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Mediative Multi-Criteria Decision Support System for Various Alternatives Based on Fuzzy Logic

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Abstract- ***Present research paper, deals with a new method for solving multi-criteria decision-making problems, in an environment where conflict of interest exits. In this present paper we have defined the Mediative fuzzy point operators. By using Mediative fuzzy logic and Mediative fuzzy point operators we have established some new generalized mathematical results of Intuitionistic fuzzy logic theory in the form of Mediative fuzzy logic theory. Second, we have applied these results to multi-criteria for making it into Mediative multi-criteria for decision support system where multiple alternatives and various criteria's exits. Numerical examples have also been carried out to prove the effectiveness and advantages of this paper.***

Keywords: Evaluation function, Mediative fuzzy set, Mediative fuzzy point operator, Mediative Multi-criteria decision making, Mediative fuzzy score function.

I. INTRODUCTION

Multi-criteria decision making has been applied to various real-life problems. With the measurement of complexity in real life problems, it is very tedious for the individual decision maker to evaluate the relevant issues using with the extent of accuracy. So, to make decision more realistic, reasonable and reliable, multiple decision maker be made available to involve in a complex decision problem which appears in the form of multi-criteria decision making. Due to the complexity of subjective factors having effect, the decision maker sometimes relies on his/her own experience or intuition to evaluate complex criterion of alternatives in decision problems. This happens due to the bi-valued logic. The Fuzzy set was introduced by L. A. Zadeh in 1965 [1].

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The main advantage of fuzzy set theory over the classical set theory is that, in fuzzy set theory a membership value assign corresponds to each element *x*, from a given universal set *X* between [0, 1]. After that there are so many developed wide areas in which we have been used fuzzy logic, fuzzy set theory has been applied to handling fuzzy decision-making problems [2] and this theory has increases over the years. Later on, the concept of non-membership value was introduced, which shows the evidence against *x* and the sum of membership value and non-membership value of an element from universal set is less than and equals to 1. This theory shows the lack of knowledge about an element, *i.e.*, in the cases where the sum of membership and non-membership value is less than 1. In this situation lack of knowledge about the belongingness of an element *x* arises. Then a possible solution to handle that kind of situation is to use Intuitionistic fuzzy set (IFS). IFS were introduced by Atanassov in 1986 [3], it is an extension of fuzzy set. Vague set and IFS were also having been used to decision-making problem [4, 5, and 6] over the years. Since intuitionistic fuzzy set at the same deals with membership and non-membership, so it is more flexible and realistic than fuzzy logic in dealing with haziness and uncertainty. As the special case of IFSs stated over a realistic set, the triangular intuitionistic fuzzy numbers [7 and 8], the trapezoidal intuitionistic fuzzy numbers [9 and 10], trapezoidal and interval valued trapezoidal intuitionistic fuzzy numbers [11 and 12] have been supplied in the field of multi-criteria decision making and multi-criteria group decision making.

But what will happen if there is a contradiction in expert's knowledge which cannot be handling by Intuitionistic fuzzy set theory. Then Mediative fuzzy logic (MFL) comes under the study. It was firstly introduced by Montiel in 2008 [13]. MFL is a novel method that can handle imperfect knowledge as Intuitionistic fuzzy logic does (IFL). MFL manage the contradictory and non-contradictory information [14 and 15] provided by an expert, who provides a mediated solution, hence the name Mediative Fuzzy Logic, in this work we will focus on the application of Mediative fuzzy set to handle Multi-Criteria Decision Making (MCDM). When we talk about the MCDM is to choose the optimal solution by the decision makers. In case of single criteria, the optimal solution easily exits, but it is difficult to handle the situation, in which we have more than one criterion, MCDM is the process which involves two or more alternative and each alternative are calculated under some criteria depends upon the decision maker choice.

When there is a conflict of interest in choosing alternative for Multi-Criteria Decision Making Problem (MCDM) we cannot use MCDM in present form then we have to consider the counter part of alternative to take the decision, so the present theory will be replaced by a new theory for MCDM which is known as Mediative MCDM based on Fuzzy logic.

In this present research paper, we defined a Mediative fuzzy logic approach to handle Multi-Criteria Decision-Making Problems (MCDM). To manage the decision-making-problem, we have to use an evaluation function in mediative environment; the aim of evaluation function is to measure the degree to which alternative satisfies the decision maker requirement. Further in this present work, we introduced mediative fuzzy point operator (MF point operator) and defined a mediative fuzzy score function based of MF point operator for solving multi-criteria decision-making problem. Furthermore, we have also defined some theorems based on MF operators.

The research paper is divided into five sections, in the second section we have introduced and proved some new results based on mediative fuzzy point operators. In the section III of the research paper we have applied the concept of mediative fuzzy logic to Multi-criteria decision-making to convert it into Mediative multi-criteria fuzzy decision making. In this section we have also generalized the results and concept of weight score function [4], degree of accuracy [5] and score function of Li *et. al*. [16 and 17], we have also defined an evaluation function for mediative fuzzy logic environment. In the section IV we have summed up all the steps of multi-criteria decision making and have used two numerical examples to support our mediative fuzzy decision system. The last and final section of the research paper is conclusion.

II. BASIC CONCEPTS

A. Mediative fuzzy set

Let X is a Universal set and A ⊆ X, then a Mediative fuzzy set ‘A’ on X is defined in the following form

$$A = \{(x, \mu_A(x), \nu_A(x)) / \ x \in X\} \qquad (1)$$

with $\zeta_A(x) = \min(\mu_A(x), \nu_A(x)))$ called contradictory factor of A in X and

$$\mu_A(x): X \to [0,1]$$
$$\nu_A(x): X \to [0,1]$$

with $(\gamma_A(x) + \frac{\zeta_A(x)}{2}) = 1 - \mu_A(x) - \nu_A(x)$ and $0 \le (\gamma_A(x) + \frac{\zeta_A(x)}{2}) \le 1$ with $-0.25 \le \gamma_A \le 1$, then A is called Mediative fuzzy set defined on X and denoted by MFS(X) or simply MFS.

$\mu_A(x)$ called degree of membership and $\nu_A(x)$ called non-membership function and $\gamma_A(x) + \frac{\zeta_A(x)}{2}$ called the index of uncertainty with contradiction about ‘x’, if $\pi_A(x) + \frac{\zeta_A(x)}{2}$ is small then we know more about ‘x’, whether x belongs to X or x does not belong to X, if $\gamma_A(x) + \frac{\zeta_A(x)}{2}$ is large then we know less about ‘x’, whether x belongs to X or x does not belong to X, and if $\gamma_A(x) + \frac{\zeta_A(x)}{2}$ =0, then MFS coverts to traditional fuzzy set, and if $\mu_A(x)$ and $1 - \nu_A(x)$ both are equals to 0 or 1, then it will convert into bi-valued logic .

B. Mediative fuzzy point operators

For any mediative fuzzy set A defined on a universal set X i.e.,$A \in MFS(X)$ and $\alpha \ and \ \beta \in [0,1]$with $\alpha + \beta \le 1$, then

a) $M_\alpha(A) = \{(x, \mu_A(x) + \alpha(\gamma_A(x) + \frac{\zeta_A(x)}{2}), \nu_A(x) + (1-\alpha)(\gamma_A(x) + \frac{\zeta_A(x)}{2})) \ / \ x \in X\}$ (2)

b) $M_{\alpha,\beta}(A) = \{(x, \mu_A(x) + \alpha(\gamma_A(x) + \frac{\zeta_A(x)}{2}), \nu_A(x) + \beta(\gamma_A(x) + \frac{\zeta_A(x)}{2})) \ / \ x \in X\}$ (3)

Two operators $M_\alpha(A)$ and $M_{\alpha,\beta}(A)$ defined are called Mediative fuzzy point operators.

Now, for each A∈ $MFS(X)$, and$x \in X$ with $\alpha_x, \beta_x \in [0,1]$ with the condition that $0 \le \alpha_x + \beta_x \le 1$, then we defined the operatorM_{α_x,β_x}: MFS(X)→MFS(X)as given below,

c) $M_{\alpha_x,\beta_x}(A) = \{(x, \mu_A(x) + \alpha_x(\gamma_A(x) + \frac{\zeta_A(x)}{2}), \nu_A(x) + \beta_x(\gamma_A(x) + \frac{\zeta_A(x)}{2})) \ / \ x \in X\}$ (4)

We will prove some new results based on these concepts;

Theorem 1

Let A, B ∈ MFS(X), $x \in X$, $with \alpha_x and \beta_x \in [0,1]$, $0 \le \alpha_x + \beta_x \le 1$

a) If A∈ FS(X): All the fuzzy sets on universal set X, then $M_{\alpha_x,\beta_x}(A)$ = A

b) $(M_{\alpha_x,\beta_x}(A)^c)^c$= M_{α_x,β_x}(A)

c) $if \ \mu_A(x) + \nu_A(x) \neq 0 \ \forall x \in X and \ \ \alpha_x = \frac{\mu_A(x)}{\mu_A(x)+\nu_A(x)} and \beta_x = \frac{\nu_A(x)}{\mu_A(x)+\nu_A(x)}$ then $M_{\alpha_x,\beta_x}(A) = \{(x, \alpha_x, \beta_x) / x \in X\}$ and $M_{\alpha_x,\beta_x}(A) \in FS(X)$

Proof:

a) It is obvious.

b) We have $A = \{(x, \mu_A(x), \nu_A(x)) \ / \ x \in X\}$, therefore

$$A^c = \{(x, \nu_A(x), \mu_A(x)) \ / \ x \in X\}$$

Now we have,

$$M_{\alpha_x,\beta_x}(A) = \{(x, \mu_A(x) + \alpha_x(\gamma_A(x) + \frac{\zeta_A(x)}{2}), \nu_A(x) + \beta_x(\gamma_A(x) + \frac{\zeta_A(x)}{2})) \ / \ x \in X\}$$

So, we get

$(M_{\alpha_x,\beta_x}(A^c)^c = complement \ of \ \{(x, \nu_A(x) + \alpha_x(\gamma_A(x) + \frac{\zeta_A(x)}{2}), \mu_A(x) + \beta_x(\gamma_A(x) + \frac{\zeta_A(x)}{2})) \ / \ x \in X\}$

$$= \{(x, \mu_A(x) + \beta_x(\gamma_A(x) + \frac{\zeta_A(x)}{2}), \nu_A(x) + \alpha_x(\gamma_A(x) + \frac{\zeta_A(x)}{2})) \ / \ x \in X\}$$
$$= M_{\beta_x,\alpha_x}(A)$$

So, $(M_{\alpha_x,\beta_x}(A)^c)^c = M_{\beta_x,\alpha_x}(A)$

c) $M_{\alpha_x,\beta_x}(A) = \{(x, \mu_A(x) + \alpha_x(\gamma_A(x) + \frac{\zeta_A(x)}{2}), \nu_A(x) + \beta_x(\gamma_A(x) + \frac{\zeta_A(x)}{2})) \ / \ x \in X\}$

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$$= \{(x, \mu_A(x) + \frac{\mu_A(x)}{\mu_A(x)+\nu_A(x)}(\gamma_A(x) + \frac{\zeta_A(x)}{2}), \nu_A(x) + \frac{\nu_A(x)}{\mu_A(x)+\nu_A(x)}(\gamma_A(x) + \frac{\zeta_A(x)}{2})) / x \in X\}$$

$$= \{(x, \frac{\mu_A(x)}{\mu_A(x)+\nu_A(x)}, \frac{\nu_A(x)}{\mu_A(x)+\nu_A(x)})\}$$

(By putting $(\gamma_A(x) + \frac{\zeta_A(x)}{2}) = 1 - (\mu_A(x) + \nu_A(x))$

$M_{\alpha_x,\beta_x}(A) = \{(x, \alpha_x, \beta_x)\}$

Also $\alpha_x + \beta_x = \frac{\mu_A(x)}{\mu_A(x)+\nu_A(x)} + \frac{\nu_A(x)}{\mu_A(x)+\nu_A(x)} = 1 \forall x \in X$

So, $M_{\alpha_x,\beta_x}(A) \in FS(X)$

Theorem 2

If $M_{\alpha_x,\beta_x}: MFS \to MFS$, i.e., M_{α_x,β_x} transforms a MFS into another MFS, then show that the mediative fuzzy index of $M^n{}_{\alpha_x,\beta_x}(A)$ is $(1-\alpha_x-\beta_x)^n(\gamma_A(x) + \frac{\zeta_A(x)}{2})$.

Proof:

We have,

$$M_{\alpha_x,\beta_x}(A) = \{(x, \mu_A(x) + \alpha_x(\gamma_A(x) + \frac{\zeta_A(x)}{2}), \nu_A(x) + \beta_x(\gamma_A(x) + \frac{\zeta_A(x)}{2})) / x \in X\}$$

then the mediative fuzzy index of $M_{\alpha_x,\beta_x}(A)$ is given as

$$(\gamma_{M_{\alpha_x,\beta_x}(A)} + \frac{\zeta_{M_{\alpha_x,\beta_x}(A)}}{2})(x) = 1 - (\mu_A(x) + \alpha_x(\gamma_A(x) + \frac{\zeta_A(x)}{2})) - (\nu_A(x) + \beta_x(\gamma_A(x) + \frac{\zeta_A(x)}{2})) = (1-\alpha_x-\beta_x)(\gamma_A(x) + \frac{\zeta_A(x)}{2})$$

Clearly, $(\gamma_{M_{\alpha_x,\beta_x}(A)} + \frac{\zeta_{M_{\alpha_x,\beta_x}(A)}}{2}) \leq (\gamma_A(x) + \frac{\zeta_A(x)}{2}) \forall x \in X$

i.e. The operator M_{α_x,β_x} transform an MFS with large MF index into another MFS with small MF index.

Now, $M^2{}_{\alpha_x,\beta_x}(A) = M(M_{\alpha_x,\beta_x}(A))$

$$= \{(x, \mu_A(x) + \alpha_x(\gamma_A(x) + \frac{\zeta_A(x)}{2}) + \alpha_x(1-\alpha_x-\beta_x)(\gamma_A(x) + \frac{\zeta_A(x)}{2}), \nu_A(x) + \beta_x(\gamma_A(x) + \frac{\zeta_A(x)}{2}) + \beta_x(1-\alpha_x-\beta_x)(\gamma_A(x) + \frac{\zeta_A(x)}{2})) / x \in X\}$$

then the mediative fuzzy index of $M^2{}_{\alpha_x,\beta_x}(A)$ is given as

$$(\gamma_{M^2{}_{\alpha_x,\beta_x}(A)} + \frac{\zeta_{M^2{}_{\alpha_x,\beta_x}(A)}}{2})(x) = 1 - [\mu_A(x) + \alpha_x(\gamma_A(x) + \frac{\zeta_A(x)}{2}) + \alpha_x(1-\alpha_x-\beta_x)(\gamma_A(x) + \frac{\zeta_A(x)}{2})] - [\nu_A(x) + \beta_x(\gamma_A(x) + \frac{\zeta_A(x)}{2}) + \beta_x(1-\alpha_x-\beta_x)(\gamma_A(x) + \frac{\zeta_A(x)}{2})$$

$$(\gamma_{M^2{}_{\alpha_x,\beta_x}(A)} + \frac{\zeta_{M^2{}_{\alpha_x,\beta_x}(A)}}{2})(x) = (1-\alpha_x-\beta_x)^2(\gamma_A(x) + \frac{\zeta_A(x)}{2})$$

In general, we have

$M^n{}_{\alpha_x,\beta_x}(A) = M(M^{n-1}{}_{\alpha_x,\beta_x}(A))$

$$= \{(x, \mu_A(x) + \alpha_x(\gamma_A(x) + \frac{\zeta_A(x)}{2}) \dots \alpha_x(1-\alpha_x-\beta_x)^{n-1}(\gamma_A(x) + \frac{\zeta_A(x)}{2}), \nu_A(x) + \beta_x(\gamma_A(x) + \frac{\zeta_A(x)}{2}) + \cdots \beta_x(1-\alpha_x-\beta_x)^{n-1}(\gamma_A(x) + \frac{\zeta_A(x)}{2})) / x \in X$$

$$= \{(x, \mu_A(x) + \alpha_x(\gamma_A(x) + \frac{\zeta_A(x)}{2})\frac{1-(1-\alpha_x-\beta_x)^n}{\alpha_x+\beta_x}, \nu_A(x) + \beta_x(\gamma_A(x) + \frac{\zeta_A(x)}{2})\frac{1-(1-\alpha_x-\beta_x)^n}{\alpha_x+\beta_x} / x \in X\}$$

With

$$(\gamma_{M^n{}_{\alpha_x,\beta_x}(A)} + \frac{\zeta_{M^n{}_{\alpha_x,\beta_x}(A)}}{2})(x) = (1-\alpha_x-\beta_x)^n(\gamma_A(x) + \frac{\zeta_A(x)}{2})$$

Where

$$\alpha_x+\beta_x \neq 0 \quad \forall x \in X$$

$if\, \alpha_x+\beta_x = 0$ i.e. $\alpha_x = \beta_x = 0$, for some x∈X

Then $\mu_{M^n{}_{\alpha_x,\beta_x}(A)}(x) = \mu_A(x)$

$\nu_{M^n{}_{\alpha_x,\beta_x}(A)}(x) = \nu_A(x)$

On the basis of above theorem, we can easily observe that If A∈MFS (X) and $\alpha_x and \beta_x \in [0,1]$ and $\alpha_x+\beta_x \leq 1$ Then for each $x \in X$ we define

$$\lim_{n\to\infty} M^n_{\alpha_x,\beta_x}(A) = \{(x, \lim_{n\to\infty}\mu_{M^n_{\alpha_x,\beta_x}(A)}(x), \lim_{n\to\infty}\nu_{M^n_{\alpha_x,\beta_x}(A)}(x)) / x \in X\}$$

Theorem 3

For any mediative fuzzy set A defined on a universal set X i.e., $A \in MFS(X)$ and $\alpha_x and \beta_x \in [0,1]$ with $0 \leq \alpha x + \beta x \leq 1$ and for each $x \in X$ then

a) Show that the limiting case of $M^n_{\alpha_x,\beta_x}$ (A) is $M_{\frac{\alpha_x}{\alpha_x+\beta_x}}$(A) i.e.,

$$\lim_{n\to\infty} M^n_{\alpha_x,\beta_x}(A) = M_{\frac{\alpha_x}{\alpha_x+\beta_x}}(A)$$

b) Show that Mediative fuzzy index of $M^n{}_{\alpha_x,\beta_x}(A)$ is zero as n approaches to infinity *i.e.*,

$$\lim_{n\to\infty}\left(\gamma_{M^n{}_{\alpha_x,\beta_x}(A)}+\frac{\zeta_{M^n{}_{\alpha_x,\beta_x}(A)}}{2}\right)(x)=0$$

Proof:

a) $\lim_{n\to\infty}\mu_{M^n_{\alpha_x,\beta_x}(A)}(x)=\lim_{n\to\infty}\left(\mu_A(x)+\alpha_x\frac{1-(1-\alpha_x-\beta_x)^n}{\alpha_x+\beta_x}\left(\gamma_A(x)+\frac{\zeta_A(x)}{2}\right)\right)$

$$=\mu_A(x)+\frac{\alpha_x}{\alpha_x+\beta_x}\left(\gamma_A(x)+\frac{\zeta_A(x)}{2}\right)$$

and

$$\lim_{n\to\infty}\nu_{M^n_{\alpha_x,\beta_x}(A)}(x)=\lim_{n\to\infty}\left(\nu_A(x)+\beta_x\frac{1-(1-\alpha_x-\beta_x)^n}{\alpha_x+\beta_x}\left(\gamma_A(x)+\frac{\zeta_A(x)}{2}\right)\right)$$

$$=\nu_A(x)+\frac{\beta_x}{\alpha_x+\beta_x}\left(\gamma_A(x)+\frac{\zeta_A(x)}{2}\right)$$

So, we have

$\lim_{n\to\infty}M^n_{\alpha_x,\beta_x}(A)=$

$\{(x,\lim_{n\to\infty}\mu_{M^n_{\alpha_x,\beta_x}(A)}(x),\lim_{n\to\infty}\nu_{M^n_{\alpha_x,\beta_x}(A)}(x))/\ x\in X\}$

$=\left\{\left(x,\mu_A(x)+\frac{\alpha_x}{\alpha_x+\beta_x}\left(\gamma_A(x)+\frac{\zeta_A(x)}{2}\right),\nu_A(x)+\frac{\beta_x}{\alpha_x+\beta_x}\left(\gamma_A(x)+\frac{\zeta_A(x)}{2}\right)\right)/\ x\in X\right\}$

$=\left\{\left(x,\mu_A(x)+\frac{\alpha_x}{\alpha_x+\beta_x}(\gamma_A(x)+\right.\right.$

ζAx2,νAx+ (1−αxαx+βx)(γAx+ζAx2))/ x∈X }

$$\lim_{n\to\infty}M^n_{\alpha_x,\beta_x}(A)=M_{\frac{\alpha_x}{\alpha_x+\beta_x}}(A)$$

b) $\lim_{n\to\infty}\left(\gamma_{M^n{}_{\alpha_x,\beta_x}(A)}+\frac{\zeta_{M^n{}_{\alpha_x,\beta_x}(A)}}{2}\right)(x)=\lim_{n\to\infty}(1-\alpha_x-$ *βx)n(γAx+ζAx2)*

=0, since $\alpha_x+\beta_x\le 1$

which tends to zero as $n\to\infty$.

The operator M_{α_x,β_x} transform an MFS with large MF index into another MFS with small MF index, in fact the MF index firstly divided into three parts

$\left(\gamma_A(x)+\frac{\zeta_A(x)}{2}\right)=\alpha_x\left(\gamma_A(x)+\frac{\zeta_A(x)}{2}\right)+\beta_x\left(\gamma_A(x)+\frac{\zeta_A(x)}{2}\right)+(1-\alpha_x-\beta_x)\left(\gamma_A(x)+\frac{\zeta_A(x)}{2}\right)$, with $\alpha_x,\beta_x\in[0,1]$ *and* $\alpha_x+\beta_x\le 1$, these three components on RHS contains parts of membership, non-membership and uncertainly or contradictory respectively. and we add factors $\alpha_x\left(\gamma_A(x)+\frac{\zeta_A(x)}{2}\right)$ *and* $\beta_x\left(\gamma_A(x)+\frac{\zeta_A(x)}{2}\right)$ into membership and non-membership part by using the MF operator M_{α_x,β_x} as shown by (5), that means we can find something new about '*x*' from known or contradictory information, And the above theorem 3 that in limiting case of operator $M^n{}_{\alpha_x,\beta_x}$ converted into ordinary fuzzy set with MF index $\gamma_A(x)+\frac{\zeta_A(x)}{2}$ is changed into zero.

III. MEDIATIVE MULTI-CRITERIA FUZZY DECISION-MAKING (MMCFDM)

Let A = $\{A_1,A_2\ldots A_m)$ a set of alternative and C = $\{C_1, C_2,\ldots C_n\}$ be the set of criteria.

Step1

Assume that the characteristics of the alternative A_i are presented by the Mediative fuzzy sets shown as follows:

$A_i=\{C_1,\ \mu_{i1},\nu_{i1}),(C_2,\mu_{i2},\nu_{i2}),$
$(C_3,\mu_{i3},\nu_{i3})\ldots\ldots\ldots(C_n,\ \mu_{in},\nu_{in})\}$ $i = 1, 2 \ldots\ m,$

where μ_{ij} denotes the degree to which alternative A_i satisfies criterion C_j, ν_{ij} denotes the degree to which the alternative A_i does not satisfy criterion C_j (*i = 1,...,m, j = 1,...,n;).*

Step2

Let us consider a criterion C_j and C_k and $\ldots C_p$ or C_s, for a decision maker who want to choose an alternative which satisfy that considered criteria. And this requirement may be represented as C_j and C_k and $\ldots$ and C_p or C_s;

Step3

First, we generalized the Chen and Tan [4]by defining an evaluation function E over a mediative fuzzy set, we have modified this evaluation function by using mediative fuzzy sets to measure the degrees to which the alternative A_i satisfies or does not satisfy the decision-makers choice.

$$E'(A_i)=\{((\mu_{ij},\nu_{ij})\vee(\mu_{ik},\nu_{ik})\vee\ldots\ldots(\mu_{ip},\nu_{ip}))\wedge(\mu_{is},\nu_{is})\}\}=(\mu_{A_i},\nu_{A_i})\qquad(5)$$

where μ_{A_i}=max $\{\min(\mu_{ij},\mu_{ik},\ldots\ldots\mu_{ip}),\mu_{is}\}$ and ν_{A_i}=max $\{\min((\nu_{ij},\nu_{ik}\ldots\ldots\nu_{ip}),\nu_{is}\}$

where $\wedge$ and $\vee$ denote the infimum and supremum respectively.

We have also generalized the measure of suitability over the mediative based evaluation function to which extent A_i satisfies C_j is denoted by the score function $S'(E'(A_i)$ defined as;

$$S'(E'(A_i)=\mu_{A_i}-\nu_{A_i}\in[-1,1],\qquad(6)$$

If $S'(E'(A_i)$ is the largest value for some '*i*' then A_i will be the best alternative for decision makers

Step4

We have also generalized the degree of accuracy for the evaluation function provided by Hong and Choi [5], by taking mediative based evaluation function, and this accuracy is denoted by $H'(E'(A_i))$ and defined by;

$$H'(E'(A_i))=\mu_{A_i}+\nu_{A_i},\qquad(7)$$

If $H'(E'(A_i))$ is the largest value for some '*i*' then A_i will be the best alternative, we also extends score functions $S_1 and S_2$ provided by Li et al. [16 and 17]over mediative based evaluation function to measure the degree of suitability as;

$$S'_1(E'(A_i)=\mu_{A_i},\ S'_2(E'(A_i)=1-\nu_{A_i}\qquad(8)$$

$$\text{or } S'_1(E'(A_i)=\mu_{A_i}-\nu_{A_i},\ S'_2(E'(A_i)=1-\nu_{A_i}\qquad(9)$$

Firstly, we draw a conclusion by the value of S'_1, if S'_1 is the largest value for some '*i*' then A_i will be the best alternative. But if S'_1 of all A_i are equal then we use the value of S'_2 to draw any conclusion.

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Step5

We have presented this new technique to solve the multi criteria decision making problems. By introducing a score function based on mediative fuzzy point operator;

A. Mediative fuzzy Score function

$$J'_n(E'(A_i)) = \mu_{M^n_{\alpha_x,\beta_x}(E'(A_i))}(x)$$
$$= \mu_{A_i} + \alpha_x(\gamma_{E'(A_i)} + \frac{\zeta_{E'(A_i)}}{2}) + \cdots \alpha_x(1 - \alpha x - \beta x)n-1(\gamma E'Ai+\zeta E'Ai2)$$

$$J'_n(E'(A_i)) = \mu_{A_i} + \frac{\alpha_x(1-(1-\alpha_x-\beta_x)^n)}{(\alpha_x+\beta_x)}(\gamma_{E'(A_i)} + \frac{\zeta_{E'(A_i)}}{2}) \quad (10)$$

And

$$J'_\infty(E'(A_i)) = \mu_{\lim_{n\to\infty} M^n_{\alpha_x,\beta_x}(E'(A_i))}(x)$$
$$= \mu_{M_{\frac{\alpha_x}{\alpha_x+\beta_x}}(E'(A_i))}(x)$$
$$J'_\infty(E'(A_i)) = \mu_{A_i} + \frac{\alpha_x}{(\alpha_x+\beta_x)}(\gamma_{E'(A_i)} + \frac{\zeta_{E'(A_i)}}{2}), \quad \alpha_x + \beta_x \neq 0 \quad (11)$$

Where α_x, $\beta_x \in [0,1]$ with $\alpha_x + \beta_x \leq 1$ and

$$\gamma_{E'(A_i)} + \frac{\zeta_{E'(A_i)}}{2} = 1 - \mu_{A_i} - \nu_{A_i}$$

If $J'_n(E'(A_i))$ is large, then alternative A_i will be the best choice for to satisfied decision maker's requirements, i.e. if there exist to $i_1 \, and i_2$ such that $J'_n\left(E'(A_{i_1})\right) < J'_n\left(E'(A_{i_2})\right)$ then A_{i_2} will be the best choice then A_{i_2} . if $\gamma_{E'(A_i)} + \frac{\zeta_{E'(A_i)}}{2} = 0$ then we may write $J'_n(E'(A_i)) = (J'_n(E'(A_i)))^+$ if there are two alternative $A_1 and A_2$ such that $J'_n(E'(A_1)) = 0.3 \; and J'_n(E'(A_2)) = 0.3^+$ then A_2 will be best choice then A_1.

The following example describes that the score function J'_n and J'_∞ defined above improves that have been given by the previous studies.

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IV. NUMERICAL COMPUTATION

In this section, we will use the mathematical concepts which we have defined for Mediative fuzzy sets in section II and applied the concepts for multi-criteria decision making in section III for developing the MMCFDM. We will use two numerical examples for decision making support system and various evidence for take-up decision for verifying our results by using this method defined for mediative fuzzy set we have shown over concept with the help of these examples

Example 1

Let A = $\{A_1,A_2 A_3, A_4, A_5)$ a set of alternative and C = $\{C_1, C_2, C_3\}$ be the set of criteria. And the characteristic of alternative in terms of criteria defined as

$$A_1 = \{(C_1, 0.2, 0), (C_2, 0.3,0.1), (C_3, 0.2, 0)\}$$
$$A_2 = \{(C_1, 0.3, 0.1), (C_2, 0.2,0.2), (C_3, 0.3, 0.1)\}$$
$$A_3 = \{(C_1, 0.4, 0.2), (C_2, 0.3,0.3), (C_3, 0.5, 0.4)\}$$
$$A_4 = \{(C_1, 0.5,0.3), (C_2, 0.4,0.4), (C_3, 0.4, 0.2)\}$$
$$A_5 = \{(C_1, 0.1,0.4), (C_2, 0.2,0.4), (C_3, 0.6, 0.4)\}$$

Suppose there is a decision maker who wants to choose alternative which satisfies the criteria C_1 and C_2 or C_3.

Now, by equation (5), we have $E'(A_1) = (0.2, 0)$

$$E'(A_2) = (0.3, 0.1)$$
$$E'(A_3) = (0.5, 0.3)$$
$$E'(A_4) = (0.4, 0.2) \text{ And } E'(A_5) = (0.6, 0.4)$$

Now by generalized our formula we have the degrees of suitability to which the alternatives satisfy the decision maker's choice obtained by the score function, from equation (6) we have

$S'(E'(A_1)) = S'(E'(A_2)) = S'(E'(A_3)) = S'(E'(A_4)) = S'(E'(A_5))$= 0.2

Now, it is to be noted that all the values of degree of suitability are equal, so we cannot conclude any conclusion, so we need to apply our generalized formula for mediative based evaluation function, then we have the degree of accuracy calculated by equation (7) as,

$H'(E'(A_1)) = 0.2, H'(E'(A_2))$ =0.4, $H'(E'(A_3))$ =0.8, $H'(E'(A_4))$ =0.6, $H'(E'(A_5))$=1

Which of the above is best, it's depends upon the decision maker choice, stable person might choice the alternative A_5 but an aggressive person may choice A_1

If we apply our extended methods using equation (8) then we have A_5 will be the best choice by first method since $S'_1(E'(A_5))$ is largest value. But if we use equation (9) second method then A_1 will be the best choice since in this case $S'_2(E'(A_1))$ will be the largest value, so we cannot conclude anything from here. So we need to calculate the mediative fuzzy score function J_n and J_∞ by taking $\alpha_x = \mu_{A_i} \, and \, \beta_x = \nu_{A_i}$ by using equation (10) and (11) we have,

$$J'_n(E'(A_1)) = 0.2 + \frac{0.2(1-(1-(0.2))^n)}{0.2}(0.8);$$

n=1: $J'_1(E'(A_1))$ = 0.36; n=2: $J'_2(E'(A_1))$ = 0.488;
n=3: $J'_3(E'(A_1))$ = 0.5904; n=4: $J'_4(E'(A_1))$ = 0.6723;
n=5: $J'_5(E'(A_1))$ = 0.7379; n=∞: $J'_\infty(E'(A_1))$ = 1;

$$J'_n(E'(A_2))=0.3+\frac{0.3(1-(1-(0.4))^n)}{0.4}(0.6);$$

n=1: $J'_1(E'(A_2))$ =0 .48; n=2: $J'_2(E'(A_2))$ = 0.588;
n=3: $J'_3(E'(A_2))$ = 0.628; n=4: $J'_4(E'(A_2))$ = 0.6917;
n=5: $J'_5(E'(A_2))$ = 0.7150; n=∞: $J'_\infty(E'(A_2))$ = 0.75;

$$J'_n(E'(A_3))=0.5+\frac{0.5(1-(1-(0.8))^n)}{0.8}(0.2);$$

n=1; $J'_1(E'(A_3))$ = 0.6; n=2; $J'_2(E'(A_3))$ = 0.58;
n=3: $J'_3(E'(A_3))$ = 0.564; n=4: $J'_4(E'(A_3))$ = 0.5512;
n=5: $J'_5(E'(A_3))$ = 0.54096; n=∞: $J'_\infty(E'(A_3))$ = 0.625;

$$J'_n(E'(A_4))=0.4+\frac{0.4(1-(1-(0.6))^n)}{0.6}(0.4);$$

n=1: $J'_1(E'(A_4))$ = 0.56; n=2: $J'_2(E'(A_4))$ = 0.624;
n=3: $J'_3(E'(A_4))$ = 0.6496; n=4: $J'_4(E'(A_4))$ = 0.6598;
n=5: $J'_5(E'(A_4))$ = 0.6639; n=∞: $J'_\infty(E'(A_4))$ = 0.6667;

$J'_n(E'(A_5))=0.6+\frac{0.6(1-(1-(1))^n)}{1}(0);$

n=1; $J'_1(E'(A_5)) = 0.6+$; n=2:$J'_2(E'(A_5)) = 0.6+$;
n=3:$J'_3(E'(A_5)) = 0.6+$; n=4:$J'_4(E'(A_5)) = 0.6+$;
n=5:$J'_5(E'(A_5)) = 0.6+$; n=∞:$J'_\infty(E'(A_5)) = 0.6+$;

we get the score of alternative A = $\{A_1, A_2, A_3, A_4, A_5\}$ as follows:

n=1: $A_5 > A_3 > A_4 > A_2 > A_1$;
n=2: $A_4 > A_5 > A_2 > A_3 > A_1$;
n=3: $A_4 > A_2 > A_5 > A_1 > A_3$;
n=4: $A_2 > A_1 > A_4 > A_5 > A_3$;
n=5: $A_1 > A_2 > A_4 > A_5 > A_3$;
n→ ∞: $A_1 > A_2 > A_4 > A_3 > A_5$;

Therefore, we now draw a conclusion on the behalf of these score function J_∞ and J_n, $A_3$is the grade 1 choice, A_5 is the grade 2 choice, A_4 is the grade 3 risky choice,A_2 is the grade 4 choice and $A_1$is grade 5 choice for the decision makers.

Example 2

Let consider an example in which a group of four experts monitoring several symptoms as per their knowledge-base to find that which kind of disease the patient is suffering from. Let us consider a group of four possible diseases like; Pneumonia, Malaria, Typhoid and Dengue and denoted as A = $\{A_1, A_2 A_3, A_4\}$ set of diseases. The experts select three parameters of symptoms to evaluate the three diseases. And let C = $\{C_1, C_2, C_3\}$ is the set of parameters of symptoms, representing the headache, temperature, and chest pain respectively.

And the characteristic of alternative in terms of criteria defined as

$$A_1 = \{(C_1, 0.2, 0.1), (C_2, 0.2, 0.1), (C_3, 0.2, 0)\}$$
$$A_2 = \{(C_1, 0.5, 0.2), (C_2, 0.5, 0.2), (C_3, 0.5, 0.4)\}$$
$$A_3 = \{(C_1, 0.2, 0.2), (C_2, 0.2, 0.4), (C_3, 0.2, 0.3)\}$$
$$A_4 = \{(C_1, 0.4, 0.2), (C_2, 0.4, 0.5), (C_3, 0.5, 0.5)\}$$

Suppose there is a decision maker who wants to choose alternative which satisfies the criteria C_1 and C_2 orC_3.

Now, $E'(A_1) = (0.2,0)$; $E'(A_2) = (0.5,0.2)$
$E'(A_3) = (0.2,0.3)$ and $E'(A_4) = (0.5,0.5)$

Now we use the score function J_n and $J_\infty, \alpha_x = \mu_{A_i}$ *and* $\beta_x = \nu_{A_i}$

Then we have,

$$J'_n(E'(A_1)) = 0.2 + \frac{0.2(1-(1-(0.2))^n)}{0.2}(0.8);$$

n=1:$J'_1(E'(A_1)) = 0.36$; n=2: $J'_2(E'(A_1)) = 0.488$;
n=3:$J'_3(E'(A_1)) = 0.5904$; n=4: $J'_4(E'(A_1)) = 0.6723$;
n=∞: $J'_\infty(E'(A_1)) = 1$;

$J'_n(E'(A_2))=0.5+\frac{0.5(1-(1-(0.7))^n)}{0.7}(0.3);$

n=1:$J'_1(E'(A_2)) = 0.65$; n=2:$J'_2(E'(A_2)) = 0.605$;
n=3: $J'_3(E'(A_2)) = 0.5735$; n=4: $J'_4(E'(A_2)) = 0.55145$;
n=∞:$J'_\infty(E'(A_2)) = 0.714$;

$J'_n(E'(A_3))=0.2+\frac{0.2(1-(1-(0.5))^n)}{0.5}(0.5);$

n=1:$J'_1(E'(A_3)) = 0.3$; n=2:$J'_2(E'(A_3)) = 0.25$;
n=3: $J'_3(E'(A_3)) = 0.225$; n=4:$J'_4(E'(A_3)) = 0.2125$;
n=∞:$J'_\infty(E'(A_3))$ =0.6667;

TABLE-I: VALUES OF SCORE FUNCTIONS OF ALTERNATIVES

	$J'_i(E'(A_1))$	$J'_i(E'(A_2))$	$J'_i(E'(A_3))$	$J'_i(E'(A_4))$
n=1	0.36	0.56	0.3	0.5+
n=2	0.488	0.52	0.25	0.5+
n=3	0.5904	0.5057	0.225	0.5+
n=4	0.6723	0.5017	0.2125	0.5+
n→ ∞	1	0.714	0.6667	0.5+

$J'_n(E'(A_4))=0.5+\frac{0.5(1-(1-(1))^n)}{1}(0);$

n=1:$J'_1(E'(A_4))$ =0.5+; n=2:$J'_2(E'(A_4)) = 0.5+$;
n=3:$J'_3(E'(A_4))$ =0.5+; n=4: $J'_4(E'(A_4)) = 0.5+$;
n=∞:$J'_\infty(E'(A_4))$ =0.5+;

From the Table-I, we get the score of alternative A= $\{A_1, A_2, A_3, A_4\}$ as follows

n=1:$A_2 > A_4 > A_1 > A_3$; n=2:$A_2 > A_4 > A_1 > A_3$;
n=3:$A_1 > A_2 > A_4 > A_3$; n=4:$A_1 > A_2 > A_4 > A_3$;
n→ ∞: $A_1 > A_2 > A_3 > A_4$;

Therefore, we now draw a conclusion on the behalf of these score function J_∞ and J_n, $A_3$is the grade 1 risky choice, A_4 is the grade 2 risky choice, A_2 is the grade 3 risky choice and A_1 is the grade 4 risky choice, so Pneumonia is the top most risky grade choice for the decision makers.

V. CONCLUSION

In this paper, we have defined a new technique to handle the multi-criteria decision-making problem as the fact that, our proposed technique deals with the Mediative fuzzy logic rather than fuzzy or Intuitionistic fuzzy logic. In this work we have defined a mediative fuzzy set which provides a mediate solution to the problem. When conflict of interest exits and the decision maker is not in the position to take any decision due to the hesitation, then we have to use the mediative fuzzy logic to deal the MCDM in the form of MMCFDM in which the mediative fuzzy logic is used to characterized the alternatives and give the best approach without changing the underlying value and motivational behavior of the criteria's. We have also defined the concept of Mediative fuzzy point operators (MF Point Operator) and on the behalf of MF point operator we have constructed a new score function for multi-criteria decision making problem know as mediative score function and we will calculate the mediative fuzzy score of alternative, which is very useful to the decision makers to choose the one alternative among the set of alternative which satisfies the different set of criteria. Examples have also been given to illustrate the mediative multi-criteria decision-making process in the numerical computation section. These examples also demonstrate a new approach to make comprehensive decision for the decision makers in the light of mediative fuzzy logic.

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