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# A Compromise Decision Support Multi Objective Travelling Salesman Problem

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***Abstract: The multi objective travelling salesman problem simultaneously optimizes several objectives. It is also called as shortest cyclic route model with multiple objectives provides the shortest route. In this article, the compromised decision support solutions are processed for a multi objective travelling salesman problem. The dynamic programming approach for optimal path with state space tree is used to get the shortest route for the objectives. Based on decision maker's preference, the compromised solution for the multi objective travelling salesman problem is obtained. The proposed methodology is very simple and easy way to get the shortest route which is illustrated with an example.***

***Keywords: Multi objective travelling salesman problem; Dynamic programming; shortest route; preferred solution.***

## I. INTRODUCTION

The Travelling Salesman Problem focus to determine the order or sequence that the salesman should visit so that the total distance travelled or cost of travelling or time of travelling is minimum, with the constraint that the salesman should visit each city exactly once and return back to the initial city. The multi objective travelling salesman problem (MOTSP) aims to simultaneously optimize several objectives, such as shortest travelling distance, minimum time, minimum cost and lower risk. For a multiple objective problem, there cannot be a single optimum solution which simultaneously optimizes all objectives, because an improvement in one objective will cause atleast another objective not being able to be optimized. Therefore the optimal solution can only be a set of non-dominated and trade-off solutions. For the multiple objectives, the notion of optimal solution does not exist generally anymore, hence based on dominance relation of Pareto, the notion of optimal solution can be replaced by the notion of efficient or Pareto optimal solution. The bounds on the Pareto-optimal set in the objective space can be defined by the ideal point and the nadir point. The ideal objective vector is an array of the lower bound of all objective functions. For each of the k objectives, there exists one different optimal solution. An objective vector constructed with these individual optimal objective values constitutes the ideal objective vector. The nadir objective vector represents the upper bounds of each objective in the entire "Pareto-optimal set. The goal of solving multi-objective problem is to find the Pareto-optimal set for the decision-maker to choose the most preferred solution. A solution selected by the decision-maker always represents a compromise between the different objectives." Few applications in literature are, urban and rural road networks, printing press scheduling problem, school bus routing problem, crew scheduling problem, mission planning problem, design of global navigation satellite system surveying networks. Over the years, the work of a considerable number of researchers has produced an important number of techniques to deal with multi-objective optimization problems.

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## II. REVIEW OF LITERATURE AND ADVANCEMENT

Borges and Hansen [3] proposed a study to confirm existence of global convexity for a multi objective travelling salesman problem. The approximate set of efficient solution using Meta heuristics and two faces Pareto local search was discussed by Lust and Teghem [10]. The approximation algorithm for several variants of the travelling salesman problem with multiple objective functions was analyzed by Manthey and Ram [11]. Huang and Raguraman [9] explained a generic approach to selecting a MOTSP route by using GIS and a bi-level GA. Fischer and Richter [8] used a branch and bound approach to solve a TSP. Paquete and stutzle [14], proposed the two-phase local search procedure to tackle bi-objective TSP. Weiqi Li [15] shows that the use of simple local search together with an effective data structure, can identify high quality pareto optimal solutions. Yan, Zhang and Kang [16] used an evolutionary algorithm to solve multi objective TSP. Arindam and Kajal [1], explains, multi-objective linear programming effectively deals with flexible aspiration levels or goals and enhances the effectiveness of solutions with acceptable solutions through fuzzy constraints. Bektas, T. [2] discussed the exact and heuristic solution procedures proposed for this problem and highlight some formulations of the problem and its practical applications. Chang and Yen[4], formulates the city-courier routing and scheduling problem as a multi-objective multiple traveling salesman problem with strict time windows and various new and improved sub-procedures are embedded in the solution framework.

In this manuscript, we propose a simple and new methodology to obtain the compromised decision support solution of multi objective travelling salesman problem. The dynamic programming approach for optimal path with state space tree is used to obtain the shortest route for each objective. The solution for the multi objective problem would be found by forming a composite objective function as the weighted sum of the objectives, where a weight for an objective is proportional to the preference factor assigned to that particular objective. The rest of the paper is organized as, section 2, we conversed the literature review. In section 3, basic definitions and mathematical model of multi objective travelling salesman problem (MOTSP) are discussed. In section 4, we propose a new algorithm for the compromised solution of MOTSP. In section 5, a numerical example is provided to illustrate the efficiency of the proposed methodology.

## III. MULTI OBJECTIVE TRAVELLING SALESMAN PROBLEM

### A. Mathematical Formulation of Multi Objective Travelling Salesman Problem

The Mathematical formulation of Multi Objective Travelling Salesman Problem is,

$$Minimize\ \ F(x) = \left(F_1(x), F_2(x), .........F_m(x)\right)$$

$$where\ \ F_1(x) = \sum_{i=1}^{n-1} D^1\left(x_i, x_{i+1}\right) + D^1\left(x_n, x_1\right),$$

....,

$$F_m(x) = \sum_{i=1}^{n-1} D^m\left(x_i, x_{i+1}\right) + D^m\left(x_n, x_1\right)$$

subject to $\sum_j x_{ij} = 1,\ \ j \neq i$ for each i

and $\sum_i x_{ij} = 1,\ \ i \neq j$ for each j

where m is the number of objective functions, n is the number of cities, $D^m$ is the travelling measure, x is the decision vector or solution and $X \in \Re^n$ is the n-dimensional decision space, consisting of a finite set of feasible solutions and F(x) is the objective function, $F \in \Re^m$, the m-dimensional objective space, where m is the number of objectives. The single-objective problem is typically studied in decision space, whereas a multi objective optimization is mostly studied in objective space. The image of a solution in the objective space is a point, $F = [F_1, F_2, \ldots F_m]$. A point F is attainable, if there exists a solution $x \in X$ such that F = F(x). The set of all attainable points is denoted as F. The ideal objective vector F* is defined as $F^* = [optF_1(x), optF_2(x), \ldots, optF_m(x)]$, which is obtained by optimizing each of the objective functions individually.

Definition: Any solution x that satisfies all constraints and variable bounds, then the solution is called as feasible solution.

Definition: A feasible vector $x^0 \in X$ (X is the feasible region) yields a non-dominated solution, if and only if, there is no other feasible vector $x \in X$, such that,

$$\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^k\, x_{ij} \le \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^k\, x_{ij}^0 \text{ , for all k}$$

$$\text{and } \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^k\, x_{ij} < \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^k\, x_{ij}^0 \text{ , for some k,}$$

$k = 1, 2, ......., K$.

Definition: A point $x^0 \in X$ is efficient if and only if, there does not exist another $x \in X$, such that,

$$\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^k\, x_{ij} \le \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^k\, x_{ij}^0 \text{ , for all k}$$

and

$$\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^k\, x_{ij} \neq \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^k\, x_{ij}^0 \text{ , for some k .}$$

Definition: A feasible vector $x^* \in X$ is called a compromise solution if and only if $x^* \in E$ and $Z(x^*) \le \Lambda_{x \in X} Z(x)$ , where $\Lambda$ stands for "minimum" and E is the set of efficient solutions.

Definition: If the compromise solution satisfies the decision maker's preferences, then the solution is called the preferred compromise solution.

### B. Dynamic Programming Approach

The Dynamic Programming formulation of optimal path problem is $g(i, S) = \min_{k \in S}\left\{C_{ik} + g\left(k, S - \{k\}\right)\right\}$ where i represent the initial node, X is the set of all nodes except initial, $C_{ik}$ is the cost (distance or time) incurred between nodes. The shortest route for each objective is obtained using this formulation. This methodology can easily understand and apply using state space tree, which will give all possible routes for each objective, taking any one node as initial node.

### C. Composite objective function

The multi objective optimization problem is compounded into a single objective optimization problem by linear combination of the multiple objectives with weights, i.e., form a composite objective function as the weighted sum of the objectives, where the objective is to minimize the linear combination of the multiple objectives with weights which minimizes a positively weighted convex sum of the objective.

$$Minimize\ \sum_{i=1}^{m} \alpha_i\ F_i(x) \quad and \quad \sum_{i=1}^{m} \alpha_i = 1,$$

$$where\ \alpha_i > 0,\ \text{weightage for the } i^{th}\ objective$$

$$and\ \ x \in X.$$

The solution of this single objective function is an efficient solution for the original multi objective problem.

## IV. ALGORITHM FOR MULTI OBJECTIVE TRAVELLING SALESMAN PROBLEM

Consider a Multi Objective Travelling Salesman Problem.

Step 1: Each objective is operated with the dynamic programming formulation for optimal path, taking any one node as initial node.

The formulation portrays the shortest route for each objective, which can easily understand with state space tree.

Step 2: According to each solution and value for each objective, we can find a pay-off matrix, where x(1), x(2),...,x(m ) are the optimal solutions of the m different problems for m different objective functions, $F_{ij} = F_j(x_i)$, (i = 1,2,..., m; j = 1,2,..., m) be the $i^{th}$ row and $j^{th}$ column element of the pay-off matrix.

Step 3: Evaluate each objective function at all these m optimum solutions leads to set of Pareto optimum solutions. The components of nadir objective vector, supremum of $F_i(x)$ and ideal objective vector, infimum of $F_i(x)$, depicts the lower and upper bound for each objective.

Step 4: Using the above mentioned composite objective function, the problem is now represented as a single objective optimization problem. According to the decision maker's preference various weights are assigned to the objective function and the preferred compromised solution is obtained, which lie between the nadir and ideal objective vector.

The solution of this optimization problem is an efficient solution for the original multi objective problem.

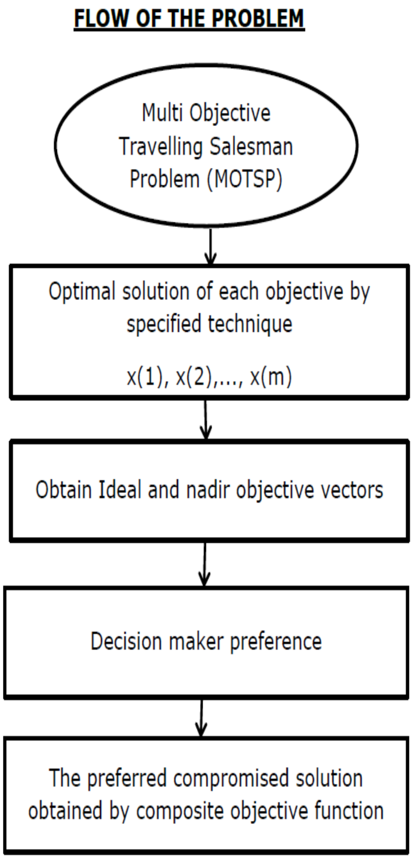


**Fig. 1. Description of the problem**

## V. NUMERICAL EXAMPLE

Consider a multi objective travelling salesman problem with three objectives such as cost, distance and time for distributing things at the time of disasters using road transport or drone like machines.

Cost

$$C = \begin{array}{c c} & \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} \\ \begin{array}{c} 0 \\ 1 \\ 2 \\ 3 \end{array} & \begin{bmatrix} \infty & 20 & 15 & 11 \\ 20 & \infty & 30 & 10 \\ 15 & 30 & \infty & 20 \\ 11 & 10 & 20 & \infty \end{bmatrix} \end{array}$$

Distance

$$D = \begin{array}{c c} & \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} \\ \begin{array}{c} 0 \\ 1 \\ 2 \\ 3 \end{array} & \begin{bmatrix} \infty & 5 & 5 & 3 \\ 5 & \infty & 5 & 3 \\ 5 & 5 & \infty & 10 \\ 3 & 3 & 10 & \infty \end{bmatrix} \end{array}$$

Time

$$T = \begin{array}{c c} & \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} \\ \begin{array}{c} 0 \\ 1 \\ 2 \\ 3 \end{array} & \begin{bmatrix} \infty & 4 & 5 & 2 \\ 4 & \infty & 3 & 3 \\ 5 & 3 & \infty & 2 \\ 2 & 3 & 2 & \infty \end{bmatrix} \end{array}$$

where $c_{ij}$ represents cost of travel from node i to node j, $d_{ij}$ represents distance of travel from node i to node j and $t_{ij}$ represents time of travel from node i to node j.

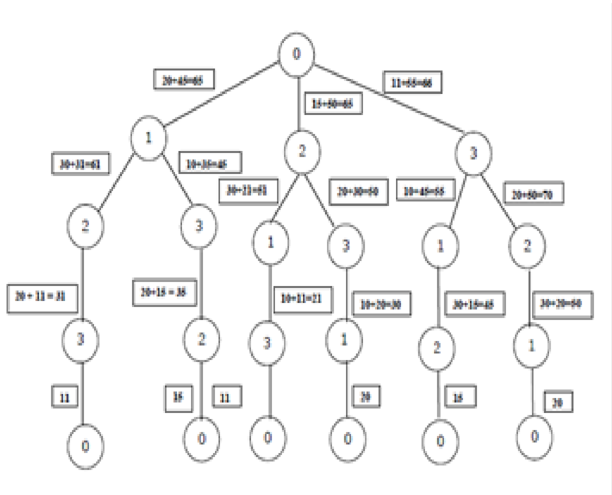

**Fig. 2. Various possible routes for cost matrix**

Fig.2 shows the various possible routes for one of the objective and the cost of travel for each route. Similarly, the various possible routes for distance and time matrix can be found and the shortest route can easily obtained.

The dynamic programming formulation for optimal path problem with '0' as initial vertex,

$$g(0,\{1,2,3\}) = \min_{k\in\{1,2,3\}} \{c_{0k} + g(k,\{1,2,3\}-\{k\})\}$$

$$g(0,\{1,2,3\}) = \min \begin{Bmatrix} C_{01} + g(1,\{2,3\}), \\ C_{02} + g(2,\{1,3\}), \\ C_{03} + g(3,\{1,2\}) \end{Bmatrix}$$

$$= \min(20+45, 15+50, 11+55)$$

$$= 65$$

Therefore the shortest route is $0 \to 1 \to 3 \to 2 \to 0$.

Hence the minimum cost is 65.

Similarly, the shortest route for 2nd objective (Distance) is $1 \to 2 \to 0 \to 3 \to 1$.

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Therefore, the minimum distance = 16.

The shortest route for 3rd objective (Time) is $2 \to 1 \to 0 \to 3 \to 2$ .

Therefore, the minimum time = 11.

Evaluating each objective function at all these optimal solutions, we get the set of pareto optimal solutions with the nadir objective vector (the upper bound) and the ideal objective vector (the lower bound) for each objective.

The bounds of Pareto optimal solution for each objective is $65 \le F_1 \le 81,\ 16 \le F_2 \le 23,\ 11 \le F_3 \le 14$

Using a composite objective function as the weighted sum of the objectives, where the objective is to minimize the linear combination of the multi objective with weights, which minimizes a positively weighted convex sum of the objective. According to the decision marker preference, for various weights, we will get different set of preferred solutions for the single objective weighted sum travelling salesman problem, which are displayed in the following table.

**Table 1. Preferred Compromised Solutions**

| $\alpha_1$ | $\alpha_2$ | $\alpha_3$ | Shortest Route | Objective values | | |
|---|---|---|---|---|---|---|
| | | | | $F_1$ | $F_2$ | $F_3$ |
| 0.4 | 0.3 | 0.3 | 0→2→1→3→0 | 66 | 16 | 13 |
| 0.8 | 0.1 | 0.1 | 0→1→3→2→0 | 65 | 23 | 14 |
| 0.5 | 0.3 | 0.2 | 0→2→1→3→0 | 66 | 16 | 13 |
| 0.3 | 0.4 | 0.3 | 0→2→1→3→0 | 66 | 16 | 13 |
| 0.3 | 0.2 | 0.5 | 0→2→1→3→0 | 66 | 16 | 13 |

## VI. RESULT AND DISCUSSION

The proposed formulation for multi objective travelling salesman problem is a simple and easy way to identify the shortest route of the problem, taking in to consideration the revelent parameters. The developed algorithm is tested on a number of randomly generated instances. The possible trade-offs between the objectives is evident from the tabulated results in Table 1. The Pareto set of solutions are given for the problem instances.

## VII. CONCLUSION

In this paper we have discussed a simple methodology to obtain the preferred solution to a multi objective travelling salesman problem. The proposed algorithm provides a set of Pareto optimal solutions with various objective values for the optimal compromised solutions. Further, the minimum objective values obtained by the proposed algorithm are better than the existing methods. The decision maker can select the appropriate compromise solution according to his satisfaction level. Also this method is providing more alternatives for the decision maker to select the preferred solution.

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