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Models for Optimisation of Delivery Timing and Volumes under Uncertainty Encompassing Penalty and Customer Attrition Risks

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***Abstract**:The paper proposes a model for inventory management and specifically for determining the optimum volume and timing of deliveries, encompassing the uncertainty of demand. The criteria of efficiency are the minimisation of integral costs and maximisation of profit with due regard for the risks of penalties and customer attrition. The triangular distribution is a reference for the distribution pattern of the stochastic demand and delivery timing fluctuations as it is one of the most common choices in case of insufficient statistical data.*

***Keywords**: inventory management, cost minimisation, delivery timing, delivery volume, uncertainty of demand, triangular distribution.*

I. INTRODUCTION

The main optimisation variables in inventory management models are delivery volume and timing. The input information in inventory management models includes average projected demand for some of the periods or demand forecast for a period of time, which are approached as being determinate though they are not actually determinate. The same applies to the delivery timing, which is, in most cases, not determinate as well. It is particularly true for trade organisations oriented at import supplies, as the procurement process, in that case, consists of many operations, such as rehandling, customs procedures, refitting at intermediary warehouses, etc. This increases the stochasticity in respect of delivery timing. Modelling taking into account uncertainty factors helps to find the most efficient inventory management in such uncertain conditions. Various models encompassing uncertainty were specifically discussed in [1]-[6] and many other scientific publications. However, the problem setting proposed in this paper has never been approached earlier.

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II. PROPOSED METHODOLOGY

A. Models for Optimisation of Delivery Timing Under Conditions of Uncertainty Encompassing the Risk of Application of Penalties

Supplies are assumed to take place at intervals. The volumes of deliveries are determinate, as they are calculated based on the known delivery schedule, which sets the timing and volumes. Assume $t_1, t_2, \dots, t_n$ refers to consecutive delivery timings over the analysed period as set forth in the schedule and $S_1, S_2, \dots, S_n$ – to the respective delivery volumes, which are also set forth in the schedule. Then, delivery volume Q for the analysed period is determined as follows:

$$Q = \sum_{i=1}^{n} S_i \quad (1)$$

The timing of delivery in the model is a stochastic variable. Uncertainty of the actual arrival timing of deliveries to the warehouse x is expressed by the following equation:

$$x = t^* + \Delta t \quad (2)$$

where t^* is the appointed delivery moment; Δ is the stochastic value describing the discrepancy of the actual timing from the projected timing and following a known distribution pattern, with probability density $f(x)$, probability distribution function $\Phi(x)$ and $x \epsilon [a, b]$ as the known range of values. The optimisation variable is the appointed delivery timing t^*. It is also assumed that the supplier's contractual obligations are encumbered with penalty terms in respect of missed schedules or volumes of deliveries. Such penalties may be fines determined as fixed additional payments in case of breach of the delivery terms, which are usually calculated as a fixed percentage of the total amount of the delivery. Penalties may also take the form of default interest being additional payments at fixed rates depending on the volume and timing of underdelivery.

The models also take into account additional costs of storage for the delivered merchandise in case of early delivery until the time of the first contractual delivery, which are proportionate to the volume of merchandise and timing of the early delivery storage at the rate H, $H \geq 0$. Apart from direct storage costs, the storage rate may also include financial losses on frozen funds, especially if the trade company uses a loan facility for working capital.

B. Model for minimisation of additional costs adjusted for penalty application risks

This model pins its target function to the expected value of integral additional costs including storage costs until the beginning of contract deliveries and the costs of penalty payments for breaching contract and delivery terms. Assume $V_1, V_2, \dots, V_n$ refer to the absolute values of fixed fines for breaching the respective delivery timings.

First, determine the acceptable region for the optimisation variable t^*. Note that $t_1 - b \le t^*$. It follows from the fact that lower values t^* exclude the risk of penalties while the additional storage costs rise and, thus, these values cannot be an optimum appointed delivery timing. Note further that $t^* \le t_1 - a$. It follows from the fact that higher values t^* exclude the risk of additional storage costs while additional penalty-related costs rise and, thus, these values cannot be an optimum appointed delivery timing as well. Thus, the target appointed delivery timing fits the following two-sided inequality:

$t_1 - b \le t^* \le t_1 - a$, whence $a \le t_1 - t^* \le\ b$ (3)

Given that $a \le \Delta t \le\ b$, the mathematical rendering of the target function of the model is expressed as follows:

$$F(t^*) = \int_a^{t_1-t^*} HQ(t_1 - t^* - \Delta t) f(\Delta t) d\Delta t + \sum_{i=1}^{n} p_i(t^*) V_i \ (4)$$

where $p_1(t^*)$, $p_2(t^*)$, …, $p_n(t^*)$ refer to the probabilities of payments of the respective fines depending on the appointed delivery timing t^* and Q is the volume of delivery in the analysed period according to relation (1). The probabilities of fines for the set delivery timing can be expressed using the probability distribution function as follows:

$$p_i(t^*) = \{\int_{t_i-t^*}^{b} f(\Delta t) d\Delta t, t_i \le t^* + b, (5) 0, t_i > t^* + b$$

or using the probability distribution function as follows:

$$p_i(t^*) = \{\Phi(b) - \Phi(t_i - t^*), t_i \le t^* + b, (6) 0, t_i > t^* + b$$

The condition for probabilities of fines being non-zero is $t_i \le t^* + b$. Given that $t^* \le t_1 - a$, the model can sufficiently take into account only the fines fitting the inequality $t_i \le t_1 + (b - a)$. Assume the above condition is true for i = 1, 2, …, K_{max}. For a specific value t^*, the maximum index of non-zero probabilities $K(t^*)$ can be different from K_{max}. In particular, as can be seen, given $t^* = t_1 - b$, the condition $t_i \le t^* + b$ is only true for i=1. Therefore, in general, $1 \le K(t^*) \le K_{max}$. For the minimisation of function F(t^*), differentiate it using the Leibniz rule for differentiation under the integral sign with variable upper limit.

$$\frac{dF(t^*)}{dt^*} = -HQ \int_a^{t_1-t^*} f(x) dx + \sum_{i=1}^{K(t^*)} f(t_i - t^*) V_i \ (7)$$

or

$$\frac{dF(t^*)}{dt^*} = -HQ\Phi(t_1 - t^*) + \sum_{i=1}^{K(t^*)} f(t_i - t^*) V_i \ (8)$$

Consider the derivative values at the ends of the analysed segment of the optimisation variable. With $t^* = t_1 - b$, the corresponding result at the left end of the segment is as follows:

$$\frac{dF(t^*)}{dt^*} = -HQ \int_a^{b} f(x) dx + \sum_{i=1}^{K(t^*)} f(b + (t_i - t_1)) V_i = -HQ \ < 0 \ (9)$$

Accordingly, with $t^* = t_1 - a$, the corresponding result at the right end of the segment is as follows:

$$\frac{dF(t^*)}{dt^*} = -HQ \int_a^{a} f(x) dx + \sum_{i=1}^{K(t^*)} f(a + (t_i - t_1)) V_i = \sum_{i=1}^{K(t^*)} f(a + (t_i - t_1)) V_i \ \ge 0 \ (10)$$

Taking into account the continuity of the discussed function and the derivative and its non-positivity at the left end of the segment and non-negativity at the right end, there is arguably a stationary point in the said segment that is the solution to the set problem. Finding it, in general, relies on the application of numerical methods of root-finding for the equation.

$$-HQ\Phi(t_1 - t^*) + \sum_{i=1}^{K(t^*)} f(t_i - t^*) V_i = 0 \ (11)$$

Next, solve this problem assuming that the stochastic variable describing the discrepancy of the actual delivery timing from the projected timing follows the triangular distribution pattern in the segment $[a, b]$. To be specific, assume K_{max} =2.

Step 1. For each i = 1, …, K_{max}, the triade of values is calculated as follows: $t_i - b, t_i - b, t_i - a.$

Step 2. Values calculated at step 1 are analysed to exclude those exceeding the value $(t_1 - a)$. The remaining values are arranged in ascending order. Assume the following sequence was arrived at in the analysed case: $t_1 - b \le t_2 - b \ \le \ t_1 - c \ \le \ t_2 - c \le t_1 - a$. Thus, the segment of acceptable values of variable t^* is broken up into 4 segments. Next, the root-finding problem for equation (11) is solved, i.e. stationary points of function (4) are found in each of them.

Step 3.1 $t_1 - b \le t^* \le t_2 - b$. Equation (11) takes on the following form:

$$-HQ\left(1 - \frac{(b - t_1 + t^*)^2}{(b - a)(b - c)}\right) + \frac{2(b - t_1 + t^*)}{(b - a)(b - c)} V_1 = 0 \ (12)$$

Equation (12) refers to the root-finding problem for a quadratic equation in a set segment, which is solved analytically.

Step 3.2 $t_2 - b \le t^* \le t_1 - c$. Equation (11) takes on the following form:

$$-HQ\left(1 - \frac{(b - t_1 + t^*)^2}{(b - a)(b - c)}\right) + \frac{2(b - t_1 + t^*)}{(b - a)(b - c)} V_1 + \frac{2(b - t_2 + t^*)}{(b - a)(b - c)} V_2 = 0 \ (13)$$

Equation (13) refers to the root-finding problem for a quadratic equation in a set segment, which is solved analytically.

Step 3.3 $t_1 - c \le t^* \le t_2 - c$. Equation (11) takes on the following form:

$$HQ\left(\frac{(t_1-t^*-a)^2}{(b-a)(c-a)}\right)+\frac{2(t_1-t^*-a)}{(b-a)(c-a)}V_1+\frac{2(b-t_2+t^*)}{(b-a)(b-c)}V_2=0 \ (14)$$

Equation (14) refers to the root-finding problem for a quadratic equation in a set segment, which is solved analytically.

Step 3.4 $t_2-c\le t^*\le t_1-a$. Equation (11) takes on the following form:

$$-HQ\left(\frac{(t_1-t^*-a)^2}{(b-a)(c-a)}\right)+\frac{2(t_1-t^*-a)}{(b-a)(c-a)}V_1+\frac{2(t_2-t^*-a)}{(b-a)(c-a)}V_2=0 \ (15)$$

Equation (15) refers to the root-finding problem for a quadratic equation in a set segment, which is solved analytically.

Step 4. The union of sets of solutions to problems 3.1-3.4 is a non-empty set due to the above justification. If only one stationary point is found, it is the solution. Otherwise, the value of function (4) is calculated in each stationary point using analytical methods and the point corresponding to the minimum function value is identified, which is the solution to the problem.

C. Model for minimisation of additional costs encompassing default interest accrual risks

This model pins its target function to the expected value of integral additional costs including storage costs until the beginning of contract deliveries and costs of default interest payments for breaching contract and delivery terms. Assume default interest accrues for each day of delay of the delivery as a fixed percentage of the total delivery amount. Assume $m_1, m_2, \dots, m_n$ refer to the values of fixed percentages of the total delivery values in charging default interest for missed schedule of the respective deliveries. Assume $W_1, W_2, \dots, W_n$ refer to the total amount of the respective deliveries.

The inferences in the previous model with regard to the acceptable region for the optimisation variable t^* hold true to the extent the risks of penalties are replaced with the risks of default interest payments and lead to a similar result $t_1-b\le t^*\le t_1-a$. Given that $a\le \Delta t\le b$, the mathematical rendering of the target function of the model is expressed as follows:

$$F(t^*)=\int_a^{t_1-t^*} HQ(t_1-t^*-\Delta t)f(\Delta t)d\Delta t+\sum_{i=1}^{n} m_i W_i P_i(t^*) \ (16)$$

where $P_1(t^*)$, $P_2(t^*)$, …, $P_n(t^*)$ refer to the expected value of the days of delay in breach of the respective delivery depending on the appointed delivery timing t^*, calculated according to (17), and Q is the volume of delivery in the analysed period determined according to relation (1).

$$P_i(t^*)=\begin{cases}\int_{t_i-t^*}^{b}(t^*+\Delta t-t_i)f(\Delta t)d\Delta t, & t_i\le t^*+b, \\ 0, & t_i>t^*+b\end{cases} \ (17)$$

The reasoning in paragraph 2.3.1 concerning the determination of value K_{max} holds true for this model, as well as the overall conclusion expressed as two-sided inequality $1\le K(t^*)\le K_{max}$. For the minimisation of function F(t^*), differentiate it using the Leibniz rule.

$$\frac{dF(t^*)}{dt^*}=-HQ\int_a^{t_1-t^*} f(x)dx+\sum_{i=1}^{K(t^*)} m_i W_i p_i(t^*) \ (18)$$

where functions $p_i(t^*)$ are determined according to expressions (5) or (6). An equivalent notation is the following expression:

$$\frac{dF(t^*)}{dt^*}=-HQ\Phi(t_1-t^*)+\sum_{i=1}^{K(t^*)} m_i W_i p_i(t^*) \ (19)$$

Consider the derivative values at the ends of the analysed segment of the optimisation variable. With $t^*=t_1-b$, the corresponding result at the left end of the segment is as follows:

$$\frac{dF(t^*)}{dt^*}=-HQ\int_a^{b} f(x)dx+\sum_{i=1}^{K(t^*)} m_i W_i p_i(t_1-b) = -HQ<0. \ (20)$$

Accordingly, with $t^*=t_1-a$, the corresponding result at the right end of the segment is as follows:

$$\frac{dF(t^*)}{dt^*}=-HQ\int_a^{a} f(x)dx+\sum_{i=1}^{K(t^*)} m_i W_i p_i(t_1-a)=\sum_{i=1}^{K(t^*)} m_i W_i p_i(t_1-a)\ge 0 \ (21)$$

Taking into account the continuity of the discussed function and the derivative and its non-positivity at the left end of the segment and non-negativity at the right end, there is arguably a stationary point in the said segment that is the solution to the set problem. Finding it, in general, relies on the application of numerical methods of root-finding for equation (21).

$$-HQ\Phi(t_1 - t^*) + \sum_{i=1}^{K(t^*)} m_i W_i p_i (t_1 - a) = 0 \ (22)$$

Next, solve this problem assuming that the stochastic variable describing the discrepancy of the actual delivery timing from the projected timing follows the triangular distribution patterns in the segment $[a, b]$. To be specific, assume K_{max} =2.

Step 1. For each i = 1, …, K_{max}, the triade of values is calculated as follows: $t_i - b, t_i - b, t_i - a$.

Step 2. Values calculated at step 1 are analysed to exclude those exceeding the value $(t_1 - a)$ and arranged in ascending order. Assume the following sequence was arrived at in the analysed case: $t_1 - b \le t_2 - b \le t_1 - c \le t_2 - c \le t_1 - a$. Thus, the segment of acceptable values of variable t^* is broken up into 4 segments. Next, the root-finding problem for equation (22) is solved, i.e. stationary points of function (16) are found in each of them.

Step 3.1$t_1 - b \le t^* \le t_2 - b$. Equation (22) takes on the following form:

$$-HQ\left(1 - \frac{(b - t_1 + t^*)^2}{(b-a)(b-c)}\right) + m_1 W_1 \frac{(b - t_1 + t^*)^2}{(b-a)(b-c)} = 0 \ (23)$$

Equation (23) refers to the root-finding problem for a quadratic equation in a set segment, which is solved analytically.

Step 3.2$t_2 - b \le t^* \le t_1 - c$. Equation (21) takes on the following form:

$$-HQ\left(1 - \frac{(b - t_1 + t^*)^2}{(b-a)(b-c)}\right) + m_1 W_1 \frac{(b - t_1 + t^*)^2}{(b-a)(b-c)} + m_2 W_2 \frac{(b - t_2 + t^*)^2}{(b-a)(b-c)} = 0 \ (24)$$

Equation (24) refers to the root-finding problem for a quadratic equation in a set segment, which is solved analytically.

Step 3.3$t_1 - c \le t^* \le t_2 - c$. Equation (22) takes on the following form:

$$-HQ\left(\frac{(t_1 - t^* - a)^2}{(b-a)(c-a)}\right) + m_1 W_1 \left(1 - \frac{(t_1 - t^* - a)^2}{(b-a)(c-a)}\right) + m_2 W_2 \frac{2(b - t_2 + t^*)}{(b-a)(b-c)} = 0 \ (25)$$

Equation (25) refers to the root-finding problem for a quadratic equation in a set segment, which is solved analytically.

Step 3.4$t_2 - c \le t^* \le t_1 - a$. Equation (22) takes on the following form:

$$-HQ\left(\frac{(t_1 - t^* - a)^2}{(b-a)(c-a)}\right) + m_1 W_1 (1 - \frac{(t_1 - t^* - a)^2}{(b-a)(c-a)}) + m_2 W_2 (1 - \frac{(t_2 - t^* - a)^2}{(b-a)(c-a)}) = 0 \ (26)$$

Equation (26) refers to the root-finding problem for a quadratic equation in a set segment, which is solved analytically.

Step 4. The union of sets of solutions to problems 3.1-3.4 is a non-empty set due to the above justification. If only one stationary point is found, it is the solution. Otherwise, the value of function (16) is calculated in each stationary point using analytical methods and the point corresponding to minimum function value is chosen, which is the solution to the problem.

III. RESULT ANALYSISA

A. Models for Optimisation of Delivery Volumes Under Conditions of Uncertainty in Line with The Criterion of Maximisation of Projected Profit

Assume deliveries are conducted at intervals exactly on time as scheduled, i.e. no uncertainty exists with regard to the delivery timing. The volume to be delivered at the beginning of each period is a variable being the optimisation variable in this model. The demand in each period is a stochastic variable following a known distribution pattern, with probability distribution density $f(x)$ and a known set of values $x\epsilon[0, XM]$, where XM is the maximum possible demand in the given period. The problem is solved autonomously for each delivery period and thus the parameter XM and distribution pattern of f(x) can vary with periods. Assume the profit on unit sales is known, v. Assume that $v \ge 0$. Total profit is proportionate to the volume of sold merchandise.

If demand turns out to be higher than the volume in stock, various financial risks may emerge, taking the form of contractual default interest, fines, other types of penalties, reputation risks or customer attrition risks.

Customer attrition risks may also have quantitative monetary evaluation. Using the regression coefficient of similar models attached to the variable "volume of unmet demand" and comparing its influence on customer attrition probability with the data of customer history, one may derive the specific relative coefficient characterising the company's average financial losses in case of unmet demand per unit. This coefficient is denoted w. Assume that $w \leq 0$.

If merchandise in stock for a specific period outweighs the demand, an unused excess emerges, causing additional storage costs and financial losses on funds frozen in such merchandise.

In such case, costs of storage of the unwanted excess supplies should be estimated for the given period in proportion to the amount of unwanted excess with a coefficient. Assume that $h \leq 0$. Note that costs are only calculated for merchandise in stock over the period, in which merchandise is not realised. Storage costs for merchandise realised during the said period are considered to be included in profit margin on the merchandise, v.

B. Models for optimisation of delivery volume encompassing additional storage costs and customer attrition risks for durable goods

The optimisation criterion in this model is the expected value of profit on realisation of the merchandise less the two described types of costs, specifically, storage costs (which emerge in case of unwanted excess merchandise) and costs of customer attrition (which emerge in case of failure to meet the demand for the period in full). The target function of the problem is expressed as follows:

$$F(Q) = \int_0^Q \left(vx + h(Q-x)\right)f(x)dx + \int_Q^{XM} \left(vQ + w(x-Q)\right)f(x)dx \ (27)$$

The problem is to maximise function (27) for variable Q in the acceptable region, i.e. the segment $[0, XM]$. The following transformations of function (27) are further accomplished:

$$F(Q) = v\int_0^Q xf(x)dx + hQ\int_0^Q f(x)dx + h\int_0^Q xf(x)dx + vQ\int_Q^{XM} f(x)dx + w\int_Q^{XM} xf(x)dx - wQ\int_Q^{XM} f(x)dx$$

For the minimisation of function F(x), differentiate it using the Leibniz rule.

$$\frac{dF(Q)}{dQ} = h\int_0^Q f(x)dx + v\int_Q^{XM} f(x)dx - w\int_Q^{XM} f(x)dx \ (28)$$

Given that

$$\int_Q^{XM} f(x)dx = 1 - \int_0^Q f(x)dx \ (29)$$

and setting the derivative to zero, the resulting equation in respect of Q is as follows and it is analysed for $x\epsilon[0, XM]$:

$$\int_0^Q f(x)dx = \frac{w-v}{h+w-v} = \frac{1}{\frac{h}{w-v}+1} = k_1 \ (30)$$

The second decomposition in relation (30) and the signs of the above parameters h, w, v imply $0 \leq k_1 \leq 1$. As the function of variable Q in the left part of equation (30) is a monotonous continuous non-decreasing function varying in the analysed segment from 0 to 1, equation (30) possesses a solution according to the Weierstrass theorem. In general, root-finding of equation (30) is conducted following the known numerical methods. Assume Q^* is the root of equation (30). Note further that the derivative (28) is a non-increasing function in the analysed segment and with that

$$\frac{dF(0)}{dQ} = v - w \geq 0, \frac{dF(XM)}{dQ} = h \leq 0$$

Thus, in point Q^* the derivative changes the sign from + to - and consequently, the point Q^* is the point of the sought maximum and there is no need to calculate the value of function F(x) at the ends of the segment.

Next, this problem is solved assuming that the stochastic variable describing the demand follows the triangular distribution pattern in the segment $[a, b]$. In such case, the probability distribution function f(x) is expressed as follows:

$$f(x) = \{0, if\ x < a; \frac{2(x-a)}{(b-a)(c-a)}, if\ a \leq x < c; \frac{2}{(b-a)}, if\ x = c; \frac{2(b-x)}{(b-a)(b-c)}, if\ c < x \leq b; 0, if\ b < x (31)$$

With that, the probability distribution function takes on the following form:

$$\Phi(x) = \{0, if\ x < a; \frac{(x-a)^2}{(b-a)(c-a)}, if\ a \leq x \leq c; 1 - \frac{(b-x)^2}{(b-a)(b-c)}, if\ c \leq x \leq b; 0, if\ b < x (32)$$

Whence the analytical solution to the problem for the case when the stochastic demand variable is described by the triangular distribution is expressed by the following relations:

$$Q^* = \{a + \sqrt{k_1(b-a)(c-a)}, 0 \leq k_1 \leq \frac{c-a}{b-a}, b + \sqrt{(1-k_1)(b-a)(c-a)}, \frac{c-a}{b-a} \leq k_1 \leq 1 (33)$$

Finally, note that the model allows limiting the scope of the problem setting to fit the case when some kind of costs discussed in it is not applicable.

If storage costs do not apply, i.e. h=0, then k_1=1 and Q^*=XM. The economic sense of this result is clear as the risk of additional storage costs is non-applicable in the first place, the risk of customer attrition is neutralised if merchandise is delivered to meet the maximum potential demand and unsold merchandise is carried over for the next period without any additional costs.

If customer attrition costs are not applicable, i.e. w=0, then

$$k_1 = \frac{v}{v-h}$$

As can be seen, $0 \le k_1 \le 1$ and equation (30) is still solvable. In such case, the solution is non-trivial, as there is a balance between profit on sales and costs of storage of excesses.

If neither storage costs nor customer attrition costs are applicable, i.e. h=0, w=0, then k_1=1 and Q^*=XM. The economic sense of this result is also clear as the risk of additional storage costs and the risk of customer attrition are non-applicable in the first place, the unsold merchandise is carried over for the next period without any additional costs and procurement of maximum volume increases projected sales.

C. Models for optimisation of delivery volume encompassing additional storage costs and customer attrition risks for semidurable goods

The optimisation criterion in this model is the expected value of profit on realisation of the merchandise less the two described types of costs, specifically, storage costs (which emerge in case of unwanted excess merchandise) and costs of customer attrition (which emerge in case of failure to meet the demand for the period in full). In contrast to the above model, the costs of procurement incurred on the merchandise delivered in the analysed period are also subtracted. The rationale is that while the unsold excesses in the previous model were carried over to the next period and factored in as the opening balance in determining the requirements of such next period. This model is based on the assumption that the unsold excess for the period is considered to be expired and is not considered further, i.e., it is excluded from the further cycle. The target function of the problem is expressed as follows:

$$F(Q) = \int_0^Q \left(c_2 x + h(Q-x)\right) f(x)dx + \int_Q^{XM} \left(c_2 Q + w(x-Q)\right) f(x)dx - c_1 Q \quad (34)$$

where c_1 is the purchasing price of the merchandise and c_2 is the selling price of the merchandise. Assume that

$$0 \le c_1 \le c_2$$

The task is to maximise function (33) for variable Q in the acceptable range, i.e. the segment $[0, XM]$. Next, the following transformations of function (34) are conducted:

$$F(Q) = c_2 \int_0^Q xf(x)dx + hQ\int_0^Q f(x)dx + h\int_0^Q xf(x)dx + c_2 Q\int_Q^{XM} f(x)dx + w\int_Q^{XM} xf(x)dx - wQ\int_Q^{XM} f(x)dx - c_1 Q$$

For the minimisation of function F(x), differentiate it using the Leibniz rule.

$$\frac{dF(Q)}{dQ} = h\int_0^Q f(x)dx + c_2\int_Q^{XM} f(x)dx - w\int_Q^{XM} f(x)dx - c_1 \quad (35)$$

Taking into account the relation (29) and setting the derivative to zero, the resulting equation in respect of Q is as follows and is analysed for $x\epsilon[0, XM]$:

$$\int_0^Q f(x)dx = \frac{w + c_1 - c_2}{h + w - c_2} = \frac{1 + \frac{c_1}{w - c_2}}{1 + \frac{h}{w - c_2}} = k_2 \quad (36)$$

Taking into account the signs of parameters w, h and relation $0 \le c_1 \le c_2$, the following set of inequalities is true: $c_1 \le c_2 - w$, $w - c_2 < 0$, $-1 \le \frac{c_1}{w-c_2} \le 0, 0 \le 1 + \frac{c_1}{w-c_2} \le 1$,

$0 \le \frac{h}{w-c_2}, 1 \le 1 + \frac{c_1}{w-c_2}$ and, subsequently, $0 \le k_2 \le 1$.

As the function of variable Q in the left part of equation (36) is a monotonous continuous non-decreasing function varying in the analysed segment from 0 to 1; equation (36) possesses a solution according to the Weierstrass theorem. In general, root-finding of equation (36) is conducted following the known numerical methods. Assume Q^* is the root of equation (36). Note further that derivative (35) is a non-increasing function in the analysed segment and with that

$$\frac{dF(0)}{dQ} = c_2 - w - c_1 \ge 0, \frac{dF(XM)}{dQ} = h - c_1 \le 0$$

Thus, in point Q^* the derivative changes the sign from + to - and consequently, the point Q^* is the point of the sought maximum and there is no need to calculate the value of function F(x) at the ends of the segment.

Next, this problem is solved assuming that the stochastic variable describing the demand follows the triangular distribution pattern in the segment $[a, b]$. The analytical solution to the problem is expressed by the following relations:

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$$Q^* = \{a + \sqrt{k_1(b-a)(c-a)}, 0 \leq k_2 \leq \frac{c-a}{b-a}, b + \sqrt{(1-k_1)(b-a)(c-a)}, \frac{c-a}{b-a} \leq k_2 \leq 1 \quad (37)$$

IV. CONCLUSION

A number of papers have addressed the problem in a similar setting though for the case where the stochastic variable describing the discrepancy between the actual delivery timing from the projected timing is considered as following the normal distribution. Firstly, it is not always true and, secondly, companies often have insufficient statistics to test the samples of stochastic variable realisations for normalcy. Therefore, a key point is the practical viability of the results as evaluations of the parameters of the triangular distribution can be conducted by expertise in case of insufficient data availability. Moreover, the use of the triangular distribution enables analytical solutions to the above problems, which considerably simplifies the practical application of the developed optimisation models.

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