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Optimization of Foreign Exchange using Kelly Criteria Model

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Abstract*: : This research is expected to be able to analyze the foreign exchange assets of investors by using Kelly criteria optimally, so that investors can obtain the expected benefits with minimal risk. The analysis used in this study is quantitative with the following steps: (i) determine the value of foreign exchange asset returns, (ii) determine the variance of foreign exchange assets, (iii) determine the optimal portfolio using Kelly criteria. The results of the analysis obtained weight allocation that provides return and risk to the portfolio that has been formed optimally. EUR is the most optimal portfolio with a weighting of 63% of the five foreign currencies invested. The results of this study can be used as a consideration for investors in loading the right investment decisions.

***Keywords* : Optimal portfolio, Return, risk, Kelly strategy.**

I. INTRODUCTION

T he increasing needs and the more diverse financial products offered, the public interest in investing more and more. The public increasingly understands that to prepare for financial needs in the future, it is necessary to conduct investment activities, both in the form of real assets and in the form of securities / other securities. Foreign exchange is one alternative for investing because profits can be obtained from investment returns and also from the exchange rate difference with the rupiah when buying or selling.

Foreign exchange or exchange rates are the prices of one currency expressed in another currency [13]. Risk is one of the influential components in investing. Many studies show that one way to control risk is to diversify [1], which is to form an investment portfolio. The portfolio selection theory proposed by Markowitz in 1952, this methodology has become a benchmark in portfolio management [8]. According to Markowitz's Modern Portfolio Theory (MPT), the optimal portfolio is based on a balance between risk and return, where risk gives the investor maximum return at a certain risk level, or the lowest risk level at a certain rate of return [2].

Markowitz's Mean-Variance Model has been widely discussed and widely used to solve investment portfolio optimization problems. Models like Kelly's criteria are also used for the same purpose. Models like Kelly's criteria are also used for the same purpose. But Kelly's criteria received less attention than the first in academia and finance. But Kelly's criteria received less attention than the first in academia and finance. Now, it is believed that Kelly's criteria provide investors with a better investment performance or equivalent to the Markowitz model [11]. There have been several previous studies related to portfolio optimization using Kelly criteria. There have been several previous studies related to portfolio optimization using Kelly criteria. Simtek [20] assumes that any financial innovation on portfolio risk tends to lead to speculation rather than risk sharing due to investors' motives in the market Simtek [20] assumes that any financial innovation on portfolio risk tends to lead to speculation rather than risk sharing due to investors' motives in the market [21]. Peterson [16] in his research shows how Kelly's criteria can be combined into a portfolio optimization model that considers the risk function. Hasilnya menunjukkan bahwa kriteria The results show that Kelly's criteria can be used to calculate optimal returns and can produce a portfolio that is similar to the results from the Mean-Variance model. Kim [10] conducted research on the optimal hedging model for foreign exchange risk following portfolio theory. The research, resulted that a lower level of risk can be achieved from the expected return with the optimization model.

This paper discusses the optimal portfolio of foreign exchange using the Kelly criteria model. This study discusses the optimal portfolio of foreign exchange using the Kelly criteria model. The objects to be analyzed are all foreign currencies traded on the Foreign Exchange market in Indonesia, including Kuwaiti Dinar (KWD), British Pound Sterling (GBP), Euro (EUR), Japanese Yen (JPY) and Swiss Franc (CHF).

II. BASIC CONCEPT

In every investment, an analysis of expected returns and risks is assumed to be a fundamental step. Investments in financial assets are no exception [8]. The selection of assets is made with the aim of giving investors maximum returns with a certain level of risk or ensuring minimal risk for certain returns [9].

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The higher the weight of an asset, the higher the expected profit [12]. The efficient allocation of assets in an uncertain environment is one of the main problems in finance [6]. Investors have several options to choose from to save their wealth. Therefore, it becomes important to analyze the investment process and investment decision making in a much broader context [15]. Foreign exchange or exchange rates are the prices of one currency expressed in another currency [13]. In understanding the behavior of investors who invest their funds in the form of foreign exchange purchases, money is seen as an asset. In other words, currencies can now be considered as financial assets, similar to stocks and bonds [17] so money is also used for investment. The most important part of portfolio management is the execution step in which an appropriate portfolio is built. This procedure considers asset allocation, security analysis, and investor requirements [5]

Kelly's criteria relate to the optimal allocation of investment capital to maximize geometric return expectations on long-term investments, maximizing geometric return expectations being an objective function of the Kelly portfolio model subject to the following two constraints. The first obstacle in the model below states that investment capital can be allocated in whole or in part to a portfolio. Such limits apply with the assumption that short-term loan and sale activities are prohibited, which corresponds to the second. This section, the investment ratio becomes the decision variable model.

$$Maximize\ p\sum_{i=1}^{N} exp\left\{\ln(1+W_i f_i)-(1-p)\sum_{i=1}^{N}\left(f_i^2 M_{ii}+2\sum_{i=1}^{N} f_i f_j\ M_{ij}\right)\right\} \quad (1)$$

III. METHOD

A. Return of Exchange Rate

An asset's return can be referred to as a measure of the total profit from an investment over a certain period of time, with respect to both changes in market value and cash distribution [19]. Return of an asset is defined as follows. *Return* [3] R_{it} of an asset i at time t (without dividends) is defined in the equation (2), P_{it} is the price of the asset i at time t and P_{t-1} is the prie asset i at time $t-1$.

$$R_{it} = \frac{P_{it}-P_{it-1}}{P_{it-1}} \quad (2)$$

The expected return of an asset is expressed by $E[R_i]$ is the expected level of profit in the future, and is defined as follows:

$$E(R_i) = \sum_{i=1}^{n}\left(w_i E(R_i)\right) \quad (3)$$

where w_i shows the weights to predict the expected return expectations on i. Equation (3) is used as a means of estimating population averages $\mu_i = E[R_i]$ [4].

In measuring risk, an investor observation is a view about the asymmetric mean [7]. Next, return variance (R) stated in the definition stated with $Var\ (R)$ as follows [18]:

$$Var\ (R) = E[(R-\mu)^2] \quad (4)$$

Markowitz concluded that portfolio variance is influenced by the historical covariance of each asset pair in the portfolio. For example return R_i and R_j of two different assets with their respective expectations μ_i dan μ_j, then the covariance of both returns is

$$\sigma_{ij} = cov\ (R_i, R_j) = E[(R_i-\mu_i)(R_j-\mu_j)] \quad (5)$$

where $\sigma_{ij} = cov\ (R_i, R_j)$ is the return covariance R_i and market returns R_j and $\sigma_i^2 = Var\ (R)$ is the variance of market returns R_j [22], [23].

B. Return and Risk Portfolio

A portfolio's return is the weighted average of each asset's return, expressed as follows:

$$\begin{aligned} R_t &= w_1R_1 + w_2R_2 + \cdots + w_nR_n \\ &= \sum_{j=1}^{N} w_jR_j \\ &= \begin{pmatrix} w_1 & w_2 & \cdots & w_n \end{pmatrix}\begin{pmatrix} R_1 \\ R_2 \\ \vdots \\ R_n \end{pmatrix} \\ &= \mathbf{w}'R \end{aligned} \quad (6)$$

where $\mathbf{w}'$ is transpose of $\mathbf{w}$ portfolio weight vector for assets to [14] [24], with

$$\sum_{i}^{n} w_i = 1. \quad (7)$$

Risk in investment analysis is the uncertainty of future returns from investments. The concept of risk can be defined as the possibility that the actual return may not be the same as expected [14]. Portfolio risk is the variance in the return of assets (securities) that make up the portfolio. One measure of risk is standard deviation, which is the square root of variance. Variants of a portfolio are stated as follows:

$$\sigma_p^2 = \sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j \sigma_{ij} \quad (8)$$

Furthermore, the covariance of all asset returns can be expressed in the form of a matrix of equation (8) as follows:

$$\sigma_p^2 = \mathbf{w}'\Sigma\mathbf{w} \quad (9)$$

where $\Sigma = \begin{pmatrix} \sigma_{11} & \sigma_{12} & \cdots & \sigma_{1n} \\ \sigma_{21} & \sigma_{22} & \cdots & \sigma_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{n1} & \sigma_{n2} & \cdots & \sigma_{nn} \end{pmatrix}$ and $\mathbf{w} = \begin{pmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \end{pmatrix}$

C. Kelly Criterion

Kelly's Criteria is a simple formula for allocating assets in maximizing the average return of the amount of capital invested. If Kelly's criteria are applied to investment portfolios consisting of stocks and bonds, it produces a return r_s with f_s and r_b with f_b, can be stated as follows:

$$g(f) = p\ln(1+r_s f_s + r_b f_b) + q\ln(1-r_s f_s + r_b f_b) \quad (10)$$

By using the notation above, equation (10) can be stated as follows:

$$g(f) = p\ln(\beta_s f_s + \beta_b f_b) + q\ln(2-\beta_s f_s + \beta_b f_b) \quad (11)$$

where $\beta_s = 1 + r_s$ and $\beta_b = 1 + r_b$ and limited by $f_s + f_b = 1$.

Optimization of Kelly Criteria can be shown in the

following formula:

$$Maximize\ p\sum_{i=1}^{N} exp\left\{\ln(1+W_i f_i)-(1-p)\sum_{i=1}^{N}\left(f_i^2 M_{ii}+2\sum_{i=1}^{N} f_i f_j\ M_{ij}\right)\right\}$$

with obstacles

$$\sum_{i=1}^{N} f_i \leq 1$$

In matrix vector equations disusun as follows:

$$\max p(1+\mathbf{f}^{\mathbf{T}}\boldsymbol{\mu})-(1-p)\mathbf{f}^{\mathbf{T}}\mathbf{M}\mathbf{f} \quad (12)$$

With obstacles

$$\mathbf{e}^{\mathbf{T}}\mathbf{f} \leq \mathbf{1} \quad (13)$$

From equations (12) and (13) the Lagrange multiplier function is given as follows:

$$\mathcal{L} = p + p\mathbf{f}^{\mathbf{T}}\boldsymbol{\mu} - q\mathbf{f}^{\mathbf{T}}\mathbf{M}\mathbf{f} + \lambda(\mathbf{e}^{\mathbf{T}}\mathbf{f}-\mathbf{1}) \quad (14)$$

$$\frac{\partial L}{\partial f} = p\boldsymbol{\mu} - 2q\mathbf{M}\mathbf{f} + \lambda\mathbf{e} = 0 \quad (15)$$

$$\frac{\partial L}{\partial \lambda} = \mathbf{e}^{\mathbf{T}}\mathbf{f} - \mathbf{1} = 0 \quad (16)$$

From Equation (15) obtained

$$2q\mathbf{M}\mathbf{f} = p\boldsymbol{\mu} + \lambda\mathbf{e} \quad (17)$$

from Equation (17) times by M^{-1}, was obtained

$$2q\mathbf{f} = p\mathbf{M}^{-1}\boldsymbol{\mu} + \lambda\mathbf{M}^{-1}\mathbf{e} \quad (18)$$

Because in Eq (18), 2q is a scalar, then it is obtained

$$\mathbf{f} = \frac{1}{2q}[p\mathbf{M}^{-1}\boldsymbol{\mu} + \lambda\mathbf{M}^{-1}\mathbf{e}] \quad (19)$$

Then, substitute Equation (19) to Equation (16), so that it is obtained

$$\mathbf{e}^{\mathbf{T}}\left(\frac{1}{2q}[p\mathbf{M}^{-1}\boldsymbol{\mu} + \lambda\mathbf{M}^{-1}\mathbf{e}]\right) = 1$$

$$\lambda = \frac{2q}{\mathbf{e}^{\mathbf{T}}\mathbf{M}^{-1}} - \frac{p\mathbf{e}^{\mathbf{T}}\mathbf{M}^{-1}\boldsymbol{\mu}}{\mathbf{e}^{\mathbf{T}}\mathbf{M}^{-1}\mathbf{e}}$$

IV. RESULT AND DISCUSSION

A. Data Analysis

The object of this research is the closing price data of the foreign exchange rate from the daily exchange rate from the period of January 1 to December 31, 2018, obtained through the website http://www.bi.go.id. The data used in this study are the closing price of the Kuwaiti Dinar exchange rate (KWD), the British Pound (GBP), Euro (EUR), Japanese Yen (JPY), and the Swiss Franc (CHF) exchange rate.

B. Kelly Criteria Portfolio Optimization Model

The results of the analysis of the calculation of return and portfolio variance for five foreign currencies, the data of which are shown in Table 1:

Tabel 1. Nilai prediktif Mean dan Variance Valuta Asing

Mata Uang	Ekspektasi Return μ	Variansi σ_P^2
KWD	0.00026322	0.003704569
GBP	2.85525E-05	0.005035876
EUR	8.28717E-05	0.004272966
JPY	0.000376549	0.005325621
CHF	0.000244141	0.004554123

Return and variance expectations in Table 1 obtain a maximum return of 0.000376549 by the JPY exchange rate and the lowest return of 0.0000285525 by the EUR exchange rate. While the minimum risk given by KWD is 0.003704569 and the maximum risk is 0.005325621 by JPY.

The problem of optimizing portfolio optimization is Kelly's criteria, using vectors $\boldsymbol{\mu}^{\mathbf{T}}$ and $\mathbf{e}^{\mathbf{T}}$ as well as the matrix $\boldsymbol{\Sigma}^{-1}$, weight vector $\mathbf{f}$ calculated using equations (19).

$$\boldsymbol{\Sigma} = \begin{bmatrix} 0.00137 & 0.00186 & 0.00158 & 0.00197 & 0.00168 \\ 0.00186 & 0.00253 & 0.00215 & 0.00268 & 0.00229 \\ 0.00158 & 0.00215 & 0.00182 & 0.00227 & 0.00194 \\ 0.00197 & 0.00268 & 0.00227 & 0.00283 & 0.00242 \\ 0.00168 & 0.00229 & 0.00194 & 0.00242 & 0.00207 \end{bmatrix}$$

And inverse covariance matrix $\boldsymbol{\Sigma}^{-1}$ is:

$$\boldsymbol{\Sigma}^{-1} = \begin{bmatrix} 79.521345 & -4.560513 & -5.374455 & -4.305468 & -4.895885 \\ -4.5600389 & 41.306770 & -3.781569 & -3.0294338 & -3.444836 \\ -5.3738962 & -3.781569 & 57.370441 & -3.570113 & -4.059661 \\ -4.3055689 & -3.029819 & -3.570567 & 36.879279 & -3.246697 \\ -4.9014630 & -3.443999 & -4.058675 & -3.251404 & 49.140114 \end{bmatrix}$$

Next, inverse the covariance matrix $\boldsymbol{\Sigma}^{-1}$ used for the process of calculating the composition of efficient portfolio weights carried out based on Kelly's criteria portfolio optimization model. The return and the calculation result of optimal portfolio weights are given in Table 2.

Table 2. Kelly Port Criteria Investment Optimization Process

p	q	KWD	GBP	EUR	JPY	CHF	w'e	Return Expectations μ	Variance σ_P^2	Ratio
0	1	0.328765	0.144229	0.220965	0.123735	0.182306	1	1.98E-02	5.44E-03	3.64E+00
0.01	0.99	0.324854	0.13761	0.213013	0.124851	0.179773	1	1.96E-02	5.23E-03	3.75E+00
0.02	0.98	0.320955	0.131096	0.205177	0.125919	0.177253	1	0.019433	5.03E-03	3.87E+00
0.03	0.97	0.317069	0.124686	0.197456	0.12694	0.174749	1	0.01923	4.83E-03	3.98E+00
0.04	0.96	0.313195	0.118381	0.189852	0.127913	0.172258	1	0.019026	4.64E-03	4.10E+00
0.05	0.95	0.309334	0.112181	0.182364	0.128839	0.169782	1	0.018823	4.46E-03	4.22E+00
0.10	0.90	0.290219	0.082747	0.146663	0.132754	0.157617	1	0.017807	3.65E-03	4.88E+00

0.15	0.85	0.271418	0.055928	0.113862	0.135482	0.145811	1	0.016795	3.00E-03	5.60E+00
0.20	0.80	0.252932	0.031723	0.083962	0.13702	0.134363	1	0.015785	2.48E-03	6.37E+00
0.25	0.75	0.234761	0.010132	0.056962	0.13737	0.123275	1	0.014778	2.06E-03	7.17E+00
0.26	0.74	0.231165	0.006128	0.05191	0.137298	0.1211	1	0.014577	1.99E-03	7.33E+00
0.27	0.73	0.227581	0.002228	0.046974	0.137178	0.11894	1	0.014376	1.92E-03	7.48E+00
0.28	0.72	0.22401	-0.00157	0.042154	0.13701	0.116794	1	0.014175	1.86E-03	7.64E+00

The results in Table 2, by taking the value of risk tolerance only up to the value $0 \leq \tau \leq 0{,}27$. This is due to the risk tolerance value $\tau > 0{,}28$ produce negative weights.

Changes in the expected value of return and variance in portfolio along with the increase in the value of risk tolerance τ can be seen in Table 2. In this case the maximum value of risk tolerance $\tau = 0{,}028$, which produces portfolio weights with the highest expected portfolio return of 0.01984 and variance of 0.00544 as shown in Figure 1.

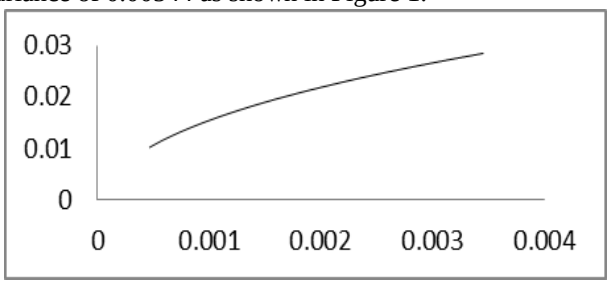

Figure 1. Efficient Frontier Portfolio Criteria Kelly in Foreign Exchange

Looking at the results in Figure 1, it appears that any increase in the value of risk tolerance will cause an increase in portfolio return expectations, and is also accompanied by an increase in portfolio variance. A series of optimal portfolios are on the efficient surface. Efficient frontier is the efficient surface where portfolio is located whose return is commensurate with the risk.

V. COUNCLUSION

In this article, we analyze Kelly's optimal foreign exchange criteria. The results of the analysis of the calculation of portfolio optimization, which was achieved optimally when the composition of the proportion of portfolio investments in KWD, GBP, EUR, JPY and CHF currencies are: 0.1406, 0.2318, 0.1769, 0.2545, and 0.1963. This article can be concluded that any increase in the value of risk tolerance will cause an increase in portfolio return expectations, and also accompanied by an increase in portfolio variance.

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