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Recognizability of Tetrahedral Picture Languages

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Abstract— In two dimensions, tiling the plane plays a vital role. Many picture generative models were attempted to tile the three dimensional space. A. Dharani et al. [6] introduced a new theoretical picture generative model to tile a three dimensional space using tetrahedral tile in two different ways namely Sequential Space Filling Grammar (SSFG) and Parallel Space Filling Grammar (PSFG). Local and recognizable tetrahedral picture languages are introduced in this paper and some of its properties are studied.

Keywords : Tetrahedral tiles, picture languages, local and recognizable tetrahedral picture languages.

I. INTRODUCTION

Generation of picture languages using various techniques such as grammars, automata, pasting systems have been studied since the seventies in Pattern recognition and image analysis [7,8]. Gimmarresi and Restivo[2] have introduced the recognizability of two dimensional picture languages as a projection of local picture languages. M. Latteux and Simplot [9] have introduced hv-local picture languages in which horizontal and vertical dominoes are involved.

K.S. Dersanambika et al. in [3] introduced the recognizability of hexagonal picture languages. Extending these ideas to triangular tiles T. Kalyani et al. [4] have introduced the recognizability of iso picture languages.

Extending these ideas to three dimensions, recognizability of three dimensional rectangular picture languages have also been introduced by D.G. Thomas et al. [1]. In [5, 6], they have introduced a new kind of picture generation using polyhedral. Space filling grammar has been defined and new patterns of picture generation using tetrahedral tiles are discussed. Here, we introduce local and recognizable tetrahedral picture languages for two types of production rules (i) vertex to edge and (ii) edge to vertex. In this paper we deal with tetrahedral pictures generated in a single layer and only the base of the tetrahedral is taken into account.

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II. PRELIMINARIES & RESULTS

We review the notion of Parallel Space Filling Grammar (PSFG) [6] ,which is used to tile a three dimensional space using polyhedral in a parallel manner.

Definition 2.1 [10] A Parallel Space Filling Grammar (PSFG) is a 5-tuple (S, S_0, Σ, P, C) where S is the set of three dimensional polyhedral. $S_0 \in$ S is the initial polyhedron. Σ is the set of alphabets representing vertices v, edge positions e_p, face positions f_p or any combination of these in a three dimensional polyhedron. P is the set of production rules of any of the following types

(i) $v \to v$ (ii) $e_p \to e_p$ (iii) $f_p \to f_p$ (iv) $v \to e_p$ (v) $f_p \to e_p$ (vi) $v \to f_p$ (vii) $e_p \to v$ (viii) $e_p \to f_p$ (ix) $f_p \to v$. C is the control language over P. The family of all possible 3D pictures generated by PSFG is denoted by L^*-PSFG. The subset of L^*-PSFG is called picture language L -PSFG.

Definition 2.2.[10] A tetrahedral tile is a polyhedral which has four vertices, four faces and six edges. Each face is an equilateral triangle. f_4 is the base of the tetrahedron. The directions along the vertices v_1, v_2 and v_3 are denoted by D_1, D_2 and D_3 respectively.

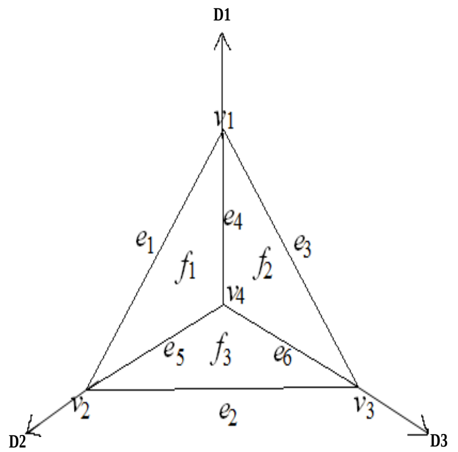
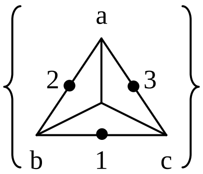


Fig. 1. A tetrahedron

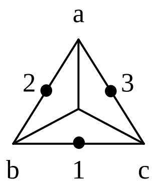
Definition 2.3. A tetrahedral picture is a picture generated by PSFG where S is the set of regular tetrahedron. S^{**T} is the set of all tetrahedral pictures over the set S. A tetrahedral picture language over S is a subset of S^{**T}.

Definition 2.4. Let the number of tetrahedral tiles along the directions D_1, D_2 and D_3 be l,m,n, then the size of the tetrahedral picture is denoted by (l,m,n). If l=m=n, then the size of the tetrahedral picture is m.

Example 2.1. We illustrate definition 2.1 by considering the base of the regular tetrahedral in particular. Consider a PSFG (S, S_0, Σ, P, C) where

S =


S contains a regular tetrahedron in which a, b, c represents vertices and 1, 2, 3 represent edge mid points.

S_0=


is the initial tetrahedron.

$\Sigma = \{a, b, c, 1, 2, 3\}$.

$P = \{a \to 1, b \to 3, c \to 2\}$ is a vertex to edge midpoint production rule.

$C = \{(P)^{2n-1}, n \geq 1\}$

The three dimensional tetrahedral picture language generated by this PSFG is given below.

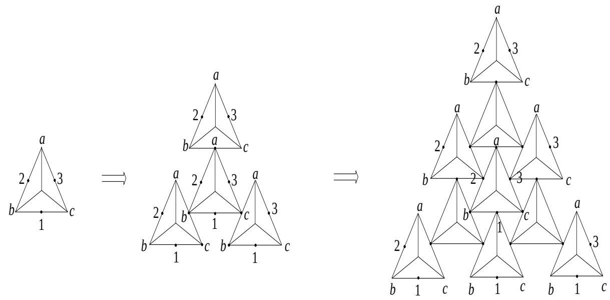
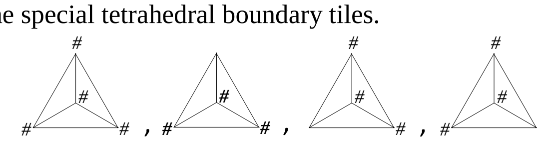


Fig. 2. A tetrahedral picture of size 3.

The number of tetrahedral tiles in a tetrahedral picture can be calculated using a centered polygon number [11]. A tetrahedral picture for the production rule $P = \{v \to e_p\}$ of size 'm' will have $\frac{1}{2}(3m^2 - 3m + 2)$ tetrahedral tiles.

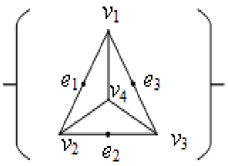
Definition 2.5. If $p \in S^{**T}$, then a picture $\hat{p}$ is the tetrahedral picture obtained by surrounding p of size 'm' with the special tetrahedral boundary tiles.


$\notin S$

The size of $\hat{p}$ is m+1.

$B_m(p)$ denotes the set of all sub pictures of p of size m.

Example 2.2. A tetrahedral picture over the set

S =


with the production rule $P = \{v \to e_p\}$ surrounded by the special boundary tiles is shown in the following figure.

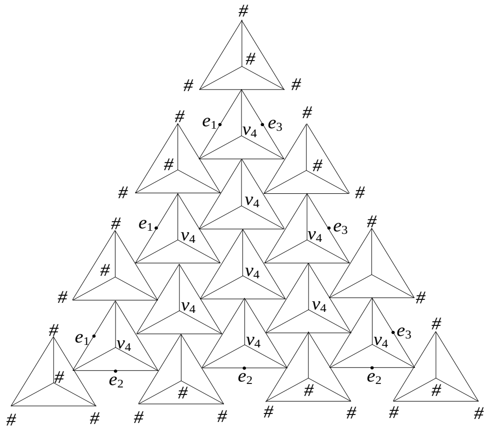
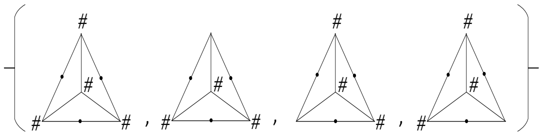


Fig. 3. A tetrahedral picture $\hat{p}$ for a tetrahedral picture p of size 3.

III. RECOGNIZABILITY OF TETRAHEDRAL PICTURE LANGUAGES OVER $v \to e_p$

3.1. Local tetrahedral picture languages over $v \to e_p$

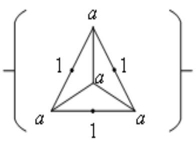
Definition 3.1. Let S be a finite set of regular tetrahedral tiles. A tetrahedral picture language over $(v \to e_p)$ $L \subseteq S^{**T}$ is called local if there exists a finite set Δ of tetrahedral pictures of size 2 over the set of tiles

$S \cup$


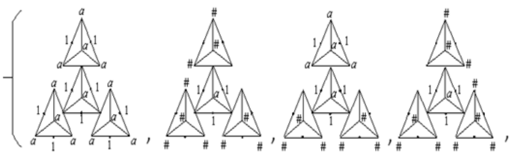
such that $L = \{p \in S^{**T} / B_2(\hat{p}) \subseteq \Delta\}$. $B_2(\hat{p})$ denotes the set of all sub pictures of $\hat{p}$ of size 2. The family of local tetrahedral picture languages will be denoted by

TrLoc($v \to e_p$).

Example 3.1. Let

S =


Where the edge midpoints are considered only for the base of the tetrahedral. Let Δ be a set of tetrahedral pictures of size 2 over S. The set Δ is shown in the following figure.

Δ =


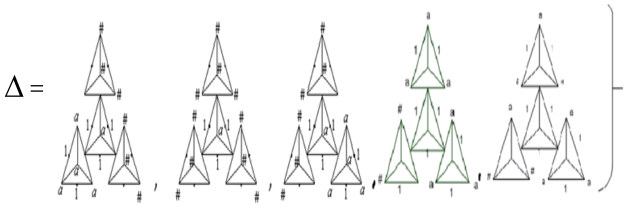


Fig. 4. Tetrahedral pictures of size 2

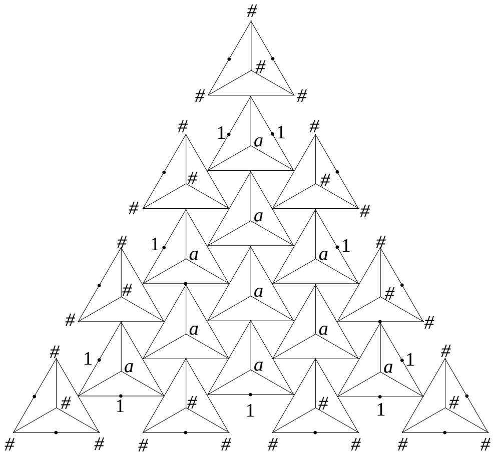
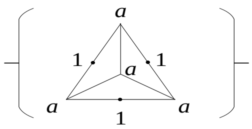


Fig. 5. A tetrahedral picture over S =



The language L = L (Δ) is the language of tetrahedral pictures. An element of which is shown in Fig.5. The tetrahedral picture language over two letter alphabets with all sides of equal length is not local, since Δ also generates tetrahedral pictures of not equal size, i.e., sides of tetrahedral pictures of different length.

Another example of a local tetrahedral picture language is given below.

Example 3.2. Let

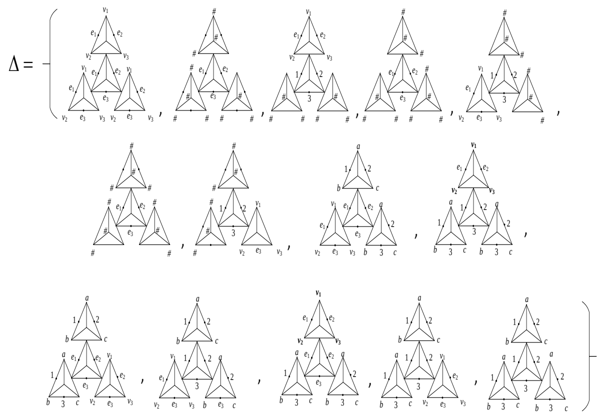
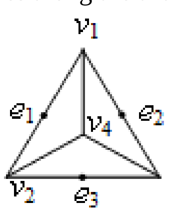
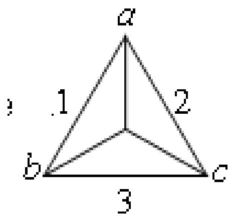


Fig. 6. Tetrahedral pictures of size 2

The language L = L (Δ) is the language of tetrahedral pictures. A member of which is shown in the following figure. The tetrahedral tiles along the triad are



And the tetrahedral tiles in the remaining places are



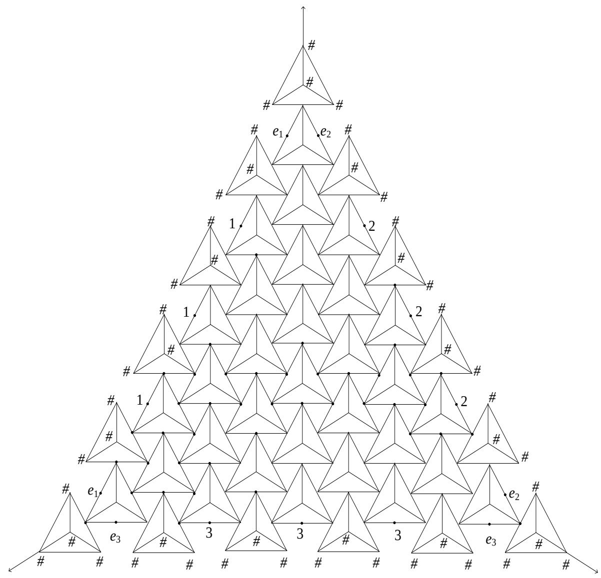


Fig. 7. A tetrahedral picture of the language in Example 3.2

Applying a coding that replaces the vertex set $\{v_1, v_2, v_3, a, b, c\}$ by {a} and the edge set $\{e_1, e_2, e_3, 1, 2, 3\}$ by {1}. We obtain a tetrahedral picture language over the vertex set {a} and edge set {1} with equal sides.

Definition 3.2. Let S and S′ be two finite sets of tetrahedral tiles and let Γ and Σ be the finite set of vertices and edges of tetrahedral tiles in S′ and S respectively. Let $\pi : \Gamma \rightarrow \Sigma$ be a mapping. The projection of the tetrahedral picture language p $\in S^{**T}$ is the tetrahedral picture language $p' \in S'^{**T}$ such that $\pi(\alpha) = v$, $\pi(\beta) = e$, that is every vertex ´α´ of the tetrahedral tiles in the set S′ is replaced by the vertex ´v´ of the tetrahedral tiles in the set S and each and every edge label ´β´ of the tetrahedral tiles in the set S′ is replaced by the edge label ´e´ of the tetrahedral tiles in the set S.

Definition 3.3. Let $L \subseteq S'^{**T}$ be a tetrahedral picture language. The projection by mapping π of L is the language $L' = \{p' \,/\, p' = \pi(p), \forall\, p \in L\} \subseteq S^{**T}$.

Definition 3.4. Let S be a finite set of tetrahedral tiles. A tetrahedral language $L \subseteq S^{**T}$ is called recognizable tetrahedral language over the production rule $v \rightarrow e_p$ if there exists a local tetrahedral picture language L ′ over the set of tetrahedral tiles S′ and a mapping $\pi : \Gamma \rightarrow \Sigma$ such that $L = \pi(L')$,where Γ and Σ contains the edge labels and vertices of tetrahedral tiles in S and S′ respectively.

Example 3.3. Let Σ = {a, 1} where a is the vertex and 1 is the edge label of tetrahedral tiles over the set S. Let L be the language of tetrahedral picture languages of equal sides. Here we consider the local language L′ of example 3.2 and apply the coding $\pi : \Gamma \rightarrow \Sigma$ given by $\pi(v_1) = \pi(v_2) = \pi(v_3) = \pi(a) = \pi(b) = \pi(c) = a$ and $\pi(e_1) = \pi(e_2) = \pi(e_3) = \pi(1) =$

$\pi(2) = \pi(3) = 1$. Thus we have L= π(L′) and L is a recognizable tetrahedral picture language.

The family of all recognizable tetrahedral picture languages will be denoted by TrREC.

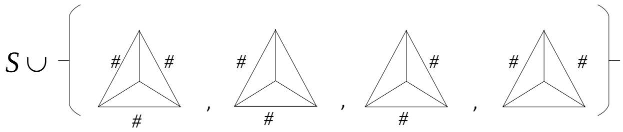
Definition 3.5. A tetrahedral tiling system T is a 6-tuple (S, S', Σ, Γ, π, θ) where Sand S' are finite sets of tetrahedral tiles and Σ and Γ are two finite sets of symbols representing vertices v, edge positions e_p, and face positions f_p are any combinations of these. π : Γ→Σ is a projection and θ is a set of tetrahedral pictures of size 2 over the alphabet Γ∪ {#}.

Definition 3.6. The tetrahedral picture language $L \subseteq S^{**T}$ is tiling recognizable if there exists a tetrahedral tiling system T = (S, S', Σ, Γ, π, θ) such that L = π(L′ (θ)). TrREC is exactly the family of tetrahedral picture languages recognizable by tetrahedral tiling systems (TrTS).

IV. LOCAL AND RECOGNIZABLE TETRAHEDRAL PICTURE LANGUAGES OVER THE PRODUCTION RULE $e_p \rightarrow v$

In this section we consider local and recognizable tetrahedral picture languages over the rule $e_p \rightarrow$ v.

Definition 4.1. Let S be a finite set of tetrahedral tiles. A tetrahedral picture language L over the production rule $e_p \rightarrow$ v is called local if there exists a finite set Δ of tetrahedral pictures of size 2 over the set of tiles



such that L = {p ∈ S^{**T} / $B_2(\hat{p}) \subseteq \Delta$}. $B_2(\hat{p})$ denotes the set of all sub pictures of $\hat{p}$ of size 2. The family of all tetrahedral local picture languages will be denoted by TrLOC ($e_p \rightarrow$ v).

Example 4.1. Consider the set Δ as in the following figure

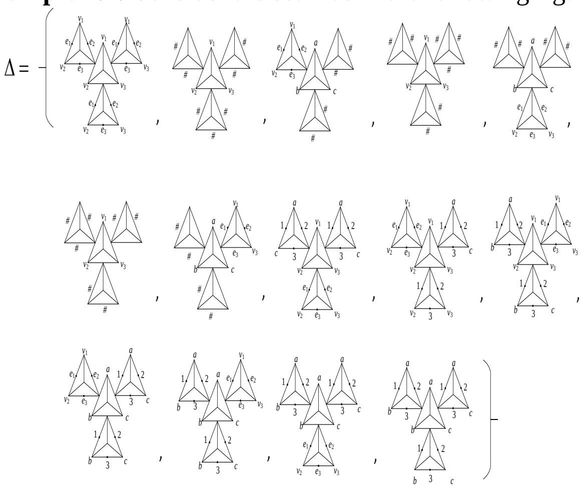


Fig. 8. Tetrahedral pictures of size 2

The language L = L (Δ) is the language of tetrahedral pictures a member of which is shown in the following figure.

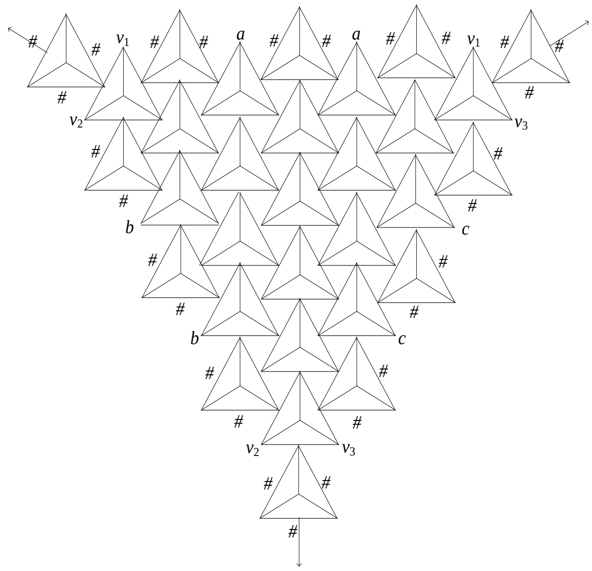
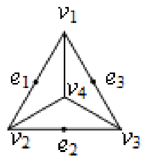
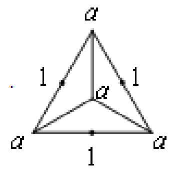


Fig. 9. A tetrahedral picture of the language in Example 4.1

The language L = L (Δ) is the local language, where the tetrahedral tiles along the triad are



and the tetrahedral tiles on the remaining places are



Applying a coding that replaces the vertex set {v_1, v_2, v_3, a, b, c} by {a} and the edge set {e_1, e_2, e_3, 1, 2, 3} by {1}. We obtain a tetrahedral picture language over the vertex set {a} and edge set {1} with equal sides which is not local.

Definition: 4.2 Let S be a finite set of tetrahedral tiles. A tetrahedral language L ⊆ S**T is called recognizable tetrahedral language over the production rule $e_p \rightarrow$v if there exists a local tetrahedral picture language L′ over the set of tetrahedral tiles S′ and a mapping π: Γ→Σ such that L= π(L′), where Γ and Σ are the finite alphabets containing the vertices and edge labels of tiles in S′ and S.

Example : 4.2 Let Σ = {a, 1} where a is the vertex and 1 is the edge label of tetrahedral tiles over the set S. Let L be the language of tetrahedral picture languages of equal sides. Here we consider the local language of example 4.1 and apply the coding π : Γ→Σ given by $\pi(v_1) = \pi(v_2) = \pi(v_3) = \pi(a) = \pi(b) = \pi(c) = a$ and $\pi(e_1) = \pi(e_2) = \pi(e_3) = \pi(1) = \pi(2) = \pi(3) = 1$. Thus we have L = π(L ′) and L is a recognizable tetrahedral language over the production rule $e_p \rightarrow$ v.

Theorem : 4.1 The family TrREC is closed under projection.

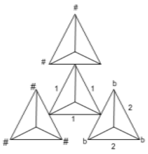
Proof. Let $L_1 \subseteq S_1^{**T}$, $L_2 \subseteq S_2^{**T}$ be two tetrahedral picture languages such that $L_2 = \Phi(L_1)$, where $\Phi: \Sigma_1 \rightarrow \Sigma_2$ is a projection. We have to prove that, if L_1 is recognizable then L_2 is recognizable too.

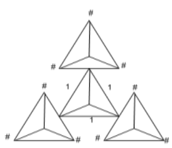
If L_1 is recognizable, then there exists a local language $L' \subseteq S_3^{**T}$ and a projection π: $\Gamma \to \Sigma_1$ such that $L_1 = \pi(L')$. Therefore the language can be obtained as $L_2 = \psi(L')$ where $\psi = \Phi \circ \pi$: $\Gamma \to \Sigma_2$ and hence L_2 is recognizable.

Theorem : 4.2 The family TrREC is closed under right and left catenations with the production rules $v \to e_p$ and $e_p \to v$.

Proof : Let $L_1 \subseteq S_1^{**T}$ and $L_2 \subseteq S_2^{**T}$ be two recognizable tetrahedral picture languages over an alphabet Σ and let $L = L_1 (/) L_2$ where L_1 and L_2 are catenated in the right of L_1 by the production rule $v \to e_p$.

Let $(S, \Gamma_1, \Theta_1, \pi_1)$ and $(S', \Gamma_2, \Theta_2, \pi_2)$ be the representations of L_1 and L_2 respectively where Γ_1 and Γ_2 are disjoint. A representation $(S'', \Gamma, \Theta, \pi)$ for L can be constructed as follows : $\Gamma = \Gamma_1 \cup \Gamma_2$. The set Θ has to contain all elements from Θ_1 but those corresponding to the right borders. More over we should add some internal tetrahedral tiles corresponding to the sides where the catenation is made. Such tetrahedral tiles contain pieces of the right border of pictures in L_1 and pieces of left border of pictures in L_2. Hence the underlying local language is defined by the following set of tiles : $\Theta = \Theta_1 \cup \Theta_2 \cup \Theta_3 \backslash \Theta_4$ where,

$\Theta_3 = \Big\{$  $, a,1 \in \Gamma_1, b,2 \in \Gamma_2 \Big\}$

$\Theta_4 = \Big\{$  $, a,1 \in \Gamma_1 \Big\}$

V. CONCLUSION

In this paper we have defined local and recognizable tetrahedral languages over the production rules $v \to e_p$ and $e_p \to v$. This work can also be extended to other production rules of PSFG. Also this work can be extended from single layer to multiple layers.

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