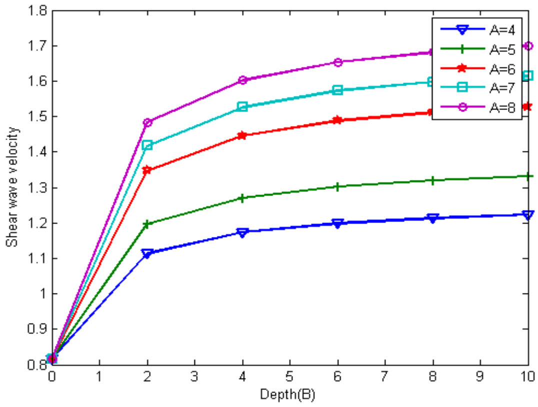

Figure 2: Changes in wave velocity at $\phi = 30^\circ$ with depth B and distinctrigidity parameter $A = 4, 5, 6, 7, 8$.

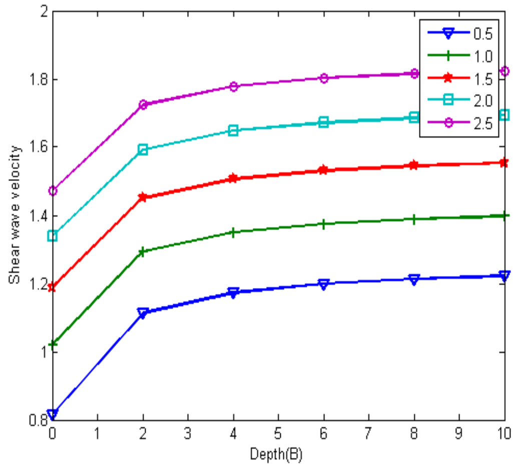

Figure 3: Changes in wave velocity at $\phi = 30^\circ$ with depth B and distinctanisotropy parameter $\overline{N} = 0.5, 1, 1.5, 2, 2.5$.

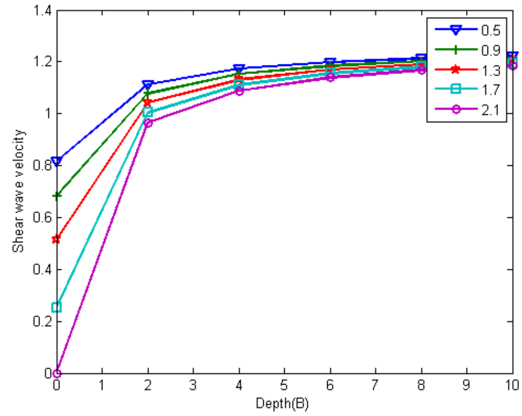

Figure 4: Changes in wave velocity at $\phi = 30^\circ$ with depth B and distinctinitial stress parameter $\overline{P} = 0.5, 0.9, 1.3, 1.7, 2.1$.

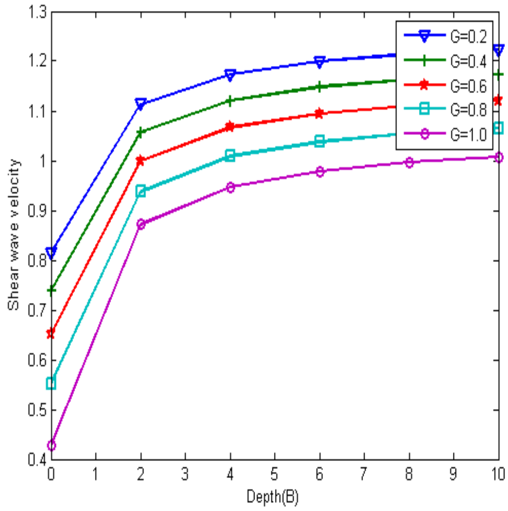

Figure 5: Changes in wave velocity at $\phi = 30^\circ$ with depth B and distinctgravity parameter $G = 0.2, 0.4, 0.6, 0.8, 1$.

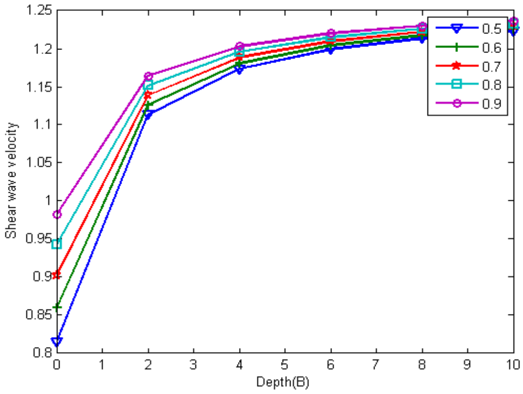

Figure 6: Changes in wave velocity at $\phi = 30^\circ$ with depth B and distinctmagnetic field parameter $\overline{H} = 0.5, 0.6, 0.7, 0.8, 0.9$.

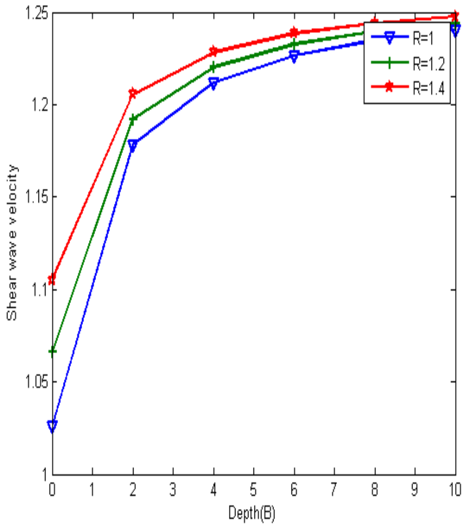

Figure 7: Changes in wave velocity at $\phi = 30^\circ$ with depth B and distinctcouple stress parameter $\Re = 1, 1.2, 1.4$.

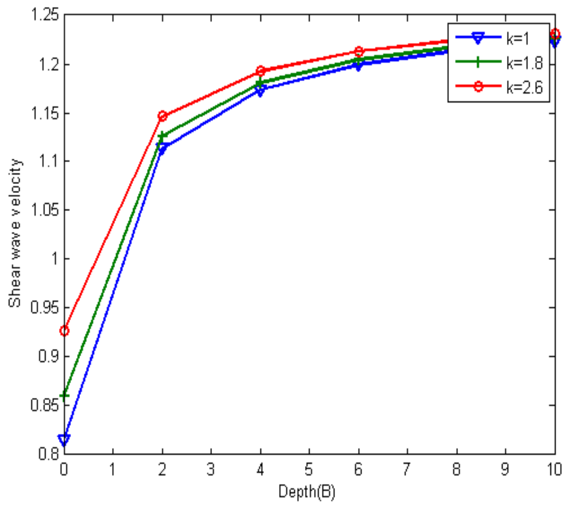

Figure 8: Changes in wave velocity at $\phi = 30^\circ$ with depth B and distinctwave numbers $\kappa = 1, 1.8, 2.6$.

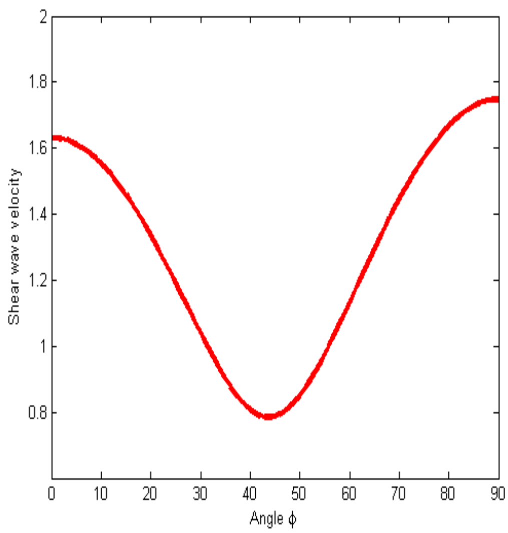

Figure 9: Changes in wave velocity at different angle.

VI CONCLUSION

The velocity V_R of the S-wave increases when the rigidity parameter A, magnetic field parameter $\overline{H}$, wave number κ as well as anisotropy parameter $\overline{N}$ increaseswith the inclusion of couple stress parameter $\Re$. Also the wave velocity V_1 decreases when the density parameter C, gravity parameter G as well as theinitial stress parameter $\overline{P}$ increaseswith the inclusion of couple stress parameter $\Re$. Finally, it is observed that the velocity decreases within the range $\phi = (0^\circ, 45^\circ)$, increases within the range $\phi = (46^\circ, 90^\circ)$ and alsothe increasing values of couple stress r_s increases the S-wave velocity..

REFERENCES

1. A. M. Abd-Alla, A. H. Hammad, Abo-Dahab, Rayleigh waves in magneto elastic half-space of orthotropic material under influence of initial stress and gravity field, Applied Mathematics and Computations, 154:583-597 (2004).
2. A. M. Abd-Alla, S. R. Mahmoud, S. M. Abo-Dahab, M. I. Helmy, Propagation of S-wave in a non-homogeneous anisotropic incompressible and initially stressed medium under influence of gravity field, Applied Mathematics and Computation, 217: 4321-4332 (2011).
3. M. A. Biot, Mechanics of Incremental Deformations, John Wiley andSons, New York, 1965.
4. M. A. Biot, A new approach to the mechanics of orthotropic multilayered plates, Int. J. Solids Struct. 8: 475-490 (1972).
5. M. A. Biot, Simplified dynamics of multilayered orthotropic viscoelastic plates, Int. J. Solids Struct. 8: 490-509 (1972).
6. S. C. Das, and S. Dey, Edge waves under initial stress, Appl. Sci. Res.22:382-389 (1970).
7. S. Dey, P. K. De, Edge wave propagation in an incompressible anisotropic initially stressed plate of finite thickness, International Journal of Computational Cognition, 7(3): 55-60 (2009).
8. P. Pal Roy, Wave Propagation in a Thinly Two-layered Laminated Medium with Stress Couples Under Initial Stresses, ActaMech, 54: 1-21 (1984).
9. Pijush Pal Roy, LokenathDebnath, Propagation of Edge Waves in a Thinly Layered Laminated Medium With Stress Couples Under Initial Stresses, Journal of Applied Mathematics and Simulation, 1(4): 271-286 (1988).
10. Rajneesh Kakar and ShikhaKakar, S-wave propagation in a non-homogeneous initial
11. stressed elastic medium under the effect of magnetic field, International Journal of
12. Current Research and Review, 4(24): 131-139 (2012).
13. M. M. Selim, Thermal Effects on Propagation of Transverse Waves in Anisotropic
14. Incompressible Dissipative Pre-Stressed Plate, Appl. Math. Inf. Sci., 10(3): 1091-1095 (2016).