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Possibilistic Distribution for Selection of Critical Path in Multi Objective Multi-Mode Problem with Trapezoidal Fuzzy Number

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Abstract: ***Execution of any project with optimum duration, cost, quality and risk is very significant for project administrators in recent very competitive commercial situation. Sometimes it is not possible to have detailed earlier statistics about project criteria. In such situations, estimation of different Decision makers are considered in linguistic variables and altered into triangular fuzzy numbers as fuzzy numbers have ability to deal with vagueness. In this paper, we frame a new multi-mode multi objective critical path problem and suggest a possibilistic methodology to find critical path for a project where three decision makers' views are considered as three modes of execution in terms of linguistic variables. In this paper have formulated model of multiple mode in project network problem and find its solution with fuzzy programming approach with exponential membership and linear membership function. The proposed approach is useful to solve multi-mode project management problem which calculates optimal critical path according to four criteria- time, cost, risk and quality with three activities modes of execution in fuzzy environment.***

Keywords**:** ***Critical Path Method, Multi-attribute decision-making (MADM), Negative Ideal Solution (NIS), Positive Ideal Solution (PIS), Trapezoidal Fuzzy Number (TFN), TOPSIS***

I. INTRODUCTION

Planning is essential for development and monitoring the various tasks convoluted in the project, before commencing any project. This will assist undertaking the project, identifying probable blocks and if necessary, preparing an alternate work-plan for the project. Network Analysis and Network Planning are the techniques of operations research used for scheduling and governing large and complex projects. All these techniques are based on the representation of the project as a Network diagram of activities. In 1956-58, PERT was developed by research team for planning and scheduling US Navy's Polaris Missile project involving very large number of activities. This technique was useful since 1958 for all jobs or projects having an element of uncertainty in the estimation of duration, just like with new types of projects.

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Such approach has never been taken up before. Critical path method (CPM) was developed independently, by E.I. Du Pont company with Remington Rand Corporation at the same time. The aim behind its development was to provide a technique for control of the maintenance of company's chemical plants. The main objective before beginning any project is to plan all mandatory activities in an proficient method so as to complete it within a definite time limit and with minimized cost for completion [1].

The time-cost compromise problem was developed and solved by heuristic algorithm and mathematical modeling by assuming a linear relation among cost as well as duration of an activity [2]. A special parametric linear program for CPM, that can be effectively solved by network flow methods was developed by the author. The model provides solutions to concerning project budget, labor requirements, procurement and plan restrictions, the results of slowdowns, and conveyance problems. A study defined the elementary Multiple Attribute Decision Making (MADM) ideas and introduced a standard notation. Authors presented a system of categorizing seventeen key MADM techniques. The mathematical calculation process of each technique is shown by explaining a modest numerical illustration [3]. A study developed an approach to give rank according to important properties [4]. Several academics have established mathematical model for the price and time compromise issues few of them are as follows: A study establish the link among project's whole duration and project's total cost [5]. In project network an exact process for the discrete cost duration compromise problem was describe [6]. A study analyzed FPNA and PERT for 32 networks in several groupings of activity symmetry, activity width and path accomplishment time inequality [7].

Fuzzy Delphi technique was applied to obtain uncertain project finishing duration and the criticality degree for each path in a project [8]. A study suggested method that includes the inexact activity durations, calculation of project arranging parameters and analysis of generated fuzzy results and compared this methodology with Monte Carlo simulation [9]. A study considered a foam block manufacture appliance plan and answered built on three approximations of duration for each activity by human sentence in place of stochastic suppositions to decide project finishing parameters in the PERT [10]. A new MCDM was presented built on the perceptions of perfect and antideal ideas. The comparative estimation values of several substitutes versus PIS are ranked to decide the finest substitute in the study [11].

A study expressed the linguistic relations into triangular FN and applied concept of TOPSIS for the ranking order of all substitutes [12]. A stepwise ranking technique was developed to define the ranking order of all distribution centre location. Here author considered criteria by linguistic variables and afterwards converted criterion in TFNs [13]. A study considered objective time as fuzzy numbers and applied Zadeh's extension principal to calculate criticality degree of all paths in project network [14]. A study defuzzified the fuzzy numbers into crisp values and applied TOPSIS for selecting plant location [15]. An enhanced fuzzy TOPSIS was developed [16]. A ranking process of the mean of the integral values to develop PIS and NIS and applied TOPSIS model for facility position selection was proposed [17].

A technique was developed technique of calculating of the fuzzy duration for floats and the maximum start and accomplishment of activities by applying the extension principle of Zadeh to the CPM [18]. TOPSIS is a multiple standard process to recognise explanations from a fixed set of substitutes built upon coinciding distance minimization from an best point and distance maximization from a base point. A fuzzy CPM built on ranking method and statistical confidence-interval was proposed to determine the order of critical paths in the network [19]. Numerous applications of TOPSIS were studied and made comparison against SMART and centroid weighting schemes [20]. Lin CT, Ying TC (2004) developed a fuzzy logic method where calculations were defined in linguistic terms and then mapped in to fuzzy values for bid decision making problem for contractor [21]. A merchant selection problem with various objectives was solved by fuzzy goal programming method [22]. A new fuzzy MCDM technique for the solution of distribution center choice problem under fuzzy situation by considering linguistic variable as triangular fuzzy numbers was developed [23]. TOPSIS was applied for suitable suppliers in supply chain system by considering many criterias like price, flexibility, quality and delivery performance [24].

TOPSIS was applied for choosing plant location under linguistic situations. This approach can handle the group decision-making complications in an extra capable manner as compare to methods available at that time [25]. A fuzzy MADM was utilized and developed an whole service performance index through TOPSIS for each pair of hotel for subjective judgments of qualitative attributes [26]. To make the ratings of applicant aircraft, TOPSIS was applied for assistance the Air Force Military institute in Taiwan to select best initial training aircraft in a fuzzy background [27]. A fuzzy number method was applied to solve structure contractor prequalification matters [28]. A study suggested an method to find CP for a project diagram having activity durations were *L-R* and *L-L* sorts fuzzy numbers by applying Yager ranking theory [29]. An interval-valued fuzzy TOPSIS was developed to deal with the situations where DMs are not able for common judgement on the scheme of describing linguistic terms built on the fuzzy sets [30]. So as to precisely attach the mathematical measures to the relative significance of the aspects and to the influence of the substitutes on these aspects in some cases the TOPSIS was extended for interval-valued fuzzy data [31].

A study categorized criteria into benefit and cost and applied ranking method to calculate PIS and NIS to solve the fuzzy TOPSIS [32]. A new fuzzy TOPSIS was developed for calculating substitutes by mixing by means of subjective and objective weight [33]. A new MADM built on the TOPSIS and ranking technique of trapezoidal interval type-2 fuzzy sets was developed [34]. A study considered weights of the choice standards as fuzzy pairwise comparison matrices and introduced a improved fuzzy TOPSIS for the issue of Energy planning [35]. A buyer based diffusion model containing of tens of thousands of interrelating buyers and applied fuzzy TOPSIS for MADM for purchasing a car was represented [36]. An effective use of fuzzy TOPSIS was presented to a real warehouse site choice problem of a big company in Iran [37]. A method was presented that combined linear programming model and Zadeh's extension principle for project planning problem in fuzzy environment [38]. A fuzzy number ranking based approach was proposed to calculate CP in trapezoidal fuzzy enviornment for a project network [39]. A ranking method was applied to present a new fuzzy CPM that was able rank all sorts of fuzzy numbers [40]. Thirty structure projects with key criteria-cost,quality, duration, environment sustainability and safety were studied [41]. *LR* flat fuzzy numbers were utilized for the first time for critical path selection in fuzzy background [42]. A method was developed based on goal programming to solve cost-duration-quality compromise problem by considering each criteria as grey number [43]. The process was describe of TOPSIS method in determining critical path with four criteria time, cost, risk and quality were considered as trapezoidal fuzzy numbers in project network [44]. A scenario-based preferences for capacity extension, strategic scheduling and operations of a shipping container port was proposed [45]. An algorithm was developed of realistic paths with the lowest cost and time for every activity [46]. An algorithm was proposed for interval networking problems to calculate CP for project network with single objective – time [47].

The originality of this paper is, we have considered three modes of execution of the project and developed a linear and exponential membership function based possibilistic approach to find optimum balanced critical path according to four criteria time, cost, risk and quality for multi-mode project management problem in trapezoidal fuzzy environment. This new approach does not require to calculate total performance score of all project path and it provide solution that maintain uncertainty in solution too.

II. FUZZY MULTI OBJECTIVE MULTI MODE CRITICAL PATH PROBLEM (FMOMMCPP) FORMULATION

The key assumptions and characteristics of the FMOMMCPP are as follows:

- All path of the project network will be considered.
- Dummy activity is considered with all objective values as zero.
- The decision making matrix should minimize Time, Cost, Risk and maximize Quality.
- Trapezoidal fuzzy numbers are considered for Linguistic variables.

III. FUZZY MULTI OBJECTIVE MULTI MODE CRITICAL PATH PROBLEM (FMOMMCPP) FORMULATION

The mathematical formulation of FMOMMCPP is made by using the following variables, parameters and the indices.

- Indices p and q defines path joining node p and q.
- "*a*" indicates mode of execution of activity.
- A = Set of arcs of the project network , $(p,q)\in A$
- M_{pq} is the set of available modes of execution for activities $p\, and\, q$, where $(p,q)\in A$
- t_{pq} indicates time acquired to complete activity (p, q)
- c_{pq} indicates cost required to complete activity (p, q)
- q_{pq} indicates the quality maintained during completion of activity (p, q)
- r_{pq} indicates the risk involved in completion of activity (p, q)

IV. DECISION VARIABLES

$$x_{pqa}=\begin{cases}1;\ if\ \text{mod}\,e\ a\ is\ assigned\ to\ activity\ (p,q)\\ 0;\ otherwise\end{cases}$$

V. FORMULATION OF OBJECTIVE FUNCTIONS

The total consumed time, total cost, total risk and total quality level are given as follows:

$$\tilde{z}_1=\sum_{p,q\in A}\sum_{a\in M_{pq}}\tilde{t}_{pqa}\,x_{pqa},\ \tilde{z}_2=\sum_{p,q\in A}\sum_{a\in M_{pq}}\tilde{c}_{pqa}\,x_{pqa},$$

$$\tilde{z}_3=\sum_{p,q\in A}\sum_{a\in M_{pq}}\tilde{r}_{pqa}\,x_{pqa},\ \tilde{z}_4=\sum_{p,q\in A}\sum_{a\in M_{pq}}\tilde{q}_{pqa}\,x_{pqa}$$

In this paper, the quality of the linguistic variable are ranked as "very low", "low"," medium low"," medium"," medium high"," high" and "very high" which are signified as trapezoidal fuzzy numbers. The six levels denote the quality of project completion, where" very low" and "very high" levels denote the least proficient and the most proficient, respectively, that is, a move from "very low" to "very high" shows that quality increases whereas the related fuzzy values decreases. Minimization of objective functions of quality is must, in order to conserve standardization objective functions.

VI. MODEL CONSTRAINTS

The constraints of FMOMMCPP are formulated as follows.

$$\sum_{q\in A, a\in M_{pq}} x_{1qa}=1 \qquad (1)$$

$$\sum_{q\in A, a\in M_{pq}} x_{pqa}=\sum_{k\in A, q\in M_{pq}} x_{kqa}\ ,n=2,3,...,k-1. \qquad (2)$$

$$\sum_{k\in A, a\in M_{pq}} x_{kpa}=1 \qquad (3)$$

$$x_{pqa}\geq 0, \forall (p,q)\in A, a\in M_{pq} \qquad (4)$$

VII. DECISION PROBLEM

The FMOMMCPP is now formulated as below:

(Model -1)

$$(\tilde{z}_1,\tilde{z}_2,\tilde{z}_3,\tilde{z}_4)=\left(\begin{array}{l}\sum_{p,q\in A}\sum_{a\in M_{pq}}\tilde{t}_{pqa}\,x_{pqa},\ \sum_{p,q\in A}\sum_{a\in M_{pq}}\tilde{c}_{pqa}\,x_{pqa},\\ \sum_{p,q\in A}\sum_{a\in M_{pq}}\tilde{r}_{pqa}\,x_{pqa},\ \sum_{p,q\in A}\sum_{a\in M_{pq}}\tilde{q}_{pqa}\,x_{pqa}\end{array}\right)$$

Subject to the constraints (1)-(4).

VIII. SOME PRELIMINARIES

To find the solution of this fuzzy project management problem some basic concepts are required which are as follows.

A. Possibilistic programming approach

Most of the time when we collect real-world problems related data then generally its include some kind of unreliability which are represented using fuzzy numbers because of their nature. Possibilistic distribution is utilized to quantify such kind of fuzzy numbers [48], [49],[50],[51],[52]. Many crucial applications have been used possibilistic programming approach for finding the solution of multi criteria based fuzzy optimization model with unspecific objective function. Hence in this paper we have utilized possibilistic programming based method to solve FMOCPP which maintain the uncertainty of the problem in real sense and converted the FMOCPP in crisp MOCPP.

B. Trapezoidal possibilistic distribution (TPD)

Trapezoidal possibility distribution [53] is used to represent the trapezoidal uncertain parameter. In particular, for the time coefficient $\tilde{t}_i=(t_i^o,t_i^m,t_i^{\bar{m}},t_i^p)$, decision maker can create the trapezoidal distribution by using $(t_i^o),([t_i^m,t_i^{\bar{m}}])$ and (t_i^p) where (t_i^o) and (t_i^p) are the most optimistic value and most pessimistic value respectively (possibility degree = 0), $([t_i^m,t_i^{\bar{m}}])$ is the interval of the most likely value that absolutely belongs to the set of available values (possibility degree = 1).

From Fig. 1, the time objective function is defined at four well-known points $(t_1^p,0),(t_1^m,1),(t_1^{\bar{m}},1)$ and $(t_1^o=0)$ it is minimized by pushing the four positions of trapezoidal possibility distribution. Since the left as vertical coordinates of the points are fixed by 1 or 0, there are only four horizontal coordinates considered.

C. α -cut sets

Several researchers [55], [56], [57] have applied α-cut concept to obtain the results for fuzzy optimization problems.

To set up a connection between traditional and fuzzy set theories, an α-cut is the extremely important concept which was introduced by Zadeh [54]. We have applied this theory in the proposed method on trapezoidal fuzzy number to conclude the assurance of the DM with respect to his fuzzy judgment.

Let $\tilde{b}=(b_1,b_2,b_3,b_4)$ be a TFN [53]. An α -cut for $\tilde{b}$, $\tilde{b}_\alpha$ is calculated as:

$$((b_2-b_1)\alpha+b_1,b_2,b_3,b_4-(b_4-b_3)\alpha)$$

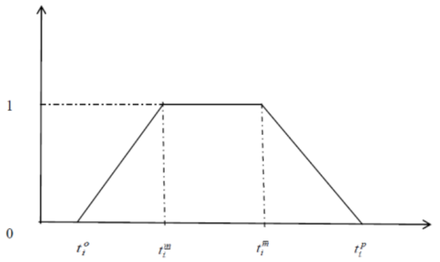

Fig. 1. Trapezoidal possibility distribution of t_i

D. Linear Membership and Exponential membership functions

Linear Membership and Exponential membership functions are defined as follow [58].

$$\mu_{z_{pq}}(x)=\begin{cases} 1 & ,if\ z_{pq}<z_{pq}^{NIS} \\ 1-\dfrac{z_{pq}-z_{pq}^{PIS}}{z_{pq}^{NIS}-z_{pq}^{PIS}} & ,if\ z_{pq}^{PIS}<z_{pq}<z_{pq}^{NIS} \\ 0 & ,if\ \ z_{pq}>z_{pq}^{NIS} \end{cases}$$

$$\mu_{z_{pq}}^{A}(x)=\begin{cases} 1 & ;if\ z_{pq}\le z_{pq}^{PIS} \\ \dfrac{e^{-s\psi_{pq}(x)}-e^{-s}}{1-e^{-s}} & ;if\ z_{pq}^{PIS}<z_{pq}<z_{pq}^{NIS} \\ 0 & ;if\ z_{pq}\ge z_{pq}^{NIS} \end{cases}$$

where $\psi_{pq}=\dfrac{z_{pq}-z_{pq}^{PIS}}{z_{pq}^{NIS}-z_{pq}^{PIS}}$ $where\, s>0(s<0)$, represents the concave (convex) nature of the membership function in $[z_{pq}^{PIS}, z_{pq}^{NIS}]$.

IX. CONSTRUCTION OF MULTI OBJECTIVE 0-1 PROGRAMMING MODEL

To transform model 1 into auxiliary multi-objective optimization model, we used Triangular possibilistic distribution (TPD) concept to study the inaccurate objectives. The Cost, time, risk and quality objective functions are described as

$$\min \tilde{z}_1=\min(z_1^o,z_1^m,z_1^{\bar{m}},z_1^p)=\sum_{p,q\in A}\sum_{a\in M_{pq}}\tilde{t}_{pqa}\,x_{pqa}$$

$$=\min\begin{pmatrix} \sum_{p,q\in A}\sum_{a\in M_{pq}} t_{pqa}^o\,x_{pqa},\ \sum_{p,q\in A}\sum_{a\in M_{pq}} t_{pqa}^m\,x_{pqa}, \\ \sum_{p,q\in A}\sum_{a\in M_{pq}} t_{pqa}^{\bar{m}}\,x_{pqa},\ \sum_{p,q\in A}\sum_{a\in M_{pq}} t_{pqa}^p\,x_{pqa} \end{pmatrix} \qquad (5)$$

(5)

where $t_{pq}=(t_{pq}^o,t_{pq}^m,t_{pq}^{\bar{m}},t_{pq}^p)$, similarly other objectives can be considered.

Using the α-cut concepts $(0\le\alpha\le1)$, each t_{ij} can be stated as.

$$(t_{pq})_\alpha=((t_{pq}^m-t_{pq}^o)\alpha+t_{pq}^0,\ t_{pq}^m,\ t_{pq}^{\bar{m}},\ t_{pq}^p-\alpha(t_{pq}^p-t_{pq}^{\bar{m}}))$$

Equation (5) can be written as:

$$(\min z_{11},\min z_{12},\min z_{13})=$$

$$\min\begin{pmatrix} \sum_{p,q\in A}\sum_{a\in M_{pq}}(t_{pqa})_\alpha^o\,x_{nma},\ \sum_{p,q\in A}\sum_{a\in M_{pq}}(t_{pqa})_\alpha^m\,x_{pqa}, \\ \sum_{p,q\in A}\sum_{a\in M_{pq}}(t_{pqa})_\alpha^{\bar{m}}\,x_{pqa},\ \sum_{p,q\in A}\sum_{a\in M_{pq}}(t_{pqa})_\alpha^p\,x_{pqa} \end{pmatrix} \qquad (6)$$

X. MULTI-OBJECTIVE 0-1 AUXILIARY MODEL

To define the most-likely, pessimistic and optimistic, situations by applying α-level set theory, the FMOMMCPP is transformed into a crisp MOMMCPP which is defined in 0-1 programming model as follows:

(Model 2)

$$(\min z_{11},\min z_{12},\min z_{13},\min z_{21},\min z_{22},\min z_{23},$$
$$\min z_{31},\min z_{32},\min z_{33},\min z_{41},\min z_{42},\min z_{43})$$

$$=\begin{pmatrix} \sum_{p,q\in A}\sum_{a\in M_{pq}}(t_{pqa})_\alpha^o\,x_{pqa},\ \sum_{p,q\in A}\sum_{a\in M_{pq}}(t_{pqa})_\alpha^m\,x_{pqa}, \\ \sum_{p,q\in A}\sum_{a\in M_{pq}}(t_{pqa})_\alpha^{\bar{m}}\,x_{pqa},\ \sum_{p,q\in A}\sum_{a\in M_{pq}}(t_{pqa})_\alpha^p\,x_{pqa}, \\ \sum_{p,q\in A}\sum_{a\in M_{pq}}(c_{pqa})_\alpha^o\,x_{pqa},\ \sum_{p,q\in A}\sum_{a\in M_{pq}}(c_{pqa})_\alpha^m\,x_{pqa}, \\ \sum_{p,q\in A}\sum_{a\in M_{pq}}(c_{pqa})_\alpha^{\bar{m}}\,x_{pqa},\ \sum_{p,q\in A}\sum_{a\in M_{pq}}(c_{pqa})_\alpha^p\,x_{pqa}, \\ \sum_{p,q\in A}\sum_{a\in M_{pq}}(r_{pqa})_\alpha^o\,x_{pqa},\ \sum_{p,q\in A}\sum_{a\in M_{pq}}(r_{pqa})_\alpha^m\,x_{pqa}, \\ \sum_{p,q\in A}\sum_{a\in M_{pq}}(r_{pqa})_\alpha^{\bar{m}}\,x_{pqa},\ \sum_{p,q\in A}\sum_{a\in M_{pq}}(r_{pqa})_\alpha^p\,x_{pqa}, \\ \sum_{p,q\in A}\sum_{a\in M_{pq}}(q_{pqa})_\alpha^o\,x_{pqa},\ \sum_{p,q\in A}\sum_{a\in M_{pq}}(q_{pqa})_\alpha^m\,x_{pqa}, \\ \sum_{p,q\in A}\sum_{a\in M_{pq}}(q_{pqa})_\alpha^{\bar{m}}\,x_{pqa},\ \sum_{p,q\in A}\sum_{a\in M_{pq}}(q_{pqa})_\alpha^p\,x_{pqa} \end{pmatrix}$$

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XI. FUZZY PROGRAMMING TECHNIQUE-BASED APPROACH TO SOLVE AUXILIARY MODEL OF FMOMMCPP

For finding the solution of the Models 2 by fuzzy programming technique first these models are solved for single objective function and for each objective function the NIS and PIS are obtained for the model. Now, by PIS and NIS describe a membership function $\mu(Z_k)$ for the k^{th} objective function. Here, linear and exponential membership functions are utilized to find an efficient solution of this MOMMCPP and by using this membership function the Models 2 is converted into the following model:

A. Model 3

Max λ,

Subject to the constraints:

$$\lambda \le \mu_{z_{pq}} \;;0 \le \lambda \le 1$$

equation (1) to equation (4).

When we utilize fuzzy linear membership function,

$$\mu_{z_{pq}}(x) = \begin{cases} 1, & ;\text{if } z_{pq} \le z_{pq}^{PIS}, \\ \dfrac{z_{pq}^{NIS} - z_{pq}}{z_{pq}^{NIS} - z_{pq}^{PIS}} & ;\text{if } z_{pq}^{PIS} < z_{pq} < z_{pq}^{NIS}, \\ 0, & ;\text{if } z_{pq} \ge z_{pq}^{NIS} \end{cases}$$

then model 3 structure is as follows:

B. Model 4

Max λ,

Subject to the constraints:

$$\lambda \le \frac{z_{pq}^{NIS} - z_{pq}}{z_{pq}^{NIS} - z_{pq}^{PIS}}$$

equation (1) to equation (4).

When we utilize exponential membership function,

$$\mu_{z_{pq}}(x) = \begin{cases} 1, & ,\text{if } z_{pq} \le z_{pq}^{PIS}, \\ \dfrac{e^{-S\Psi_k(x)} - e^{-S}}{1 - e^{-S}} & ,\text{if } z_{pq}^{PIS} < z_{pq} < z_{pq}^{NIS}, \\ 0 & ,\text{if } z_{pq} \ge z_{pq}^{NIS} \end{cases}$$

where, $\Psi_{pq}(x) \le \dfrac{z_{pq} - z_{pq}^{PIS}}{z_{pq}^{NIS} - z_{pq}^{PIS}}$

Structure of Model 3 is as follows:

C. Model 5

Max λ,

Subject to the constraints:

$$\left(e^{-S\Psi_k(x)} - e^{-S}\right) \ge \lambda\left(1 - e^{-S}\right) \quad ,$$

where $\Psi_k(x) \le \dfrac{z_k(x) - z_{pq}^{PIS}}{z_{pq}^{NIS} - z_{pq}^{PIS}}, k = 1, 2, \ldots, n$

with constraints (1) to (4).

XII. FLOW CHART

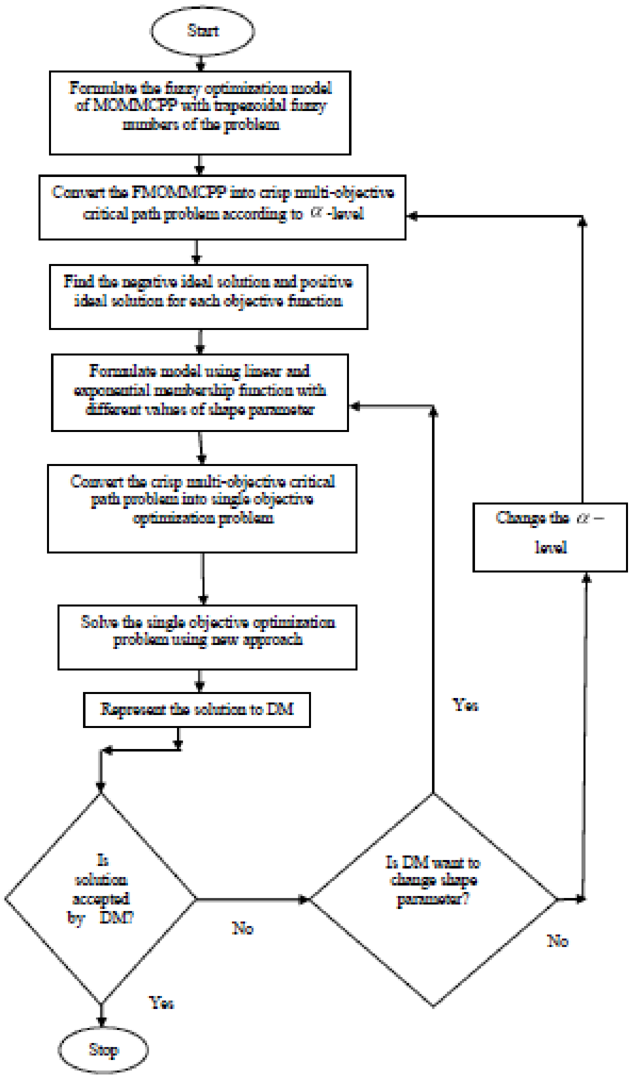

Fig. 2. Flow chart Fuzzy programming technique-based approach to solve Auxiliary model of FMOMMCPP

XIII. NUMERICAL ILLUSTRATION

In this section, multi objective multi-mode critical path problem is formulated [44]. We consider a project having 13 main activities with three modes of execution. A project network is given in Fig. 3. Mapping of linguistic variable is considered in to triangular fuzzy number. Fuzzy values for each criteria Time, Cost, Risk and Quality multiplied by corresponding weights with three modes of executions are given in Table I,II,III and IV respectively.

Table- I: Fuzzy information of activities on the time criteria with three modes of execution with $\alpha = 0.1$

Act ivity	MODE		
	M_1	M_2	M_3
1-2	(4.3,7, 10,11.8)	(7.2,9, 10,11.8)	(3.2,5 ,7,8.8)
1-3	(3.3,6, 9,11.7)	(0,0, 0,0)	(0,0, 0,0)
1-4	(2.2,4, 6,7.8)	(3.2,5, 7,8.8)	(5.2,7, 9,10.8)
2-5	(3.2,5, 7,8.8)	(0,0, 0,0)	(0,0, 0,0)
3-5	(3.1,4, 5,5.9)	(7.2,9, 10,11.8)	(0,0, 0,0)
4-6	(2.1,3, 4,4.9)	(0.1,1 ,1,1)	(7.2,9, 10,11.8)
8-9	(8.2,10, 12,13.8)	(0,0, 0,0)	(0,0, 0,0)
3-6	(2.2,4, 6,7.8)	(9,9, 10,10)	(0,0, 0,0)
5-7	(5.3,8, 11,13.7)	(0,0, 0,0)	(0,0 ,0,0)
4-8	(4.1,5, 6,6.9)	(0.1,1, 3,4.8)	(7.2,9, 10,11.8)
6-10	(3.3,6, 9,11.7)	(7.2,9, 10,11.8)	(9,9, 10,10)
7-10	(3.2,5, 7,8.8)	(0,0, 0,0)	(0,0, 0,0)
9-10	(4.1,5, 6,6.9)	(0,0 ,0,0)	(0,0, 0,0)

Table- II: Fuzzy information of activities on the cost criteria with three modes of execution with $\alpha = 0.1$

Act ivity	MODE		
	M_1	M_2	M_3
1-2	(1550,2000, 2500,2950)	(2230,2500, 3200,3740)	(1660,2200 , 2800,3430)
1-3	(1050,1500, 2000,3350)	(0,0, 0,0)	(0,0, 0,0)
1-4	(250,700, 1200,1650)	(260,800, 1250,1745)	(1850,2300 , 2900,3485)
2-5	(250,700, 1200,1650)	(0,0, 0,0)	(0,0, 0,0)
3-5	(1550,2000, 2500,2950)	(2640,3000, 3600,3960)	(0,0, 0,0)
4-6	(5550,6000, 6500,6950)	(795,1200, 1800,2250)	(2230,2500 , 3200,3740)
8-9	(1550,2000, 2500,2950)	(0,0, 0,0)	(0,0, 0,0)
3-6	(1550,2000, 2500,2950)	(2230,2500, 3200,3740)	(0,0, 0,0)
5-7	(750,1200, 1700,2150)	(0,0, 0,0)	(0,0, 0,0)
4-8	(1050,1500, 2000,2450)	(2230,2500, 3200,3740)	(2640,3000 , 3600,3960)
6-10	(1010,2000, 2500,4750)	(1020,2100, 2600,4940)	(1660,2200 , 2800,3430)
7-10	(3550,4000, 4500,4950)	(0,0, 0,0)	(0,0, 0,0)
9-10	(2550,3000, 3500,3950)	(0,0, 0,0)	(0,0, 0,0)

Table- III: Fuzzy information of activities on the risk criteria with three modes of execution with $\alpha = 0.1$

Act ivity	MODE		
	M_1	M_2	M_3
1-2	(5.2,7, 9,10.8)	(7.2,9, 10,11.8)	(3.2,5,7,8.8)
1-3	(1.2,3, 5,6.8)	(0,0, 0,0)	(0,0,0,0)
1-4	(1.2,3, 5,6.8)	(3.2,5, 7,8.8)	(5.2,7,9,10.8)
2-5	(3.2,5, 7,8.8)	(0,0, 0,0)	(0,0,0,0)
3-5	(3.2,5, 7,8.8)	(7.2,9, 10,11.8)	(0,0,0,0)
4-6	(3.2,5, 7,8.8)	(0.1,1, 1,1)	(7.2,9, 10,11.8)
8-9	(1.2,3, 5,6.8)	(0,0, 0,0)	(0,0, 0,0)
3-6	(1.2,3, 5,6.8)	(9,9, 10,10)	(0,0, 0,0)
5-7	(1.2,3, 5,6.8)	(0,0, 0,0)	(0,0, 0,0)
4-8	(5.2,7, 9,10.8)	(0.1,1, 3,4.8)	(7.2,9, 10,11.8)
6-10	(3.2,5, 7,8.8)	(7.2,9, 10,11.8)	(9,9, 10,10)
7-10	(5.2,7, 9,10.8)	(0,0, 0,0)	(0,0, 0,0)
9-10	(5.2,7, 9,10.8)	(0,0, 0,0)	(0,0, 0,0)

Table- IV: Fuzzy information of activities on the minimized quality criteria with three modes of execution with $\alpha = 0.1$

Activ ity	MODE		
	M_1	M_2	M_3
1-2	(5.2,7, 9,10.8)	(7.2,9, 10,11.8)	(3.2,5, 7,8.8)
1-3	(1.2,3, 5,6.8)	(0,0, 0,0)	(0,0, 0,0)
1-4	(1.2,3, 5,6.8)	(3.2,5, 7,8.8)	(5.2,7, 9,10.8)
2-5	(3.2,5, 7,8.8)	(0,0, 0,0)	(0,0, 0,0)
3-5	(3.2,5, 7,8.8)	(7.2,9, 10,11.8)	(0,0, 0,0)
4-6	(3.2,5, 7,8.8)	(0.1,1, 1,1)	(7.2,9, 10,11.8)
8-9	(1.2,3, 5,6.8)	(0,0, 0,0)	(0,0, 0,0)
3-6	(1.2,3, 5,6.8)	(9,9, 10,10)	(0,0, 0,0)
5-7	(1.2,3, 5,6.8)	(0,0, 0,0)	(0,0, 0,0)
4-8	(5.2,7, 9,10.8)	(0.1,1, 3,4.8)	(7.2,9, 10,11.8)
6-10	(3.2,5, 7,8.8)	(7.2,9, 10,11.8)	(9,9, 10,10)
7-10	(5.2,7, 9,10.8)	(0,0, 0,0)	(0,0, 0,0)
9-10	(5.2,7, 9,10.8)	(0,0, 0,0)	(0,0, 0,0)

XIV. FORMULATION OF CONSTRAINTS

In this problem there are multi-mode between each activity so to convert this problem in mathematical form we have rearrange the project network using dummy activity shown in figure 3 by dotted arrows.

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The weight of each dummy activity is taken to be zero that means there is no effect in optimal solution while we added a dummy activity.

Constraints of FMOMMCPP are formulated as below.

$$
\begin{aligned}
& x_{121}+x_{122}+x_{123}+x_{131}+x_{132}+x_{133}+x_{141}+x_{142}+x_{143}=1,\\
& x_{212}+x_{222}+x_{232}=x_{251}+x_{252}+x_{253},\\
& x_{313}+x_{323}+x_{333}=x_{354}+x_{355}+x_{356}+x_{361}+x_{362}+x_{363},\\
& x_{414}+x_{424}+x_{434}=x_{464}+x_{465}+x_{466}+x_{481}+x_{482}+x_{483},\\
& x_{515}+x_{525}+x_{535}+x_{545}+x_{555}+x_{565}=x_{571}+x_{572}+x_{573},\\
& x_{616}+x_{626}+x_{636}+x_{646}+x_{656}+x_{666}=x_{6101}+x_{6101}+x_{6101},\\
& x_{717}+x_{727}+x_{737}=x_{7104}+x_{7105}+x_{7106},\\
& x_{818}+x_{828}+x_{838}=x_{891}+x_{892}+x_{893},\\
& x_{919}+x_{929}+x_{939}=x_{9107}+x_{9108}+x_{9109},\\
& x_{10110}+x_{10210}+x_{10310}+x_{10410}+x_{10510}+x_{10610}+x_{10710}\\
& +x_{10810}+x_{10910}=1.\\
& x_{121}=x_{212},x_{122}=x_{222},\ x_{123}=x_{232},x_{131}=x_{313},\\
& x_{132}=x_{323},\ x_{133}=x_{333},x_{141}=x_{414},x_{142}=x_{424},\\
& x_{143}=x_{434},x_{251}=x_{515},\ x_{252}=x_{525},x_{253}=x_{535},\\
& x_{354}=x_{545},\ x_{355}=x_{555},x_{356}=x_{565},x_{361}=x_{616},\\
& x_{362}=x_{626},x_{363}=x_{636},x_{464}=x_{646},x_{465}=x_{656},\\
& x_{466}=x_{666},x_{571}=x_{717},x_{572}=x_{727},\ x_{573}=x_{737},\\
& x_{481}=x_{818},x_{482}=x_{828},x_{483}=x_{838},x_{891}=x_{919},\\
& x_{892}=x_{929},x_{893}=x_{939},x_{6101}=x_{10110},\\
& x_{6102}=x_{10210},x_{6103}=x_{10310},x_{7104}=x_{10410},\\
& x_{7105}=x_{10510},x_{7106}=x_{10610},x_{9107}=x_{10710},\\
& x_{9108}=x_{10810},x_{9109}=x_{10910}.\forall x_{pqk}\geq 0.
\end{aligned}
\tag{7}
$$

XV. SOLUTION PROCEDURE

Obtain PIS and NIS

To find the solution of this model the fuzzy programming technique based developed approach is utilized and for that at different α level the value of each objective PIS and NIS are mentioned in Table V, subject to constraints (7).

XVI. RESULT AND DISCUSSION

The solution of model 4 with PIS and NIS obtained in Table-V by using LINGO software is given in Table VI.

The solution shows (Table: VI) that at different α level the optimal degree of satisfaction and different optimal path both are different. At $\alpha = 0, 0.1, 0.5$ and 0.9 degree of satisfaction are 0.5238095, 0.5233645, 0.5315315 and 0.5828414 respectively. For $\alpha = 0.9$ maximum degree of satisfaction 0.5828414 is attained. Results shows that for different values of α levels critical path 1-3-6-10 remains same with different mode of executions. Value of time, cost, risk and quality objective functions at different α level are also mentioned in Table VI. Fig. 4 indicates values of time, cost, risk and quality objective functions at different α levels with model 4.

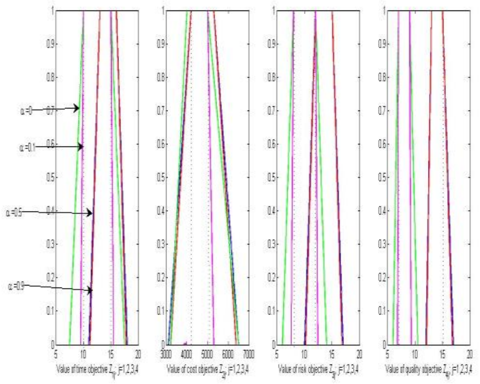

Fig. 4. Solution of Time, cost, risk and quality objectives with model 4.

Table VII indicates the solution of FMOMMCPP with exponential membership function by fuzzy programming technique (Model 5). It shows that at α level 0 and shape parameters (-1,-1,-1,-1), (-0.2,-0.3,-0.5,-0.9), (-0.1,-0.4,-0.8,-0.9) and (-0.3,-0.5,-0.6,-0.8) the optimal degree of satisfaction are 0.6450262, 0.5792443, 0.5792443 and 0.581871 respectively. The critical path 1-3-6-10 remains same with different modes of execution with for all values of shape parameter at α level 0. At α level 0.1 and shape parameters (-1,-1,-1,-1), (-0.2,-0.3,-0.5,-0.9), (-0.1,-0.4,-0.8,-0.9) and (-0.3,-0.5,-0.6,-0.8) the optimal degree of satisfaction are 0.6446083, 0.5820276, 0.5820276 and 0.5834461 respectively. The critical path 1-3-6-10 remains same with different modes of execution with for all values of shape parameter at α level 0.1. At α level 0.5 and shape parameters (-1,-1,-1,-1), (-0.2,-0.3,-0.5,-0.9), (-0.1,-0.4,-0.8,-0.9) and (-0.3,-0.5,-0.6,-0.8) the optimal degree of satisfaction are 0.6646805, 0.5938032, 0.5941355 and 0.6062139 respectively. The critical path 1-3-6-10 remains same with different modes of execution with for all values of shape parameter at α level 0.5. At α level 0.9 and shape parameters (-1,-1,-1,-1), (-0.2,-0.3,-0.5,-0.9), (-0.1,-0.4,-0.8,-0.9) and (-0.3,-0.5,-0.6,-0.8) the optimal degree of satisfaction are 0.6926805, 0.612289, 0.6070698 and 0.6278725 respectively. The critical path 1-3-6-10 remains same with different modes of execution with for all values of shape parameter at α level 0.9. With

Table- V: PIS and NIS for each objective

α level	Solutions	Objective															
		z_{11}	z_{12}	z_{13}	z_{14}	z_{21}	z_{22}	z_{23}	z_{24}	z_{31}	z_{32}	z_{33}	z_{34}	z_{41}	z_{42}	z_{43}	z_{44}
0	PIS	24	31	37	47	8900	10500	12500	15750	21	26	33	41	36	36	40	48
	NIS	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
0.1	PIS	24.7	31	37	46	9060	10500	12500	15375	21.4	26	33	40.2	36	36	40	47.2
	NIS	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.9
0.5	PIS	27.5	31	37	42	9700	10500	12500	13875	23	26	33	37	36	36	40	44
	NIS	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.5
0.9	PIS	30.3	31	37	38	10340	10500	12500	12705	25.2	26	33	33.8	36	36	40	40.8
	NIS	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.1

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Table- VI: Results for $\alpha = 0, 0.1, 0.5$ and 0.9 using model 4

α	λ	**Critical path**	**Objective values (z_1, z_2, z_3, z_4) on critical path**	**Critical path in [44]**	**Objective values (z_1, z_2, z_3, z_4) [44]**
0	0.5238095	1-3(3)-6(1)-10(3)	((11,13,16,18),(3100,4200,5300,6500),(10,12,15,17), (12,13,15,17))	1-3-6-10	((8,16,24,32),(3400,5500,7000,1150),(5,11,17,23),(8,11,14,18))
0.1	0.5233645	1-3(3)-6(1)-10(3)	((11.2,13,16,17.8),(3210,4200,5300,6380),(10.2,12,15,16.8),(12.1,13,15,16.8))		
0.5	0.5315315	1-3(3)-6(1)-10(1)	((7.5,10,15,17.5),(3200,4000,5000,6500),(6,8,12,14),(6,7,9,10.5))		
0.9	0.5828414	1-3(3)-6(1)-10(1)	((9.5,10,15,15.5),(3840,4000,5000,5300),(7.6,8,12,12.4),(6.8,7,9,9.3))		

Table –VII: Solution applying model 5 and Comparison of solution with other approach

Alpha cut α	**Shape parameter (S_1, S_2, S_3, S_4)**	**Degree of satisfaction λ**	**Critical path**	**Objective values (z_1, z_2, z_3, z_4) on critical path**	**Critical path in [44]**	**Objective values (z_1, z_2, z_3, z_4) [44]**
0	Case - 1	0.6450262	1-3(3)-6(1)-10(3)	((11,13,16,18),(3100,4200,5300,6500)),(10,12,15,17),(12,13,15,17))	1-3-6-10	((8,16,24,32),(3400,5500,7000,11500),(5,11,17,23),(8,11,14,18))
	Case - 2	0.5792443	1-3(3)-6(3)-10(3)	((9,9,10,10),(1600,2200,2800,3500),(9,9,10,10),(9,9,10,10))		
	Case - 3	0.5792443	1-3(3)-6(3)-10(1)	((3,6,9,12),(900,2000,2500,5000),(3,5,7,9),(2,3,4,5))		
	Case - 4	0.581871	1-3(3)-6(1)-10(1)	((5,10,15,20),(2400,4000,5000,8000),(4,8,12,16),(5,7,9,12))		
0.1	Case - 1	0.6446083	1-3(3)-6(1)-10(3)	((11.2,13,16,17.8),(3210,4200,5300,6380)),(10.2,12,15,16.8),(12.1,13,15,16.8))		
	Case - 2	0.5820276	1-3(3)-6(3)-10(3)	((9,9,10,10),(1660,2200,2800,3430)),(9,9,10,10),(9,9,10,10))		
	Case - 3	0.5820276	1-3(3)-6(3)-10(3)	((9,9,10,10),(1660,2200,2800,3430)),(9,9,10,10),(9,9,10,10))		
	Case - 4	0.5834461	1-3(3)-6(1)-10(3)	((11.2,13,16,17.8),(3210,4200,5300,6380)),(10.2,12,15,16.8),(12.1,13,15,16.8))		
0.5	Case - 1	0.6646805	1-3(2)-6(1)-10(1)	((7.5,10,15,17.5),(3200,4000,5000,6500),(6,8,12,14),(6,7,9,10.5))		
	Case - 2	0.5938032	1-3(2)-6(3)-10(3)	((9,9,10,10),-1900220028003150,(9,9,10,10),(9,9,10,10))		
	Case - 3	0.5941355	1-3(3)-6(1)-10(1)	((7.5,10,15,17.5),(3200,4000,5000,6500),(6,8,12,14),(6,7,9,10.5))		
	Case - 4	0.6062139	1-3(3)-6(1)-10(1)	((7.5,10,15,17.5),(3200,4000,5000,6500),(6,8,12,14),(6,7,9,10.5))		
0.9	Case - 1	0.6926805	1-3(3)-6(1)-10(1)	((9.5,10,15,15.5),(3840,4000,5000,5300)),(7.6,8,12,12.4),(6.8,7,9,9.3))		
	Case - 2	0.612289	1-3(2)-6(1)-10(1)	((9.5,10,15,15.5),(3840,4000,5000,5300)),(7.6,8,12,12.4),(6.8,7,9,9.3))		
	Case - 3	0.6070698	1-3(2)-6(3)-10(3)	((9,9,10,10),(2140,2000,2800,2870),(9,9,10,10),(9,9,10,10))		
	Case - 4	0.6278725	1-3(2)-6(1)-10(1)	((9.5,10,15,15.5),(3840,4000,5000,5300)),(7.6,8,12,12.4),(6.8,7,9,9.3))		

four cases of shape parameter we get good degree of satisfaction that helps DM to take the decisions.

The path that optimizes all four objectives in [44] is 1-3-6-10 and value of objective functions time, cost, risk and quality according to this path is ((8,16,24,32),(3400,5500,7000,1150), (5,11,17,23),(8,11,14,18)). Our new approach provides the different satisfactory objective values at $\alpha = 0, 0.1, 0.5$ and 0.9.

Our developed new approach gives the same output as closely related existing solution approach which shows that the developed solution approach provides additional optimal degree of satisfaction to take the decision to decision makers. The Fig.5 indicates the distribution of objective values with respect to exponential membership function at different α level.

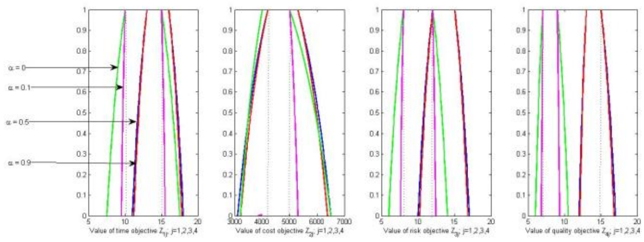

Fig. 5. Solution of time, cost, risk and quality objectives with model 5

XVII. CONCLUSION

This paper presents a new multi objective multi-mode model in fuzzy environment and methods to find the critical path when more than one mode of executions in project network are available. In this paper we have developed new approach which obtains critical path that optimize all four objective functions time, cost, risk and quality. and will give us optimum critical path.

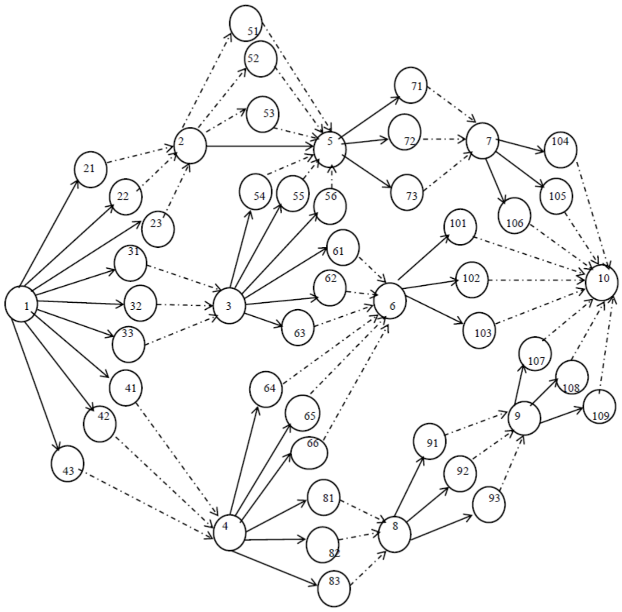

Fig. 3. Multi Mode Project Network

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