

Fractional Order PI Controller for Nonlinear Processes using Binomial Theorem Approximation

Gowtham.T, Srinivasan.K, Balaji.M, Kaushik.S, Thirukuralkani .K .N



Abstract— In Most of the process industries, Integer order Proportional Integral controller (IOPI) has been designed for different processes due to its simplicity but it will damage the final control element due to the sudden change in controller output. So, this research work proposed to eliminate the ringing phenomena by applying large pure time delay in the controller output by proposing Fractional Order Proportional Integral (FOPI) controller using Binomial theorem approximation for the nonlinear conical and spherical tank level processes. The control of liquid level in the nonlinear tank is arduous in nature due to an imbalance in the area of the transverse section of the tank. The robustness of the IOPI and FOPI controllers has demonstrated in Time Domain Specifications (TDS). The FOPI controller eliminates the peak overshoot in the process variable and ringing effect in controller output to protect the pneumatic positioned control valve.

Keywords—Fractional order Proportional Integral controller, Binomial theorem approximation, Ringing effect, and Level control.

I. INTRODUCTION

In chemical and paramedical industries the nonlinear spherical and conical tanks play a vital role to store and dispense the liquids. The applied liquid with pressure is get balanced without any tangential stress to banish the bubbles in chemical industries. Therefore uniform pressure has to be distributed into the spherical and conical tanks to avoid forming foam inside the tank. The control of liquid level in the nonlinear tanks is difficult due to their nonlinear dynamic behavior in terms of time domain specifications such as peak overshoot, settling time, peak time and rise time. The FOPI controller produces smooth controller output to eliminate the ringing phenomena and produces better peak overshoot and settling time.

Ying Luo [1] developed FOPI controller for stable first order system with delay time. The controller performance of the FOPI controller satisfies robustness and loop gain variations when comparing to the traditional PI controller.

Michael K [2] designed a Fractional order PD controller for Fractional order controlled system by approximating the Mittag-Leffler-type Laplace transform. The FOPI controller produces satisfactory control results by eliminating the critically damped oscillations presents in the process variable. Ying Luo [3] proposed tuning methods for FOPI, FO [PI] and IOPI controllers for fractional order systems. Both FOPI and FO [PI] controller improves the controller performance by eliminating peak overshoot presents in closed loop response. Zhe Gao [4] proposed the Discretized FOPI controller using Haar wavelet approximation. Integral time is divided into two parts based upon the present and past sampled data. In this approach, a 30% error is getting minimized when comparing to the IOPI controllers. N.S.Bhuvanewari [5] developed an adaptive and optimal controller for a conical tank level control system using the neural algorithm. In this type of controller tracking case of set point can be achieved quickly by improving the settling time but steady state errors have produced in the process variable. Zhang Zhi-Gang [6] implemented Dahlin's and conventional PID controllers for steam temperature system and discussed the controller output. The Dahlin's controller eliminates the ringing phenomena presents in the controller output. Puneet Mishra [7] proposed a classical PID controller structure with automatic online tuning. The Integral gain is varied in accordance with the error to avoid damage to the control valve. G.Sivagurunathan [8] modeled spherical tank level control system by black box modeling technique for the Fuzzy tuned PID controller. Po Yen Lin [9] spherical agglomeration continuous stirred tank setup is made for drug products for effect cut and sieving. This paper indicates the spherical tank is most widely used in paramedical industries. Kishore Bingi [10] designed a Fractional order PI controller using oustaloup approximation to control real time pressure plant. This paper results in better performance in terms of overshoot and rise time based upon Time Domain specifications. G.Sakthivel [11] designed an intelligent Fuzzy logic controller for spherical tank level control system. Servo response of experiment initiates slow settling time, steady state error with no overshoot. Gowtham.T [12] simulated spherical tank level control system using MATLAB Simulink. The results are compared with 3 review papers and finally proposed a nonlinear PI controller with better peak time.

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The PID controller provides desirable overshoot, rise time and settling time for large operating regions. Srinivasan.K [14] proposed the different tuning procedure for the pH control process and explained the plant behavior with Fractional Order PID Controller. Janarthanan.S [15] detailed about the design of FOPID controller using different tuning methods for the liquid level control of the conical tank system. In this research work, they explained the robust control action of FOPID controller.

Many researchers have done in spherical and conical tank level system by using a conventional PI controller it generates peak overshoot presents in the process variable [8] [13].

The contributions in this work are as follows;

- The Real time Spherical and conical tank transfer functions have acquired by Process Reaction curve method by adopting the controller in manual mode. IOPI controller is tuned by Cohen-Coon tuning settings.
- Binomial theorem approximation rule is applied to the FOPI controller to improve the closed loop response by changing the order of lambda values. Step response for IOPI and FOPI controllers have been validated for nonlinear process and compared with the reported results in the literature.

II. EXPERIMENTAL SETUP

The Spherical and Conical tank systems are made by Differential pressure transmitter (DPT), Current to Pressure Converter (I/P), Current to voltage converter (I/V), Voltage to current converter (V/I), Rotameter and pneumatic positioner valve with actuator shown in Fig.2 and Fig.3 respectively.

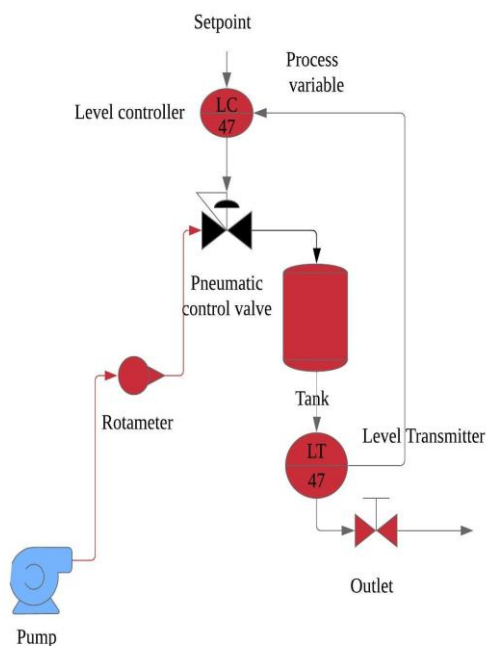


Figure 1 Level control process

The DPT is a level transmitter acts as a feedback measurement device to measure the level of the tank in terms of the pressure difference between the downstream and upstream and converts 4-20mA current. The converted current is passed to Current to Voltage converter (I/V) to

produce a voltage which is proportional to an applied current.

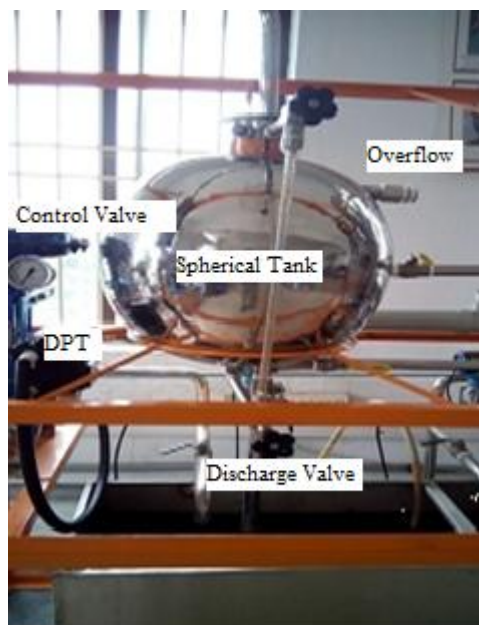


Figure 2 Real time Spherical tank system



Figure 3 Real time conical tank system

The voltage is connected to DAQ unit. The output from the DAQ is interfaced with PC to acquire the process variable. The error is getting detected with the help of the comparator unit. The controller output is generated based upon controller gain and error values to minimize the error presents in nonlinear tanks. The controller output is interfaced with DAQ in the form of 0-5 V. The voltage output is converted in terms of 4-20mA current by (V/I) converter and subsequently converted to 3-15psi by (I/P) converter to get manipulated variable to actuate the electro pneumatic positioned control valve to open and close.

III. MATHEMATICAL MODELING

The Mathematical modeling of Spherical and conical tank transfer function is described by the first order differential equation with dead time using Process Reaction Curve method is shown in Eqn. 1.

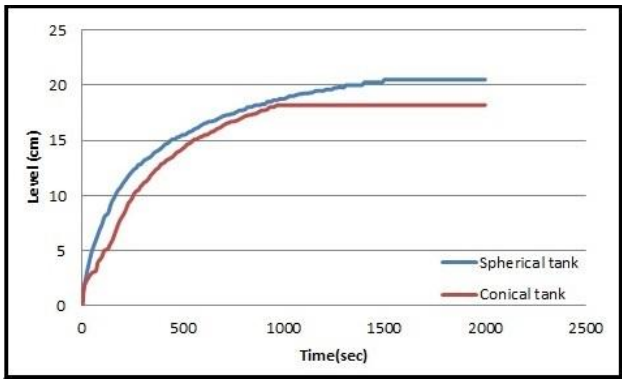


Figure 4 Open loop responses of nonlinear tank

$$\frac{dv}{dt} = Fin(t) - Fout(t) \quad (1)$$

$$V = \frac{4}{3}\pi h^3 \quad (2)$$

Where

- V = Volume of spherical tank
- F_{in} = Volumetric flow rate of spherical tank
- F_{out} = Volumetric flow rate for outlet flow rate
- h = Height of the tank in (cm)

Apply steady state values to the system from the open loop responses and substituting the equations (1) and (2) to bring the process as a linear.

$$\frac{H(s)}{Q(s)} = \frac{R_t}{\tau s + 1} \quad (3)$$

$$R_t = \frac{2hs}{F_{out}} \text{ and } \tau = 4\pi h_s R_t \quad (4)$$

$$G_p(s) = \frac{7.854e^{-0.1s}}{2022s + 1} \quad (5)$$

Eqn. 3 represents a spherical tank transfer function.

Where (R_t) is process gain, (τ) is time constant and (h_s) height of the tank at steady state. A controller output in terms of percentage values is applied to the pneumatic valve to obtain open loop responses.

$$V = \frac{1}{3}\pi h^2 \quad (6)$$

V is the volume of the conical tank system.

$$G_p(s) = \frac{K_p e^{-t_d(s)}}{\tau s + 1} \quad (7)$$

Similarly, the conical tank is modeled using Process reaction curve method where the height of the tank at steady state value is shown in Fig.4.

$$G_p(s) = \frac{1.08e^{-12s}}{30.31s + 1} \quad (8)$$

Eqn.8 represents a conical tank transfer function.

IV. DESIGN OF INTEGER ORDER PI CONTROLLER

The PI controller demands gain parameters like proportional gain (K_p) and integral time (T_i). The gain is acquired by open loop response of systems by adopting controller in manual mode.

The quarter decay ratio has applied to controller settings to obtain a controller gain by Cohen Coon tuning method and tuned gain values shown in Table 1. This tuning rule approached effective process variable than other tuning rules for the following transfer function is shown in Eqn. 5 and 8.

$$C(s) = K_p^\lambda \left(1 + \frac{K_I}{K_{ps}}\right)^\lambda \quad (9)$$

TABLE I. P-I TUNING VALUES FOR DIFFERENT REGIONS

Nonlinear systems	Tuning parameters	
	K_p	K_I
Conical tank	0.59	0.023
Spherical tank	42.97	0.249

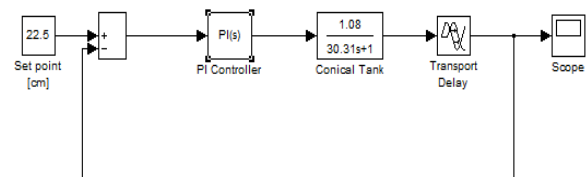


Figure 5 Simulation model for Conical Tank system

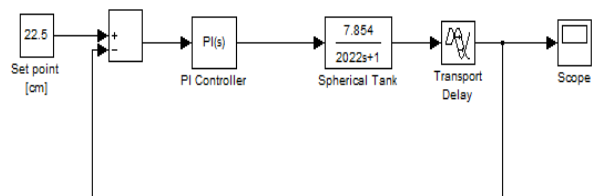


Figure 6 Simulation model for Spherical Tank system

The Integer order PI controller usually proportional and integral gain orders are $\lambda = 1$. Eqn.9 represents the final controller output for the following Fig.5 and Fig.6 respectively.

V. DESIGN OF FRACTIONAL ORDER PI CONTROLLER

In the Fractional order PI controller, the Proportional and Integral gain order λ and order of binomial theorem approximation have chosen by fmincon toolbox [15] using MATLAB Simulink tool. This proposed scheme will improve the step responses based upon the Time domain specifications.

$$C(s) = K_p^\lambda \left[1 + \frac{\lambda K_I}{K_{ps}} \frac{\lambda(\lambda - 1)}{2!} \left(\frac{K_I}{K_{ps}}\right)^2 \right] \quad (10)$$

Eqn. 10 represents second order approximation for conical tank system.

$$C(s) = 0.59^{0.99} \left[1 + \frac{0.99 \times 0.023}{\frac{0.59s}{0.99(0.99-1)} + \frac{0.023}{(0.59s)^2}} \right] \quad (11)$$

Eqn. 11 represents proportional, integral gain values and lambda values are substituted.

$$C(s) = 0.5931 \left[\frac{0.3481s^2 + 0.0134343s - 0.00000261855}{0.3481s^2} \right] \quad (12)$$

Eqn. 12 represents the final control output to actuate the final control element.

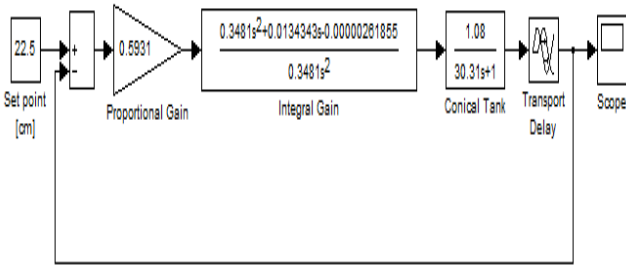


Figure 7 Simulation model for Conical Tank system using 2nd order approximation

Here $\lambda = 0.99$ and 2nd order approximation technique has chosen for conical tank level control system. The corresponding reasons were discussed in Fig.9 and Fig.10 respectively.

$$c(s) = K_p^\lambda \left[1 + \frac{\lambda K_i}{K_p s} + \frac{\lambda(\lambda-1)}{2!} \left(\frac{K_i}{K_p s} \right)^2 + \frac{\lambda(\lambda-1)(\lambda-2)}{3!} \left(\frac{K_i}{K_p s} \right)^3 \right] \quad (13)$$

Eqn. 13 represents third order approximation for spherical tank system.

$$C(s) = 42.97^{0.85} \left[1 + \frac{0.85 \times 0.249}{42.97s} + \frac{0.85(0.85-1)}{2} \left(\frac{0.249}{42.97s} \right)^2 + \frac{0.85(0.85-1)(0.85-2)}{6} \left(\frac{0.249}{42.97s} \right)^3 \right] \quad (14)$$

Eqn. 14 represents proportional, integral gain values and lambda values were substituted.

$$C(s) = 24.44 \left[\frac{79340.706s^3 + 389.899s^2 - 0.1132s + 0.00037707}{79340.706s^3} \right] \quad (15)$$

Eqn. 15 represents final control output for 3rd order approximation.

$$C(s) = 24.44 \left[\frac{1846.42s^2 + 9.0946005s - 0.002635}{18.46.42s^2} \right] \quad (16)$$

Eqn. 16 represents final control output for 2nd order approximation. Eqn.15 and Eqn.16 step responses were discussed in Fig.10.

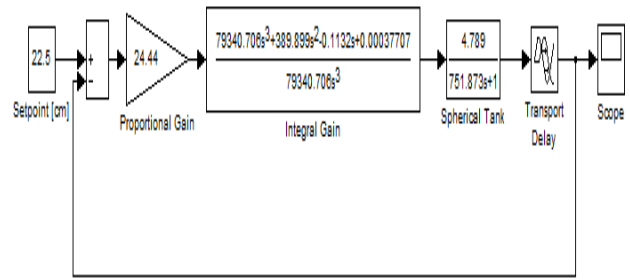


Figure 8 Simulation model for Spherical Tank system using 3rd order approximation

Here $\lambda = 0.85$ and 3rd order approximation technique has chosen for spherical tank level control system. The corresponding reasons are discussed in Fig.12 and Fig.13 respectively.

VI. EXPERIMENTAL RESULTS

The step responses of FOPI and IOPI controller are obtained for an operating region 22.5cm as shown in Fig.9.

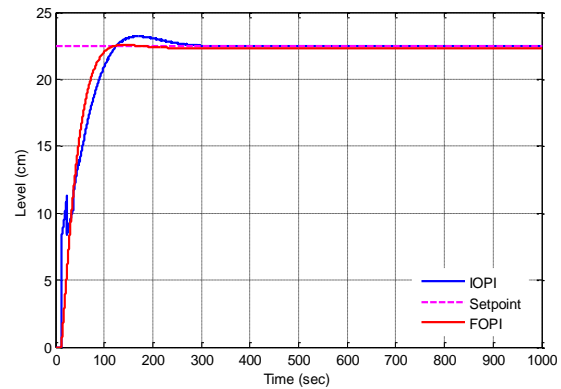


Figure 9 Step response of the conical tank process

The FOPI produces no peak overshoot, better rise and settling time than IOPI controller as shown in Fig.9.

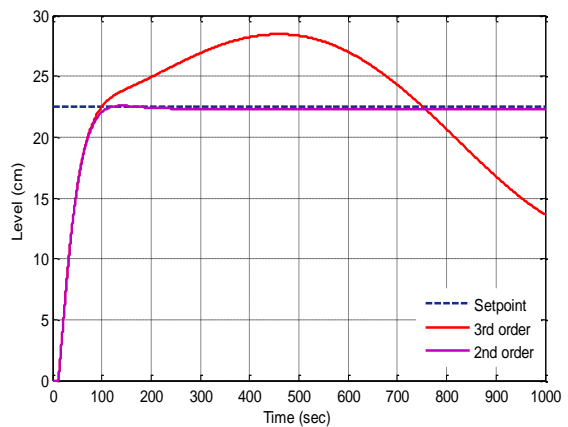


Figure 10 Order of Binomial theorem approximations for conical tank process.

Here 2nd and 3rd order approximations were applied using trial and error method. Hence 3rd order approximation responses are unstable due to the large gain value shown in Fig.10.

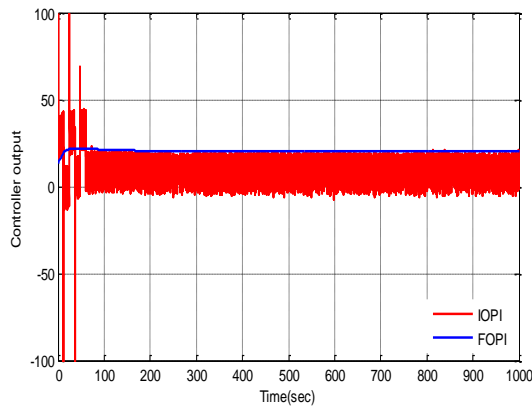


Figure 11 Controller output of conical tank process.

The FOPI and IOPI controllers generate the controller output to actuate the pneumatic positioned valve to adjust the flow rate. This ringing effect produced in the IOPI controller output (CO) could reduce the lifespan of the pneumatic control valve which is shown in Fig.11. The ringing phenomenon describes a signal with a discontinuity in the time domain infinite frequency content. In the control valve the air passes to control the liquid flow by the direction of the airflow. In this research work, disc seat poppet type valve is used for fast-short stroke to provide accurate maximum and minimum opening to control the liquid level in nonlinear tanks. Therefore FOPI controller provides longer service life than IOPI controllers.

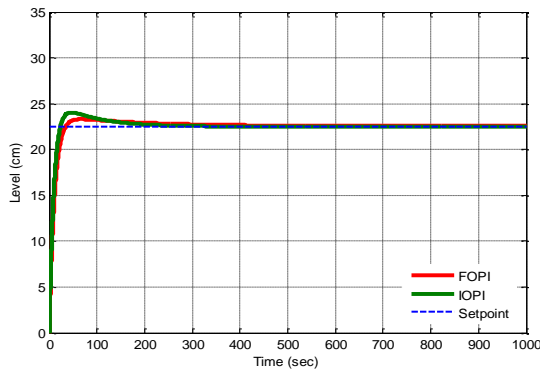


Figure 12 Step response of Spherical tank process

The step responses of FOPI controller produces better rise, settling time with no peak overshoot than IOPI controller shown in Fig.12.

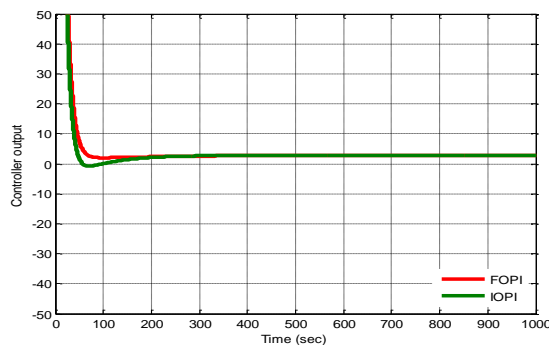


Figure 13 Controller output of spherical tank process.

Here the level of the spherical tank increases where the controller output is decreased. After settling, the process variable and controller output get maintained constantly.

IOPI controller output reaches large positive and negative value when comparing to the FOPI controller is shown in Fig.13.

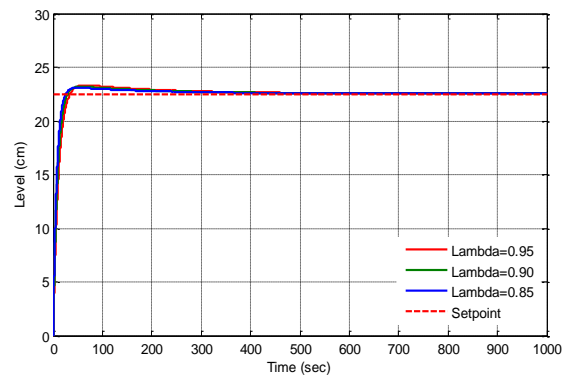


Figure 14 Step responses for different λ values of FOPI controller

The lambda values were chosen by fmincon tool box [15] and finally, the suitable lambda value selected as 0.85 to control the level of the spherical tank. This gives better control of the spherical tank level process. When lambda value increases rise time, settling time and peak overshoot are getting increased. The lambda value 0.85 results in better rise time with less overshoot shown in Fig.14.

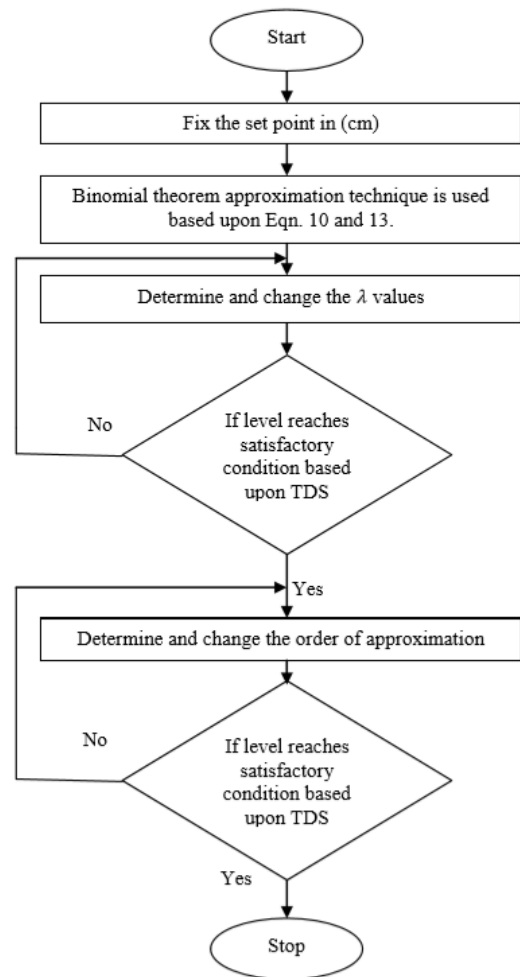


Figure 15 The Flow diagram of FOPI controller

TABLE II. COMPARATIVE ANALYSIS AND RESULTS OF CONTROL SCHEMES BASED UPON THE TIME DOMAIN SPECIFICATIONS.

Controllers	Set point (cm)	Specifications			
		Rise time (sec)	Peak time (sec)	Settling time(sec)	% overshoot
Controllers reported in Literature [Spherical tank]					
Conventional PI [8]	30	240	-	1050	10
Fuzzy Logic Controller [8]		210	-	380	0
Proposed controller results [Spherical tank]					
Conventional PI	22.5	18	25	240	4.5
Fractional order PI		17	40	237	2
Controllers reported in Literature [Conical tank]					
Conventional PI [13]	40	30	50	300	30
Proposed controller results [Conical tank]					
Conventional PI	22.5	140	170	300	4.4
Fractional order PI		70	95	95	0

VII. CONCLUSION AND FUTURE WORK

The Fractional Order PI controller using binomial theorem approximation technique has been implemented in this work. This has done by changing the lambda values and the order of approximation until it reaches the optimum values as shown in Fig.15. The time domain specification results of the FOPI and IOPI controller has shown in the Table II. These results were compared with the Fuzzy logic controller and Conventional PID controllers as described in the literature. The FOPI controller produces a better control action with quick rise and settling time when compared to the Fuzzy logic and IOPI controllers for both the conical and spherical tank level system without any steady state error. The ringing effect presents in the controller output is getting eliminated when compared to the IOPI controller. Therefore the FOPI controller gives the better control action than the intelligent controllers in terms of settling time, Rise time and Controller output of the final control element. In future FOPI controller can be implemented to the hemisphere tanks for the reason that hemisphere tanks are less expensive and easy to fabricate when comparing to spherical and conical tanks.

VIII. ACKNOWLEDGMENT

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