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Fuzzy Magic and Bi-magic Labelling of Intuitionistic Path Graph

D. Ajay, J. Joseline Charisma, P. Chellamani

***Abstract**: A graph labelling is an assignment of integers to the vertices or edges, or both, subject to certain conditions. Fuzzy labelling models precision, flexibility and compatibility to the classical models. The objective is to discuss about the magic and bi-magic labelling of fuzzy graphs. In the beginning the magic labelling of fuzzy path graph is discussed followed by the bi-magic labelling of fuzzy path graph. The next main purpose is to introduce the Intuitionistic path graph and examine the existence of fuzzy magic and bi-magic labelling.*

***Keywords**: Fuzzy path graph, fuzzy magic labelling, fuzzy bi-magic labelling, Intuitionistic fuzzy path graph.*

I. INTRODUCTION

The concept 'fuzzy' was introduced by Lotfi A Zadeh [1] in the year 1965, which gives the phenomena of uncertainty and vagueness. The classical sets are replaced by the fuzzy sets and this plays more vital role in giving accuracy and precision in theory. The fuzzy concept has widespread applications in mathematical fields. A graph is a convenient way of representing information involving the object's relationship. The vertices and edges of a graph represent the objects and their relations respectively. Graph theory is a very important tool to represent many real world problems.

The concept of fuzzy graphs was introduced by Kauffman and developed by Azirel Rosenfield [3]. Several fuzzy analogues graph theoretic concepts such as paths, cycles and connectedness have been studied. In general, a fuzzy graph model is necessary because of vagueness in describing the objects and their relationships. Crisp and fuzzy graphs are similar in structure. However, fuzzy graph is emphasized more when there is uncertainty on vertices and edges. Fuzzy graph has more application as data mining, image segmentation, clustering, image capturing, networking, planning and scheduling.The notion Graph labelling was introduced by Rosa [2]. A graph labelling is a mapping that carries a set of graph's elements onto a set of numbers called labels. Various labelling such as graceful, cordial, mean and magic have already been studied. The new concept of fuzzy labelling was introduced by A NagoorGani et al [4]. The notion of bi-magic labelling was introduced by Basker Babujee [7], in which we can find two magic values. In this paper, taking the lead of magic labelling, we have shown fuzzy magic, bi-magic labelling of Path graph and also fuzzy magic, bi-magic labelling of Intuitionistic Path graph.

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II. PRELIMINARIES

A fuzzy graph $G = (\sigma, \mu)$ is a pair of functions $\sigma: V \to [0,1]$ and $\mu: V \times V \to [0,1]$, where for all $u, v \in V$,

$$\mu(u,v) \leq \sigma(u) \wedge \sigma(v).$$

A labelling of a graph is an assignment of values to the vertices and the edges of a graph. The graph G has fuzzy labelling if it is bijective such that the membership value of edges and vertices are distinct and

$$\mu(u,v) < \sigma(u) \wedge \sigma(v) \text{ for all } u, v \in V$$

A fuzzy labelling of graph $G = (\sigma, \mu)$ is said to be a fuzzy magic labelling, if there exists an m such that

$$\sigma(x) + \sigma(y) + \mu(xy) = m \text{ for all } xy \in E \text{ and } x, y \in V$$

A fuzzy labelling graph G is said to be a fuzzy bi-magic labelling graph if there exist m_1 and m_2 such that

$$\sigma(x) + \sigma(y) + \mu(xy) = m_1 \text{ or } m_2 \text{ for all } xy \in E \text{ and } x, y \in V$$

An Intuitionistic fuzzy graph is of the form $G = (V, E)$ where $V = \{v_1, v_2, \ldots, v_n\}$ such that $\mu_1: V \to [0,1]$ and $\gamma_1: V \to [0,1]$ denote the degree of membership and non-membership of the element $v_i \in V$, respectively, and $0 \leq \mu_1(v_i) + \gamma_1(v_i) \leq 1$ for every $v_i \in V, (i = 1,2,3, \ldots, n)$, E⊆V×V where $\mu_2: V \times V \to [0,1]$ and $\gamma_2: V \times V \to [0,1]$ are such that $\mu_2(v_i, v_j) \leq min[\mu_1(v_i), \mu_1(v_j)]$ and $\gamma_2(v_i, v_j) \leq max[\gamma_1(v_i), \gamma_1(v_j)]$, $0 \leq \mu_2(v_i, v_j) + \gamma_2(v_i, v_j) \leq 1$, for every $(v_i, v_j) \in E$ $(i,j = 1,2, \ldots, n)$.

An Intuitionistic fuzzy labelling graph is an Intuitionistic fuzzy magic labelling graph if there exists an M such that

$$M = \{\mu_1(v_i) + \mu_1(v_j) + \mu_2(v_i v_j),\ \ \gamma_1(v_i) + \gamma_1(v_j) + \gamma_2(v_i v_j)\}$$

An Intuitionistic fuzzy labelling graph is an Intuitionistic fuzzy magic labelling graph if there exists M_1, M_2 such that

$$M_1 \text{ or } M_2 = \{\mu_1(v_i) + \mu_1(v_j) + \mu_2(v_i v_j),\ \gamma_1(v_i) + \gamma_1(v_j) + \gamma_2(v_i v_j)\}$$

III. FUZZY MAGIC AND BI-MAGIC LABELLING OF INTUITIONISTIC PATH GRAPH

Taking fuzzy magic labelling of simple graphs [5] as a lead, we proceed for the magic and bi-magic labelling of Path graph (P_m).

Theorem 3.1 Any Path graph P_m, $m \geq 2$ ($m \in N$), admits fuzzy magic labelling.

Proof: Let $m \in N$, and $m \geq 2$, $V = \{v_1, v_2, v_3, v_4, \dots \dots, v_m\}$ and $E = \{e_i = v_i v_{i+1} | 1 \leq i \leq m-1\}$ and Path graph be $P_m = (V, E)$.

For all i ($1 \leq i \leq m$), the membership value for the vertices $\sigma: V \rightarrow [0,1]$, is defined as follows:

$$\sigma(v_i) = \begin{cases} \dfrac{4m - (i + (m-1))}{2(2m-1)} & \text{if } i \text{ is odd} \\ \dfrac{3m - (i + (m-1))}{2(2m-1)} & \text{if } i \text{ is even and } m \text{ is odd} \\ \dfrac{3m - (i + (m-2))}{2(2m-1)} & \text{if } i \text{ is even and } m \text{ is even} \end{cases}$$

The membership of edges $\forall i, (1 \leq i \leq m-1)$, are defined by the function $\mu: E \rightarrow [0,1]$ by

$$\mu(e_i) = \left\{\frac{i}{2m-1}\right\}, \quad \text{where } e_i = v_i v_{i+1} \in E.$$

For fuzzy labelling, the main objective of the relation is that $\mu(e_i) < \sigma(v_i) \wedge \sigma(v_{i+1})$, this condition must be satisfied. To check this relation there are 4 cases:

Case (1) i – odd and m – odd

If i is odd then $\sigma(v_i) = \left\{\frac{4m-(i+(m-1))}{2(2m-1)}\right\}$ and $i = i+1$, becomes even, then $\sigma(v_{i+1}) = \left\{\frac{3m-(i+(m-1))}{2(2m-1)}\right\}$.

We know $3m < 4m$, and also

$$3m - (i + (m-1)) < 4m - (i + (m-1))$$

$$\Rightarrow \frac{3m-(i+(m-1))}{2(2m-1)} < \frac{4m-(i+(m-1))}{2(2m-1)}$$

$$\Rightarrow \sigma(v_i) \wedge \sigma(v_{i+1}) = \frac{3m - (i + (m-1))}{2(2m-1)}$$

$$i < 3m \Rightarrow i < 3m - (i + (m-1)) \quad \Rightarrow \frac{i}{2m-1} < \frac{3m-(i+(m-1))}{2(2m-1)}$$

$$\Rightarrow \mu(e_i) < \sigma(v_i) \wedge \sigma(v_{i+1})$$

Similarly, for all the other 3 cases (namely, i is even & m is odd, i is odd & m is even, i & m are even) the relation of fuzzy labelling $\mu(e_i) < \sigma(v_i) \wedge \sigma(v_{i+1})$ is verified.

Thus, Path graph is fuzzy labelling. Now we are ought to prove the existence of the magic value of Path graph.

i.e. $\sigma(v_i) + \sigma(v_{i+1}) + \mu(v_i v_{i+1}) = M$.

We have the following cases for the magic value existence

Case (i) m – odd

When m is odd, we get

$$\sigma(v_i) = \frac{4m - (i + (m-1))}{2(2m-1)} \quad \text{and} \quad \sigma(v_{i+1}) = \frac{3m - (i + (m-1))}{2(2m-1)}$$

Thus $M = \frac{4m-(i+(m-1))}{2(2m-1)} + \frac{3m-(i+(m-1))}{2(2m-1)} + \frac{i}{2m-1}$

$$M = \frac{5m+2}{2(2m-1)}$$

Case (ii) m – even

When m is even, we get

$$\sigma(v_i) = \frac{4m - (i + (m-1))}{2(2m-1)} \quad \text{and} \quad \sigma(v_{i+1}) = \frac{3m - (i + (m-2))}{2(2m-1)}$$

Thus $M = \frac{4m-(i+(m-1))}{2(2m-1)} + \frac{3m-(i+(m-2))}{2(2m-1)} + \frac{i}{2m-1}$

$$M = \frac{5m+3}{2(2m-1)}$$

Therefore, the magic value is

$$M = \begin{cases} \dfrac{5m+2}{2(2m-1)} & \text{if } m \text{ is odd} \\ \dfrac{5m+3}{2(2m-1)} & \text{if } m \text{ is even} \end{cases}$$

Example The magic value of the graph P_{12} is 1.35.

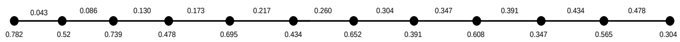

Fig. 1.A fuzzy magic labelling of path graph P_{12}

Theorem 3.2. Path graph P_m, $m \in N$ and $m \geq 3$ admits fuzzy bi-magic labelling.

Proof: Let $m \in N$, and $m \geq 3$, $V = \{v_1, v_2, v_3, v_4, \dots \dots, v_m\}$ and $E = \{e_i = v_i v_{i+1} | 1 \leq i \leq m-1\}$ and Path graph be $P_m = (V, E)$.

For all i ($1 \leq i \leq m$), the membership value for the vertices $\sigma: V \rightarrow [0,1]$, is defined as it is defined in the previous theorem (Theorem 3.1).

The membership of edges of P_m, $\forall i$, $(1 \leq i \leq m-1)$, are defined by the function $\mu: E \rightarrow [0,1]$ and classified into two mappings as follows:

$$\mu(e_i) = \begin{cases} \dfrac{i}{2m-2} & \text{if } i \text{ is odd} \\ \dfrac{i}{2m+3} & \text{if } i \text{ is even} \end{cases}$$

For fuzzy labelling, the main objective of the relation is that this condition must be satisfied $\mu(e_i) < \sigma(v_i) \wedge \sigma(v_{i+1})$. There are 4 cases:

Case (1) i is odd and m is even

Then $\sigma(v_i) = \left\{\frac{4m-(i+(m-1))}{2(2m-1)}\right\}$ and $\sigma(v_{i+1}) = \frac{3m-(i+(m-2))}{2(2m-1)}$

$$\Rightarrow \sigma(v_i) \wedge \sigma(v_{i+1}) = \frac{3m - (i + (m-2))}{2(2m-1)}$$

1a: i is odd (edge)

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$$\frac{i}{2m-2} < \frac{3m-(i+(m-2))}{2(2m-1)} \qquad \Rightarrow \mu(e_i) < \sigma(v_i) \wedge \sigma(v_{i+1})$$

1b: i is even (edge)

$$\frac{i}{2m+3} < \frac{3m-(i+(m-2))}{2(2m-1)} \qquad \Rightarrow \mu(e_i) < \sigma(v_i) \wedge \sigma(v_{i+1})$$

The same procedure is followed for all the other cases and the fuzzy labelling condition is checked.

Thus, Path graph is fuzzy labelling graph. The bi-magic value of the path graph comes under following cases:

Case (1) m – odd

If m is odd then

$$M = \sigma(v_i) + \mu(v_i v_{i+1}) + \sigma(v_{i+1})$$

$$M = \frac{4m-(i+(m-1))}{2(2m-1)} + \frac{3m-(i+(m-1))}{2(2m-1)} + \mu(v_i v_{i+1})$$

1a: i – odd (edge)

$$M_1 = \frac{5m+1-2i}{2(2m-1)} + \frac{i}{2m-2}$$

1b: i – even (edge)

$$M_2 = \frac{5m-2i+1}{2(2m-1)} + \frac{i}{2m+3}$$

Case (2) m – even

Then, magic value is

$$M = \frac{4m-(i+(m-1))}{2(2m-1)} + \frac{3m-(i+(m-2))}{2(2m-1)} + \mu(v_i v_{i+1})$$

2a: i – odd (edge)

$$M_1 = \frac{5m+2-2i}{2(2m-1)} + \frac{i}{2m-2}$$

2b: i – even (edge)

$$M_2 = \frac{5m-2i+2}{2(2m-1)} + \frac{i}{2m+3}$$

The bi-magic value of Path graph is

$$M_1, M_2 = \begin{cases} \frac{5m+1-2i}{2(2m-1)} + \frac{i}{2m-2}, \frac{5m-2i+1}{2(2m-1)} + \frac{i}{2m+3} & \text{if } m \text{ is odd} \\ \frac{5m+2-2i}{2(2m-1)} + \frac{i}{2m-2}, \frac{5m-2i+2}{2(2m-1)} + \frac{i}{2m+3} & \text{if } m \text{ is even} \end{cases}$$

Example The bi-magic value of the path graph P_6 is M_1=1.5 and M_2=1.4.

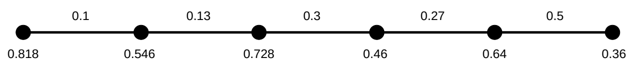

Fig. 2.A fuzzy Bi-magic labelling of path graph P_6

Theorem 3.3 Any Intuitionistic Path Graph P_m, $m \geq 2$ admits fuzzy magic labelling.

Proof: Let $m \in N$, and $m \geq 2$, $V = \{v_1, v_2, v_3, v_4, \dots \dots, v_m\}$ and $E = \{e_i = v_i v_{i+1} | 1 \leq i \leq m-1\}$ and Path graph be $P_m = (V, E)$. In Intuitionistic graphs we know that, every membership value has 2 values as membership and non-membership value. Thus, we define two different membership value for every vertex and edge.

For all i $(1 \leq i \leq m)$, the membership value for the vertices $\sigma: V \to [0,1]$ and the non-membership value $\gamma: V \to [0,1]$ is defined as follows:

$$\sigma(v_i) = \begin{cases} \frac{4m-(i+(m-1))}{2(2m-1)} & \text{if } i \text{ is odd} \\ \frac{3m-(i+(m-1))}{2(2m-1)} & \text{if } i \text{ is even and } m \text{ is odd} \\ \frac{3m-(i+(m-2))}{2(2m-1)} & \text{if } i \text{ is even and } m \text{ is even} \end{cases}$$

(membership value)

$$\gamma(v_i) = \begin{cases} \frac{4m-(i+(m-1))}{20(2m-1)} & \text{if } i \text{ is odd} \\ \frac{3m-(i+(m-1))}{20(2m-1)} & \text{if } i \text{ is even and } m \text{ is odd} \\ \frac{3m-(i+(m-2))}{20(2m-1)} & \text{if } i \text{ is even and } m \text{ is even} \end{cases}$$

(non-membership value)

The membership and non-membership of edges $\forall i, (1 \leq i \leq m-1)$, are defined by the function $\mu: E \to [0,1]$ as $\mu(e_i) = \left\{\frac{i}{2m-1}\right\}$ (membership) and $\rho: E \to [0,1]$ as $\rho(e_i) = \left\{\frac{i}{10(2m-1)}\right\}$ (non-membership), $where\ e_i = v_i v_{i+1} \in E$

For fuzzy labelling, the main objective of the relation is that this condition must be satisfied

$$\mu(e_i) < \sigma(v_i) \wedge \sigma(v_{i+1}) \text{ and } \rho(e_i) < \gamma(v_i) \vee \gamma(v_{i+1})$$

Case (1) i is even and m is odd

Membership value:

$$\sigma(v_i) = \left\{\frac{3m-(i+(m-1))}{2(2m-1)}\right\} \text{ and } \sigma(v_{i+1}) = \left\{\frac{4m-(i+(m-1))}{2(2m-1)}\right\}$$

$$\Rightarrow \sigma(v_i) \wedge \sigma(v_{i+1}) = \frac{3m-(i+(m-1))}{2(2m-1)}$$

Non-membership value:

$$\gamma(v_i) = \left\{\frac{3m-(i+(m-1))}{20(2m-1)}\right\} \text{ and } \gamma(v_{i+1}) = \left\{\frac{4m-(i+(m-1))}{20(2m-1)}\right\}$$

$$\Rightarrow \gamma(v_i) \vee \gamma(v_{i+1}) = \frac{4m-(i+(m-1))}{20(2m-1)}$$

Membership value:

$$\frac{i}{2m-1} < \frac{3m-(i+(m-1))}{2(2m-1)} \qquad \Rightarrow \mu(e_i) < \sigma(v_i) \wedge \sigma(v_{i+1})$$

Non-membership value:

$$\frac{i}{10(2m-1)} < \frac{4m-(i+(m-1))}{20(2m-1)} \qquad \Rightarrow \rho(e_i) < \gamma(v_i) \vee \gamma(v_{i+1})$$

Similarly, the same way of proving the relation of fuzzy labelling is followed for the other cases.

Now we are ought to prove the existence of the magic value of Path graph.

(i.e.) $\left(\sigma(v_i)+\sigma(v_{i+1})+\mu(v_iv_{i+1}),\ \gamma(v_i)+\gamma(v_{i+1})+\rho(v_iv_{i+1})\right) = (M, m^1)$.

The magic value existence

Membership value:

The magic value of membership value of P_n is

$$M = \begin{cases} \dfrac{5m+2}{2(2m-1)} & if\ m\ is\ odd \\ \dfrac{5m+3}{2(2m-1)} & if\ m\ is\ even \end{cases}$$

Non-membership value:

Case (i) m – odd

When m is odd, we get

$$\sigma(v_i) = \frac{4m-(i+(m-1))}{20(2m-1)} \quad and \quad \sigma(v_{i+1}) = \frac{3m-(i+(m-1))}{20(2m-1)}$$

Thus $m^1 = \frac{4m-(i+(m-1))}{20(2m-1)} + \frac{3m-(i+(m-1))}{20(2m-1)} + \frac{i}{10(2m-1)}$

$$m^1 = \frac{5m+2}{20(2m-1)}$$

Case (ii) m – even

When m is even, we get

$$\sigma(v_i) = \frac{4m-(i+(m-1))}{20(2m-1)} \quad and \quad \sigma(v_{i+1}) = \frac{3m-(i+(m-2))}{20(2m-1)}$$

Thus $m^1 = \frac{4m-(i+(m-1))}{20(2m-1)} + \frac{3m-(i+(m-2))}{20(2m-1)} + \frac{i}{10(2m-1)}$

$$m^1 = \frac{5m+3}{20(2m-1)}$$

Therefore, the magic value of non-membership value is

$$m^1 = \begin{cases} \dfrac{5m+2}{20(2m-1)} & if\ m\ is\ odd \\ \dfrac{5m+3}{20(2m-1)} & if\ m\ is\ even \end{cases}$$

Thus, the magic value of Intuitionistic Path graph is

$$(M, m^1) = \begin{cases} \dfrac{5m+2}{2(2m-1)}, & \dfrac{5m+2}{20(2m-1)} & if\ m\ is\ odd \\ \dfrac{5m+3}{2(2m-1)}, & \dfrac{5m+3}{20(2m-1)} & if\ m\ is\ even \end{cases}$$

Example The magic value of the Intuitionistic Path graph P_5 is (1.5, 0.15).

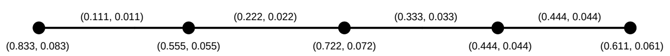

Fig. 3.A fuzzy magic labelling of intuitionistic path graph P_5

Theorem 3.4 Intuitionistic Path graph P_m, *where* $m > 2$ *and* $m \in N$ admits fuzzy bi-magic labelling.

Proof: Let $m \in N$, and $m > 2$, $V = \{v_1, v_2, v_3, v_4, \ldots \ldots, v_m\}$, $E = \{e_i = v_iv_{i+1} | 1 \le i \le m-1\}$ and Path graph be $P_m = (V, E)$.

For all $i\ (1 \le i \le m)$, the membership value and non-membership value for the vertices $\sigma: V \to [0,1]$ and $\gamma: V \to [0,1]$, is considered as it is defined in the previous theorem (Theorem 4.1).

The membership $\mu: E \to [0,1]$ and non-membership $\rho: E \to [0,1]$ of edges of P_m, $\forall i$, $(1 \le i \le m-1)$, are defined by the functions,

$$\mu(e_j) = \begin{cases} \frac{i}{2m-2} & if\ i\ is\ odd \\ \frac{i}{2m+3} & if\ i\ is\ even \end{cases}$$ (membership value) and

$$\rho(e_j) = \begin{cases} \frac{i}{10(2m-2)} & if\ i\ is\ odd \\ \frac{i}{10(2m+3)} & if\ i\ is\ even \end{cases}$$ (non-membership value)

Case (1) i is even and m is even

Membership value:

$$\sigma(v_i) = \left\{\frac{3m-(i+(m-2))}{2(2m-1)}\right\}\ and$$

$$\sigma(v_{i+1}) = \left\{\frac{4m-(i+(m-1))}{2(2m-1)}\right\}$$

$$\Rightarrow \sigma(v_i) \wedge \sigma(v_{i+1}) = \frac{3m-(i+(m-2))}{2(2m-1)}$$

1a: i is odd

$$\frac{i}{2m-2} < \frac{3m-(i+(m-2))}{2(2m-1)} \quad \Rightarrow \mu(e_i) < \sigma(v_i) \wedge \sigma(v_{i+1})$$

1b: i is even

$$\frac{i}{2m+3} < \frac{3m-(i+(m-2))}{2(2m-1)} \quad \Rightarrow \mu(e_i) < \sigma(v_i) \wedge \sigma(v_{i+1})$$

Non-membership value:

$$\gamma(v_i) = \left\{\frac{3m-(i+(m-2))}{20(2m-1)}\right\}\ and\ \gamma(v_{i+1}) = \left\{\frac{4m-(i+(m-1))}{20(2m-1)}\right\}$$

$$\Rightarrow \gamma(v_i) \vee \gamma(v_{i+1}) = \frac{4m-(i+(m-1))}{20(2m-1)}$$

1a: i is odd

$$\frac{i}{10(2m-2)} < \frac{4m-(i+(m-1))}{20(2m-1)} \quad \Rightarrow \rho(e_i) < \gamma(v_i) \vee \gamma(v_{i+1})$$

1b: i is even

$$\frac{i}{10(2m+3)} < \frac{4m-(i+(m-1))}{20(2m-1)} \quad \Rightarrow \rho(e_i) < \gamma(v_i) \vee \gamma(v_{i+1})$$

The other cases verification of fuzzy labelling relation is proceeded in the same method.

Thus, Intuitionistic Path graph attains fuzzy labelling graph.

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The bi-magic value of the path graph comes under following cases:

The bi-magic value of membership value of Intuitionistic path graph is

$$M_1, M_2 = \begin{cases} \dfrac{5m+1-2i}{2(2m-1)} + \dfrac{i}{2m-2}, \dfrac{5m-2i+1}{2(2m-1)} + \dfrac{i}{2m+3} & if\ m\ is\ odd \\ \dfrac{5m+2-2i}{2(2m-1)} + \dfrac{i}{2m-2}, \dfrac{5m-2i+2}{2(2m-1)} + \dfrac{i}{2m+3} & if\ m\ is\ even \end{cases}$$

Non-membership value:

Case (1) m – odd

If m is odd then

$$M = \sigma(v_i) + \mu(v_i v_{i+1}) + \sigma(v_{i+1})$$

$$M = \frac{4m - (i + (m-1))}{20(2m-1)} + \frac{3m - (i + (m-1))}{20(2m-1)} + \mu(v_i v_{i+1})$$

1a: i – odd (edge)

$$m^1 = \frac{5m+1-2i}{20(2m-1)} + \frac{i}{10(2m-2)}$$

1b: i – even (edge)

$$m^2 = \frac{5m-2i+1}{20(2m-1)} + \frac{i}{10(2m+3)}$$

Case (2) m – even

Then

$$M = \frac{4m - (i + (m-1))}{20(2m-1)} + \frac{3m - (i + (m-2))}{20(2m-1)} + \mu(v_i v_{i+1})$$

2a: i – odd (edge)

$$m^1 = \frac{5m+2-2i}{20(2m-1)} + \frac{i}{10(2m-2)}$$

2b: i – even (edge)

$$m^2 = \frac{5m-2i+2}{20(2m-1)} + \frac{i}{10(2m+3)}$$

The bi-magic value of non-membership value of Intuitionistic path graph is

$$(m^1, m^2) = \begin{cases} \dfrac{1}{10}\left[\dfrac{5m+1-2i}{2(2m-1)} + \dfrac{i}{2m-2}\right], \dfrac{1}{10}\left[\dfrac{5m-2i+1}{2(2m-1)} + \dfrac{i}{2m+3}\right] & if\ m\ is\ odd \\ \dfrac{1}{10}\left[\dfrac{5m+2-2i}{2(2m-1)} + \dfrac{i}{2m-2}\right], \dfrac{1}{10}\left[\dfrac{5m-2i+2}{2(2m-1)} + \dfrac{i}{2m+3}\right] & if\ m\ is\ even \end{cases}$$

The bi-magic value of Intuitionistic Path graph is

$$\begin{cases} \left(\dfrac{5m+1-2i}{2(2m-1)} + \dfrac{i}{2m-2}, \dfrac{1}{10}\left[\dfrac{5m+1-2i}{2(2m-1)} + \dfrac{i}{2m-2}\right]\right), \left(\dfrac{5m-2i+1}{2(2m-1)} + \dfrac{i}{2m+3}, \dfrac{1}{10}\left[\dfrac{5m-2i+1}{2(2m-1)} + \dfrac{i}{2m+3}\right]\right) & if\ m\ is\ odd \\ \left(\dfrac{5m+2-2i}{2(2m-1)} + \dfrac{i}{2m-2}, \dfrac{1}{10}\left[\dfrac{5m+2-2i}{2(2m-1)} + \dfrac{i}{2m-2}\right]\right), \left(\dfrac{5m-2i+2}{2(2m-1)} + \dfrac{i}{2m+3}, \dfrac{1}{10}\left[\dfrac{5m-2i+2}{2(2m-1)} + \dfrac{i}{2m+3}\right]\right) & if\ m\ is\ even \end{cases}$$

Example The bi-magic value of the Intuitionistic path graph P_8 is (1.4, 0.14) and (1.3, 0.13).

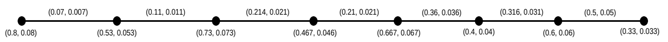

Fig. 4. A fuzzy Bi-magic labelling of intuitionistic path graph P_8

IV. CONCLUSION

The fuzzy magic and bi-magic labelling of fuzzy path graph have been discussed. First whether fuzzy path graph admits magic and bi-magic labelling is examined and the magic, bi-magic value is given with examples. The magic and bi-magic labelling of Intuitionistic path graph have been studied with proper example. Furthermore, working on magic & bi-magic labelling of Neutrosophic path, star graph and also on some more graphs.

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