

Economic and Mathematical Model for Size and Structure Optimisation of Predator and Prey Populations



A.P. Kaledin, Yu.A. Yuldashbaev, T.S. Kubatbekov, A.I. Filatov, A.M. Ostapchuk, V.M. Makeeva, M.V. Stepanova, U.A. Shergaziev

Abstract: The paper proposes an original economic and mathematical model for size and structure optimisation of Predator and Prey populations.

The most well-known mathematical model in biology for periodical dynamics of antagonistic animal species was developed independently by Alfred Lotka and Vito Volterra. This classical mathematical Predator-Prey model is known as the Lotka-Volterra model.

Keywords: Lotka-Volterra model, economic and mathematical modelling, animals.

I. INTRODUCTION

The problem setting is as follows. A closed ecological area is home to two antagonistic animal species (Predator and Prey). Prey feeds on plants available in unlimited abundance. Predator only lives off the above specific species of Prey. The task is to determine the dynamics of the Predator and Prey populations in the given ecosystem. The model assumes the probability of encounters between Predator and Prey to increase with Prey population growth, which is also followed, after a certain time lag, by increases of the Predator population. In the given setting, this classical model describes some of the scenarios of interactions between the Predator and Prey populations in nature.

The Lotka-Volterra model is widely covered in literature; therefore, there is no point to discuss it in this paper.

In modelling the actual relations between antagonistic animal species, this classic model presents several challenges. The classical Predator-Prey model is structurally unstable, as even a minor change of the right side of one of the equations may fundamentally change its phase profile. The significant wave pattern of the Predator and Prey population dynamics makes it difficult to compare model results with empirical data. Predator's mono diet means the model is a single-factor model (one Predator species and one Prey species), which significantly reduces the potential of recreating actual natural relations. It leaves no scope for Predator-Prey population size or structure optimisation in line with a set criterion. Moreover, there is no possibility to set resource constraints and take into account the sex-age structure of Predator-Prey systems.

II. PROPOSED METHODOLOGY

A. Economic and mathematical model for size and structure optimisation of Predator and Prey populations

The economic and mathematical model for size and structure optimization of Predator and Prey populations (Predator-Prey EMM) has a block-diagonal structure. Figure 1 presents the structural scheme of the Predator-Prey EMM.

Manuscript published on November 30, 2019.

* Correspondence Author

A.P. Kaledin*, Russian State Agrarian University named after K.A. Timiryazev, Moscow, Russia.

Yu.A. Yuldashbaev, Russian State Agrarian University named after K.A. Timiryazev, Moscow, Russia.

T.S. Kubatbekov, Russian State Agrarian University named after K.A. Timiryazev, Moscow, Russia.

A.I. Filatov, Russian State Agrarian University named after K.A. Timiryazev, Moscow, Russia.

A.M. Ostapchuk, Russian State Agrarian University named after K.A. Timiryazev, Moscow, Russia.

V.M. Makeeva, Moscow State University named after M.V. Lomonosov, Moscow, Russia.

M.V. Stepanova, Yaroslavl State Agricultural Academy, Yaroslavl, Russia.

U.A. Shergaziev, Kyrgyz national agrarian university named after k. I. Skryabin

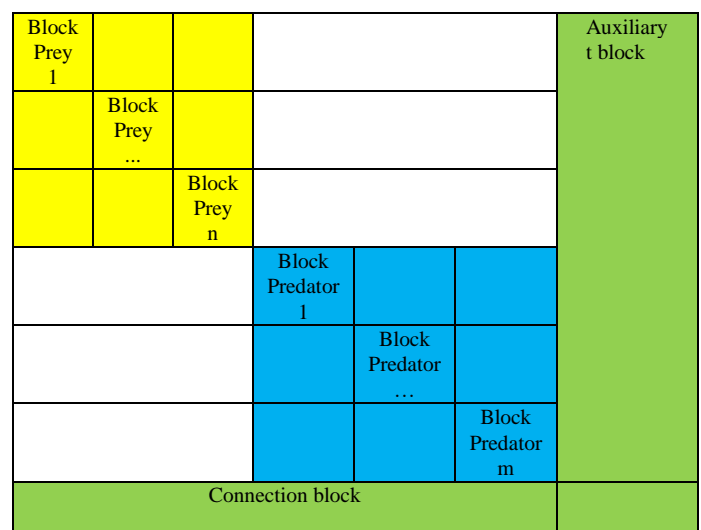


Fig.1: Structural scheme of the Predator-Prey EMM

© The Authors. Published by Blue Eyes Intelligence Engineering and Sciences Publication (BEIESP). This is an open access article under the CC-BY-NC-ND license <http://creativecommons.org/licenses/by-nc-nd/4.0/>



Economic and Mathematical Model for Size and Structure Optimisation of Predator and Prey Populations

The blocks Prey 1...n make the subsystem Prey and the blocks Predator 1...m make the subsystem Predator, which are combined in a system by the auxiliary and connection blocks.

Each of the blocks in the Prey or Predator subsystems is based on the game species population turnover block with directions of use of the population and produce.

B. Game species population turnover block

The economic and mathematical model for population turnover optimisation of game animals belongs to structural models reflecting the dynamics of sex-age groups within a set period [1]-[3]. As long as the model is a block within the Predator-Prey EMM, it does not contain several variables and constraints, as well as the target function.

The system of variables of the model is represented by a group of variables of time averages (annual averages) of the population by sex-age population groups. The variables of the population and produce distribution in dynamics are added (official harvesting, illegal hunting and losses to predation).

The model constraint system includes the following groups:

- by the relation of productive animals (females) and born animals;
- by the relation of sexes in young and old groups;

- by the relation of proximate age groups for females and males;
- by the relation of productive animals and animals of old age groups for females and males;
- product constraints by population dynamics and products of population management.

Table 1 shows the structural scheme of a segment of the population turnover block of game animals.

Legend:

W is the yield of born young animals per annual (time average) head of productive animals (females);

Cto is the coefficient of animal turnover for the age group calculated as the ratio of duration of the given period (year) to the life expectancy of the animal of the given age group;

Cp is the coefficient of persistence of the given age group;

Cd is the coefficient of decrease for the age group calculated as 1 minus the coefficient of persistence;

Cl is the coefficient of female load per male;

Table 1: Structural scheme of the game species population turnover block

Constraints	Time averages (annual averages) of the population, head												Constraint type	Constraint scope	
	Productive animals		Old group		Medium group 2		Medium group 1		Young group		Born animals 0-2				
	males	females	males	females	males	females	males	females	males	females	males	females			
	X ₁	X ₂	X ₃	X ₄	X ₅	X ₆	X ₇	X ₈	X ₉	X ₁₀	X ₁₁	X ₁₂			
Relation of productive animals (females) and born animals		W										C _{to}	C _{to}	≤	0
Relation of sexes in young groups												1	-1	=	0
Relation of sexes in old groups	Cl		1											≤	0
Females	Relation of proximate age groups											C _{to}	C _{to} * C _p	≤	0
	Relation of proximate age groups							C _{to}				C _{to} * C _p		≤	0
	Relation of proximate age groups								C _{to}			C _{to} * C _p		≤	0
	Relation of proximate age groups											C _{to} * C _p		≤	0
Males	Relation of proximate age groups											C _{to}	C _{to} * C _p	≤	0
	Relation of proximate age groups								C _{to}			C _{to} * C _p		≤	0
	Relation of proximate age groups									C _{to}		C _{to} * C _p		≤	0
	Relation of proximate age groups											C _{to}	C _{to} * C _p	≤	0



Table 3: Fragment of the Elk block of the numerical EMM

Model		Average annual population, head						Head			Meat for Wolf, kg	
		Age groups of Elk						Total	Projected harvesting, head	Illegal hunting		Wolves
		current year's brood 0-1	yearlings 1-2	two-year-olds 2-3	adults 3-9	adults 9-15						
	x1	x2	x3	x4	x5	x6	x7	x8	x9			
Elk	Born young animals	y1	15		-9.45	-1.575	-1.575					
	current year's brood 0-1	y2	-14.25	15								
	yearlings 1-2	y3		-14.25	15			0.4	0.4	0.4		
	two-year-olds 2-3	y4			-14.250	2.50		0.4	0.4	0.4		
	adults 3-9	y5				-2.375	2.5		0.2	0.2	0.2	
	9 and older	y6					1					
	Total	y7	1	1	1	1	1	-1				
	Projected harvesting, head	y8						0.03	-1			
	Illegal hunting, head	y9						0.12		-1		
	Elk meat for wolves, kg	y10									280	-1

Table 4: Fragment of the Boar block of the numerical EMM

Model		Average annual population, head					Head			Meat for Wolf, kg	
		Age groups of Boar					Total	Projected harvesting, head	Illegal hunting		Wolves
		current year's brood 0-1	piglets 1-2	adults 2-7	adults 7-12						
	x11	x12	x13	x14	x15	x16	x17	x18			
Boar	Born young animals	y11	12		-4.32	-4.32					
	current year's brood 0-1	y12	-11.4	12					0.4		
	piglets 1-2	y13		-11.400	2.400			0.75	0.75	0.4	
	adults 2-7	y14			-2.28	2.4		0.25	0.25	0.2	
	adults 7-12	y15				1					
	Total	y16	1	1	1	1	-1				
	Projected harvesting, head	y17					0.15	-1			
	Illegal hunting, head	y18					0.30		-1		
	Boar meat for wolves, kg	y19								44	-1

Table 5: Fragment of the White Hare block of the numerical EMM

Model		Average annual population, head			Head			White Hare meat for wolves, kg	
		Age groups of White Hare			Total	Projected harvesting, head	Illegal hunting		Wolves
		young hares 0-1	animals 1-6	animals 6-11					
	x20	x21	x22	x23	x24	x25	x26	x27	



White Hare	Born young animals	y20	11	-6.16	-6.16					
	young hares 0-1	y21	-10.45	2.20			0.50	0.50	0.75	
	animals 1-6	y22		-2.09	2.2		0.50	0.50	0.25	
	animals 6-11	y23			1					
	Total	y24	1	1	1	-1				
	Projected harvesting, head	y25				0.04	-1			
	Illegal hunting, head	y26				0.15		-1		
	White Hare meat for wolves, kg	y27							2.625	-1

Table 6: Fragment of the Brown Hare block of the numerical EMM

Model		Age groups of Brown Hare					Head			Brown Hare meat for wolves, kg
		young hares 0-1	animals 1-8	animals 8-15	Total	Projected harvesting, head	Illegal hunting	Wolves		
		x28	x29	x30	x31	x32	x33	x34	x35	
Brown Hare	Born young animals	y20	15	-6.857	-6.857					
	young hares 0-1	y21	-14.25	2.14			0.50	0.50	0.75	
	animals 1-6	y22		-2.035	2.142		0.50	0.50	0.25	
	animals 6-11	y23			1					
	Total	y24	1	1	1	-1				
	Projected harvesting, head	y25				0.04	-1			
	Illegal hunting, head	y26				0.15		-1		
	Brown Hare meat for wolves, kg	y27							3.25	-1

Table 7: Fragment of the Beaver block of the numerical EMM

Model		Average annual population, head					Head			Meat for Wolf, kg	
		Age groups of Beaver					Total	Projected harvesting, head	Illegal hunting		Wolves
		current year's brood 0-1	yearlings 1-2	two-year-olds 2-3	adults 3-10	adults 10-17					
		x36	x37	x38	x39	x40					
Beaver	Born young animals	y36	17		-11.9	-1.7	-1.7				
	current year's brood 0-1	y37	-16.15	17							

Economic and Mathematical Model for Size and Structure Optimisation of Predator and Prey Populations

	yearlings 1-2	y3 8		-16.15	17					0.3	0.3	0.5	
	two-year-olds 2-3	y3 9			-16.15	2.429				0.3	0.3	0.4	
	adults 3-10	y4 0				-2.30 7	2.429			0.4	0.4	0.1	
	adults 10-17	y4 1					1						
	Total	y4 2	1	1	1	1	1	-1					
	Projected harvesting, head	y4 3						0.03	-1				
	Illegal hunting, head	y4 4						0.09		-1			
	Beaver meat for wolves, kg	y4 5										12.7	-1

Table 8: Fragment of the Wolf block of the numerical EMM

Model	Average annual population, head							Total	Wolf's meat requirements, kg
	Age groups of Wolf								
	young wolves 0-1	yearlings 0-1	adults 2-3	adults 3-5	adults 5-8	adults 8-12			
	x46	x47	x48	x49	x50	x51	x52		
Born young animals	y46	12		-27	-13.5	-9	-6.75		
young wolves 0-1	y47	-6	12						
yearlings 0-1	y48		-11.4	12					
adults 2-3	y49			-11.4	6				
adults 3-5	y50				-5.7	4			
adults 5-8	y51					-3.8	3		
adults 8-12	y52						1		
Total	y53	1	1	1	1	1	1	-1	
Wolf's meat requirements, kg	y54	300	450	600	900	1200	1350		-1

The connection block of the numerical EMM is the Wolf feed (diet) constraint, which connects all blocks of the Prey subsystem and the block of the Predator subsystem.

The target function is the population persistence measure of game animals given a decline in losses due to official

harvesting, illegal hunting and losses to Wolf. For populations of the Prey subsystem bounded above and populations of the Predator subsystem bounded below, this criterion allows to calculate the optimum structure and size of each population.

III. RESULT ANALYSIS

A. Analysis of A Solution of The Predator-Prey Emm

Base possibility

The base possibility calculations under the Predator-Prey EMM assume losses to Wolf at 5% for Elk and 3% for Boar.

Age groups of Wolf	young wolves 0-1	yearlings 0-1	adults 2-3	adults 3-5	adults 5-8	adults 8-12	Total
Average annual Wolf population	2.2	1.1	1.0	2.0	2.8	1.0	10.0
Annual population, head	2.2	1.1	1.0	1.0	0.9	0.3	
Annual meat requirements per one wolf, kg	300	450	600	900	1,200	1,350	
Meat per Wolf population, kg	650.9	488.1	618.3	1,762.2	3,348.2	1,350.0	8,217.7

	Elk	Boar	White	Brown	Beaver
			Hare	Hare	
Official harvesting share, at the lowest	0.07	0.15	0.04	0.04	0.05
Illegal hunting share, at the lowest	0.14	0.20	0.15	0.15	0.10
Losses to Wolf, at the lowest	0.05	0.03	0.07	0.07	0.10
Projected harvesting, head	33.6	54.0	192.0	18.0	1.5
Illegal hunting, head	67.2	72.0	720.0	67.5	3.0
Losses to Wolf, head	24.0	10.8	336.0	31.5	3.0
Average meat weight per one head for wolves' feed, kg	280.0	44.0	2.6	3.3	12.7

Indicator	Age groups of Elk					Total
	current year's brood 0-1	yearlings 1-2	two-year-olds 2-3	adults 3-9	adults 9-15	
Average annual population, head	59.5	43.3	37.8	195.5	144.0	480.0
Annual (current) population, head	59	43	38	33	24	

Indicator	Age groups of Boar				Total
	current year's brood 0-1	piglets 1-2	adults 2-7	adults 7-12	
Average annual population, head	48.5	45.7	175.8	90.0	360.0
Annual (current) population, head	48.5	45.7	35.2	15.0	

Economic and Mathematical Model for Size and Structure Optimisation of Predator and Prey Populations

Indicator	Age groups of White Hare			Total
	young hares 0-1	animals 1-6	animals 6-11	
Average annual population, head	682.1	2,917.9	1,200.0	4,800.0
Annual (current) population, head	682.1	583.6	240.0	

Indicator	Age groups of Brown Hare			Total
	young hares 0-1	animals 1-8	animals 8-15	
Average annual population, head	42.3	250.2	157.5	450.0
Annual (current) population, head	42.3	35.7	22.5	

Indicator	Age groups of Beaver					Total
	current year's brood 0-1	yearlings 1-2	two-year-olds 2-3	adults 3-10	adults 10-17	
Average annual population, head	2.5	2.3	2.1	12.6	10.5	30.0
Annual (current) population, head	2.5	2.3	2.1	1.8	1.5	

B. Possibility No.1 in case of an increase in Wolf's feed base (share of losses to Wolf: +0.02 for Elk and +0.02 for Boar), i.e. losses of Elk and Boar to Wolf at respectively 7% and 5%.

Age groups of Wolf	young wolves 0-1	yearlings 0-1	adults 2-3	adults 3-5	adults 5-8	adults 8-12	Total
Average annual Wolf population	7.4	3.7	1.1	2.0	2.9	1.0	18.2
Annual (current) population, head	7.4	3.7	1.1	1.0	1.0	0.3	
Annual meat requirements per one wolf, kg	300	450	600	900	1,200	1,350	
Meat per Wolf population, kg	2,234.1	1,675.6	643.6	1,834.2	3,485.0	1,350.0	11222.5

	Elk	Boar	White	Brown	Beaver	Total
			Hare	Hare		
Projected harvesting share, at the lowest	0.07	0.15	0.04	0.04	0.05	
Illegal hunting share, at the lowest	0.14	0.20	0.15	0.15	0.10	

Losses to Wolf, at the lowest	0.07	0.05	0.07	0.07	0.10	
Projected harvesting, head	33.6	54.0	192.0	18.0	1.5	
Illegal hunting, head	67.2	72.0	720.0	67.5	3.0	
Losses to Wolf, head	33.6	18	336.0	31.5	3.0	
Average meat weight per one head for wolves' feed, kg	280.0	44.0	2.6	3.3	12.7	
Meat for Wolf, kg	9,408.0	792.0	882.0	102.4	38.1	11,222.5

C. Possibility No.2 in case of an increase in Wolf's feed base (share of losses to Wolf: +0.04 for Elk and +0.04 for

Boar), i.e. losses of Elk and Boar to Wolf at respectively 9% and 7%.

Age groups of Wolf	young wolves 0-1	yearlings 0-1	adults 2-3	adults 3-5	adults 5-8	adults 8-12	Total
Average annual Wolf population	9.6	4.8	1.4	2.7	3.8	1.0	23.3
Annual population, head	9.6	4.8	1.4	1.3	1.3	0.3	
Annual meat requirements per one wolf, kg	300	450	600	900	1,200	1,350	
Meat per Wolf population, kg	2,882.1	2,161.5	845.5	2,409.7	4,578.5	1,350.0	14,227.3

	Elk	Boar	White	Brown	Beaver	Total
			Hare	Hare		
Projected harvesting share, at the lowest	0.07	0.15	0.04	0.04	0.05	
Illegal hunting share, at the lowest	0.14	0.20	0.15	0.15	0.10	
Losses to Wolf, at the lowest	0.09	0.07	0.07	0.07	0.10	
Projected harvesting, head	33.6	54.0	192.0	18.0	1.5	
Illegal hunting, head	67.2	72.0	720.0	67.5	3.0	
Losses to Wolf, head	43.2	25.2	336.0	31.5	3.0	
Average meat weight per one head for wolves' feed, kg	280.0	44.0	2.6	3.3	12.7	
Meat for Wolf, kg	12,096.0	1,108.8	882.0	102.4	38.1	14,227.3

IV. CONCLUSION

The discussed Predator-Prey EMM allows for a wider scope of modelling regarding the relations of antagonistic animals (Predator-Prey model) when compared to the classical

Lotka-Volterra model. The Predator-Prey EMM enables to model a full-fledged Predator-Prey system in its full diversity, i.e. a system of several populations of Prey and Predator.



The model allows describing populations in line with the sex-age structure, which enables full-fledged modelling of a population's biological turnover. The sex-age differentiation of game animals allows to fine-tune technical and economic coefficients for relative consumption and produce output in the analysed Predator-Prey relations.

The proposed Predator-Prey EMM allows to calculate the Predator and Prey populations in sex-age structures for various possibilities of changes in the respective conditions and to track population changes for Prey and Predator in their mutual relation.

The discussed Predator-Prey EMM enables a variety of options for further development. Thus, adding feed bases to the Predator-Prey EMM would completely provide for territorial analyses of specific biosystems in their full variety and functioning.

REFERENCES

1. Tkachenko V.V., Lukyanenko T.V., Shadrina Zh.A. "A Set of Economic and Mathematical Models for Assessment of Agricultural Crop Cultivation Technologies", *International Journal of Recent Technology and Engineering*, Vol.8 (2), 2019.
2. Kiseleva I.A., Kuznetsov V.I., Sadovnikova N.A., Chernysheva E.N., Androshina I.S. "Mathematical Modeling of Investment Risks", *International Journal of Innovative Technology and Exploring Engineering*, Vol.8 (7), 2019.
3. Dagaev A.M., Novikov A.V., Afonin M.V., Maximov D.A., Golubtsova E.V. "Systems Engineering: Tax Risk Peculiarities in Project Execution", *International Journal of Engineering and Advanced Technology*, Vol.8 (5), 2019.
4. Kaledin, A.P., Filatov A.I., Ostapchuk A.M. "Osnovy okhotnichogo resursovedeniya" [Basic hunting resource studies], Reutov: ERA Publishing house, 2018, p. 344
5. Kaledin, A.P., Abdulla-Zade E.G., Ostapchuk A.M., Filatov A.I., Vachugov D.D. "Prognozirovaniye dinamiki populyatsii kabana v Podmoskovye na osnove matrichnoi modeli" [Forecasting boar population dynamics in the Moscow region based on the matrix model], *Mezhdunarodnyi nauchnyi zhurnal* [The International Scientific Journal], 3, 2016, pp. 30-35.
6. Kaledin, A.P., Abdulla-Zade E.G., Nikolaev A.A., Filatov A.I., Vachugov D.D. "Model dinamiki populyatsii losya v Podmoskovye" [Population dynamics model for elk in the Moscow region], *Mezhdunarodnyi tekhniko-ekonomicheskii zhurnal* [International Technical-Economic Journal], 3, p. 2016.
7. Kaledin, A., Filatov A., Ostapchuk A., Romanov A., Moroz S. "Modelirovaniye dinamiki chislennosti kabana kak obekta okhoty i nagruzki populyatsii na kormovuyu bazu v OOO Sknyatinskoe okhotniche khozyaistvo Tverskoi oblasti" [Modelling the dynamics of boar population as a hunting target and the population load on the feed base in Sknyatinskoe okhotniche khozyaistvo, OOO, of the Tver region], *Glavnyi zootekhnik* [Head zootechnician], 9, 2018, pp. 41-52.
8. Kaledin, A., Filatov A., Ostapchuk A. "Prognozirovaniye dinamiki chislennosti okhotnichikh zhivotnykh v Rossiiskoi Federatsii na osnove modelnykh eksperimentov" [Forecasting game animal population dynamics in the Russian Federation based on modelling experiments], *Mezhdunarodnyi nauchnyi zhurnal* [The International Scientific Journal], 2, 2017, pp. 66- 74.
9. Kaledin, A., Filatov A., Ostapchuk A., Anashkina E. "Modelirovaniye dinamiki chislennosti kabana v Yaroslavskoi oblasti na granitse depopulyatsii" [Modelling of boar population dynamic in the Yaroslavl region at the depopulation threshold], *Mezhdunarodnyi tekhniko – ekonomicheskii zhurnal* [International Technical-Economic Journal], 3, 2017, pp. 64-68.
10. Kaledin A., Filatov A., Nikolaev A., Ostapchuk A., Anashkina E. "Regionalnyi aspekt prognozirovaniya dinamiki chislennosti losya v Yaroslavskoi oblasti na osnove modelnykh eksperimentov" [Regional aspect of forecasting elk population dynamics in the Yaroslavl region based on modelling experiments], *Mezhdunarodnyi nauchnyi zhurnal* [The International Scientific Journal], 3, 2017, pp. 43-47.
11. Kaledin, A.P., Yuldashbaev Yu.A., Filatov A.I., Demin V.A., Ostapchuk A.M., Anashkina E.N., Kubatbekov T.S. "Optimizing the Economic Use of Populations of Game Animals in the Region (by the

Example of the Yaroslavl Region)", *Journal of Pharmaceutical Sciences and Research*, Vol 10 (10), 2018, pp. 2555-2558.