



# Degree of a Vertex in Max-Product of Intuitionistic Fuzzy Graph

S. Yahya Mohamed, A. Mohamed Ali

**Abstract:** Graph theory has applications in many areas of computer science, including data mining, image segmentation, clustering and networking. Product on graphs has a wide range of application in networking system, automata theory, game theory and structural mechanics. In many cases, some aspects of a graph-theoretic problem may be uncertain. Intuitionistic fuzzy models provide more compatible to the system compared to the fuzzy models. An intuitionistic fuzzy graph can be derived from two given intuitionistic fuzzy graphs using max-product. In this paper, we studied the degree of vertex in intuitionistic fuzzy graph by the max-product of two given intuitionistic fuzzy graph. Also find the necessary and sufficient condition for max-product of two intuitionistic fuzzy graphs to be regular.

**Keywords:** Intuitionistic fuzzy graph (IFG), Max-Product, Regular Intuitionistic fuzzy graph.

## I. INTRODUCTION

In 1965, Zadeh [30] represented the uncertainty as fuzzy subset of sets. Since then, the theory of fuzzy sets has become a strong area of research in different disciplines including medical and life sciences, management sciences, social sciences, engineering, statistics, artificial intelligence, signal processing, graph theory, multi-agent systems, pattern recognition, robotics, computer networks, expert systems, decision-making, automata theory, etc. The concept of intuitionistic fuzzy set was introduced by Atanassov [2]. Shannon and Atanassov [5] introduced the notion of intuitionistic fuzzy graphs in 1994. Parvathi and Karunambigai [6] defined the intuitionistic fuzzy graph as a special case of intuitionistic fuzzy graphs introduced by Atanassov and Shannon [2]. Akram and Davvaz [1] studied some operations on strong intuitionistic fuzzy graphs. Yahya Mohamed and Mohamed Ali [6, 7, 8] introduced the max-product and modular product on intuitionistic fuzzy graphs and they defined the complement of modular product of two intuitionistic fuzzy graph. Some recent works in intuitionistic fuzzy graph and interval-valued Pythagorean

fuzzy graph can be found in [9, 10, 11, 12]. In this paper, we have studied the degree of vertex in intuitionistic fuzzy graph by the max-product of two given intuitionistic fuzzy graph. Some regularity conditions for max product are stated and proved.

## II. PRELIMINARIES

Throughout this paper, assume that  $G^*$  is a crisp graph and  $G$  is an intuitionistic fuzzy graph.

### Definition 2.1

An intuitionistic fuzzy graph is of the form  $G = ((\sigma_1, \sigma_2), (\mu_1, \mu_2))$  on  $G^* = (V, E)$ , where

1.  $V = \{x_1, x_2, \dots, x_n\}$  such that  $\sigma_1: V \rightarrow [0,1]$  and  $\sigma_2: V \rightarrow [0,1]$  denote the degree of membership and non-membership of the element  $x_i \in V$  respectively, such that  $\sigma_1(x_i) + \sigma_2(x_i) \leq 1$  for all  $x_i \in V$ .
2.  $E \subseteq V \times V$  where  $\mu_1: E \rightarrow [0,1]$  and  $\mu_2: E \rightarrow [0,1]$  are defined by  $\mu_1(x_i, x_j) \leq \sigma_1(x_i) \wedge \sigma_1(x_j)$  and  $\mu_2(x_i, x_j) \leq \sigma_2(x_i) \vee \sigma_2(x_j)$  such that  $\mu_1(x_i, x_j) + \mu_2(x_i, x_j) \leq 1, \forall (x_i, x_j) \in E$ .

### Definition 2.2

An intuitionistic fuzzy graph  $G = ((\sigma_1, \sigma_2), (\mu_1, \mu_2))$  is called strong intuitionistic fuzzy graph if

$$\begin{aligned} \mu_1(x_i, x_j) &= \sigma_1(x_i) \wedge \sigma_1(x_j) \text{ and} \\ \mu_2(x_i, x_j) &= \sigma_2(x_i) \vee \sigma_2(x_j), \quad \forall (x_i, x_j) \in E, i \neq j. \end{aligned}$$

### Definition 2.3

An intuitionistic fuzzy graph  $G = ((\sigma_1, \sigma_2), (\mu_1, \mu_2))$  is called complete intuitionistic fuzzy graph if

$$\begin{aligned} \mu_1(x_i, x_j) &= \sigma_1(x_i) \wedge \sigma_1(x_j) \text{ and} \\ \mu_2(x_i, x_j) &= \sigma_2(x_i) \vee \sigma_2(x_j), \quad \forall x_i, x_j \in V, i \neq j. \end{aligned}$$

### Definition 2.4

Let  $G = ((\sigma_1, \sigma_2), (\mu_1, \mu_2))$  be an intuitionistic fuzzy graph, then the order of  $G$  is defined to be  $O(G) = (O_{\sigma_1}(G), O_{\sigma_2}(G))$  where  $O_{\sigma_1}(G) = \sum_{x \in V} \sigma_1(x)$  and  $O_{\sigma_2}(G) = \sum_{x \in V} \sigma_2(x)$ .

### Definition 2.5

Let  $G = ((\sigma_1, \sigma_2), (\mu_1, \mu_2))$  be an intuitionistic fuzzy graph, then the size of  $G$  is defined to be  $S(G) = (S_{\mu_1}(G), S_{\mu_2}(G))$

Manuscript published on November 30, 2019.

\* Correspondence Author

**S.Yahya Mohamed\***, Assistant Professor, PG and Research Department of Mathematics, Government Arts College, Affiliated to Bharathidasan University, Trichy, Tamilnadu India. Email: [yahya\\_md@yahoo.co.in](mailto:yahya_md@yahoo.co.in)

**A.Mohamed Ali\***, Assistant Professor, Department of Mathematics, G.T.N. Arts College (Autonomous), Dindigul -05.

Research Scholar, Government Arts College, Affiliated to Bharathidasan University, Trichy. Email: [mohamedali1128@gmail.com](mailto:mohamedali1128@gmail.com)

© The Authors. Published by Blue Eyes Intelligence Engineering and Sciences Publication (BEIESP). This is an open access article under the CC-BY-NC-ND license (<http://creativecommons.org/licenses/by-nc-nd/4.0/>)

## Degree of a Vertex in Max-Product of Intuitionistic Fuzzy Graph

where  $S_{\mu_1}(G) = \sum_{xy \in E} \mu_1(xy)$  and  $S_{\mu_2}(G) = \sum_{xy \in E} \mu_2(xy)$ .

### Definition 2.6

The complement of an intuitionistic fuzzy graph  $G = (V, E)$  is an intuitionistic fuzzy graph  $\bar{G} = ((\bar{\sigma}_1, \bar{\sigma}_2), (\bar{\mu}_1, \bar{\mu}_2))$ , where  $(\bar{\sigma}_1, \bar{\sigma}_2) = (\sigma_1, \sigma_2)$  and

$$\bar{\mu}_1(xy) = \sigma_1(x) \wedge \sigma_2(y) - \mu_1(xy) \text{ and}$$

$$\bar{\mu}_2(xy) = \sigma_1(x) \vee \sigma_2(y) - \mu_2(xy), \forall xy \in E.$$

### Definition 2.7

Let  $G = ((\sigma_1, \sigma_2), (\mu_1, \mu_2))$  be an intuitionistic fuzzy graph.

The degree of a vertex  $x$  in  $G$  is denoted by  $d_G(x) = (d_1^G(x), d_2^G(x))$  and defined by

$$d_1^G(u) = \sum_{x \neq y} \mu_1^G(x, y) = \sum_{(x,y) \in E} \mu_1^G(x, y) \text{ and}$$

$$d_2^G(u) = \sum_{x \neq y} \mu_2^G(x, y) = \sum_{(x,y) \in E} \mu_2^G(x, y).$$

### Definition 2.8

Let  $G_1: ((\sigma_1^{G_1}, \sigma_2^{G_1}), (\mu_1^{G_1}, \mu_2^{G_1}))$  and  $G_2: ((\sigma_1^{G_2}, \sigma_2^{G_2}), (\mu_1^{G_2}, \mu_2^{G_2}))$  be two intuitionistic fuzzy graphs. The max-product of two intuitionistic fuzzy graph  $G_1$  and  $G_2$  is denoted by  $G_1 \times_m G_2 = (V_1 \times_m V_2, E_1 \times_m E_2)$ ,  $E_1 \times_m E_2 = \{(u_1, v_1)(u_1, v_2) / u_1 = u_2, v_1 v_2 \in E_2 \text{ or } v_1 = v_2, u_1 u_2 \in E_1\}$  by  $\sigma_1^{G_1 \times_m G_2}(u_1, v_1) = \sigma_1^{G_1}(u_1) \vee \sigma_1^{G_2}(v_1)$ ,  $\sigma_2^{G_1 \times_m G_2}(u_1, v_1) = \sigma_1^{G_1}(u_1) \wedge \sigma_1^{G_2}(v_1)$ , for all  $(u_1, v_1) \in V_1 \times V_2$  and

$$\mu_1^{G_1 \times_m G_2}((u_1, v_1)(u_2, v_2)) = \begin{cases} \sigma_1^{G_1}(u_1) \vee \mu_1^{G_2}(v_1 v_2) & \text{if } u_1 = u_2, v_1 v_2 \in E_2 \\ \mu_1^{G_1}(u_1 u_2) \vee \sigma_1^{G_2}(v_1) & \text{if } v_1 = v_2, u_1 u_2 \in E_1 \end{cases}$$

$$\mu_2^{G_1 \times_m G_2}((u_1, v_1)(u_2, v_2)) = \begin{cases} \sigma_2^{G_1}(u_1) \wedge \mu_2^{G_2}(v_1 v_2) & \text{if } u_1 = u_2, v_1 v_2 \in E_2 \\ \mu_2^{G_1}(u_1 u_2) \wedge \sigma_2^{G_2}(v_1) & \text{if } v_1 = v_2, u_1 u_2 \in E_1 \end{cases}$$

### Example 2.9

Let  $G_1^* = (V_1, E_1)$  and  $G_2^* = (V_2, E_2)$  be two crisp graphs such that  $V_1 = \{u_1, u_2, u_3\}, V_2 = \{v_1, v_2\}$ ,

$E_1 = \{u_1 u_3, u_2 u_3\}, E_2 = \{v_1 v_2\}$ . Consider two intuitionistic fuzzy graphs  $G_1 = ((\sigma_1^{G_1}, \sigma_2^{G_1}), (\mu_1^{G_1}, \mu_2^{G_1}))$  and  $G_2 = ((\sigma_1^{G_2}, \sigma_2^{G_2}), (\mu_1^{G_2}, \mu_2^{G_2}))$  and  $G_1 \times_m G_2$  as follows:

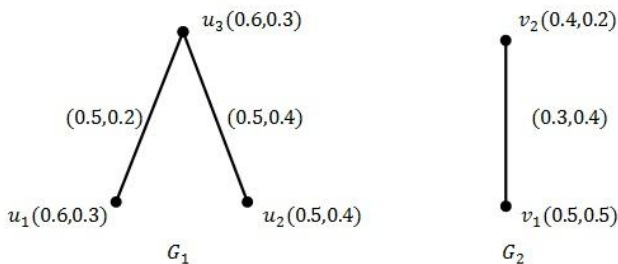


Figure - 1

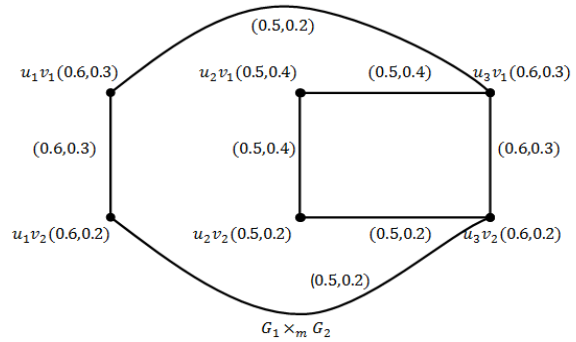


Figure - 2

## III. DEGREE OF A VERTEX IN MAX-PRODUCT OF INTUITIONISTIC FUZZY GRAPH

For any vertex  $(u_1, u_2) \in V_1 \times V_2$ . The degree of a vertex in  $G_1 \times_m G_2$  is defined as

$d^{G_1 \times_m G_2}(u_1, u_2) = (d_1^{G_1 \times_m G_2}(u_1, u_2), d_2^{G_1 \times_m G_2}(u_1, u_2))$ , where

$$d_1^{G_1 \times_m G_2}(u_1, u_2) = \sum_{(u_1, u_2)(v_1, v_2) \in E_1 \times E_2} \mu_1^{G_1 \times_m G_2}((u_1, u_2)(v_1, v_2))$$

$$= \sum_{u_1=v_1, u_2 v_2 \in E_2} \sigma_1^{G_1}(u_1) \vee \mu_1^{G_2}(u_2 v_2) + \sum_{u_2=v_2, u_1 v_1 \in E_1} \mu_1^{G_1}(u_1 v_1) \vee \sigma_1^{G_2}(u_2),$$

$$d_2^{G_1 \times_m G_2}(u_1, u_2) = \sum_{(u_1, u_2)(v_1, v_2) \in E_1 \times E_2} \mu_2^{G_1 \times_m G_2}((u_1, u_2)(v_1, v_2))$$

$$= \sum_{u_1=v_1, u_2 v_2 \in E_2} \sigma_2^{G_1}(u_1) \wedge \mu_2^{G_2}(u_2 v_2) + \sum_{u_2=v_2, u_1 v_1 \in E_1} \mu_2^{G_1}(u_1 v_1) \wedge \sigma_2^{G_2}(u_2).$$

## IV. REGULARITY CONDITION ON $G_1 \times_m G_2$

In this section, we derived the necessary and sufficient of conditions for the max product of two given intuitionistic fuzzy graph  $G_1 \times_m G_2$  to be regular.

### Theorem 4.1

Let  $G_1: ((\sigma_1^{G_1}, \sigma_2^{G_1}), (\mu_1^{G_1}, \mu_2^{G_1}))$  and  $G_2: ((\sigma_1^{G_2}, \sigma_2^{G_2}), (\mu_1^{G_2}, \mu_2^{G_2}))$  be two intuitionistic fuzzy graphs with underlying crisp graphs  $G_1^*$  and  $G_2^*$  are regular. If  $\sigma_1^{G_1} \leq \mu_1^{G_2}$ ,  $\sigma_2^{G_1} \geq \mu_2^{G_2}$ ;  $\sigma_1^{G_2} \geq \mu_1^{G_1}$ ,  $\sigma_2^{G_2} \leq \mu_2^{G_1}$  and  $\sigma_1^{G_1}, \sigma_2^{G_2}$  are constant functions, then  $G_1 \times_m G_2$  is regular if and only if  $G_2$  is regular intuitionistic fuzzy graph.

**Proof:**

Let  $G_1:((\sigma_1^{G_1}, \sigma_2^{G_1}), (\mu_1^{G_1}, \mu_2^{G_1}))$  and  $G_2:((\sigma_1^{G_2}, \sigma_2^{G_2}), (\mu_1^{G_2}, \mu_2^{G_2}))$  be two intuitionistic fuzzy graphs with underlying crisp graphs  $G_1^*$  and  $G_2^*$  are regular with  $d^{G_1^*} = r_1$ ;  $d^{G_2^*} = r_2$  and  $\sigma_1^{G_1} \leq \mu_1^{G_2}$ ,  $\sigma_2^{G_1} \geq \mu_2^{G_2}$ ;  $\sigma_1^{G_2} \geq \mu_1^{G_1}$ ,  $\sigma_2^{G_2} \leq \mu_2^{G_1}$  and  $\sigma_1^{G_2}, \sigma_2^{G_2}$  are constant functions of values  $k_1$  and  $k_2$  respectively.

Assume that  $G_2$  is  $(p, q)$  - regular intuitionistic fuzzy graph. Then, the degree of any vertex in max-product of intuitionistic fuzzy graph is given by

$$\begin{aligned} d^{G_1 \times_m G_2}(u_1, u_2) &= (d_1^{G_1 \times_m G_2}(u_1, u_2), d_2^{G_1 \times_m G_2}(u_1, u_2)), \text{ where} \\ d_1^{G_1 \times_m G_2}(u_1, u_2) &= \sum_{(u_1, u_2), (v_1, v_2) \in E_1 \times E_2} \mu_1^{G_1 \times G_2}((u_1, v_1)(u_2, v_2)) \\ &= \sum_{u_1=v_1, u_2=v_2 \in E_2} \sigma_1^{G_1}(u_1) \vee \mu_1^{G_2}(u_2 v_2) \\ &+ \sum_{u_2=v_2, u_1 v_1 \in E_1} \mu_1^{G_1}(u_1 v_1) \vee \sigma_1^{G_2}(u_2) \\ &= \sum_{u_1=v_1, u_2 v_2 \in E_2} \mu_1^{G_2}(u_2 v_2) + \sum_{u_2=v_2, u_1 v_1 \in E_1} \sigma_1^{G_2}(u_1) \\ &= d_1^{G_2}(u_2) + d^{G_1^*}(u_1) \sigma_1^{G_2}(u_1) \\ &= p + r_1 k_1 \end{aligned}$$

and

$$\begin{aligned} d_2^{G_1 \times_m G_2}(u_1, u_2) &= \sum_{(u_1, u_2), (v_1, v_2) \in E_1 \times E_2} \mu_2^{G_1 \times G_2}((u_1, v_1)(u_2, v_2)) \\ &= \sum_{u_1=v_1, u_2 v_2 \in E_2} \sigma_2^{G_1}(u_1) \wedge \mu_2^{G_2}(u_2 v_2) \\ &+ \sum_{u_2=v_2, u_1 v_1 \in E_1} \mu_2^{G_1}(u_1 v_1) \wedge \sigma_2^{G_2}(u_2) \\ &= \sum_{u_1=v_1, u_2 v_2 \in E_2} \mu_2^{G_2}(u_2 v_2) + \sum_{u_2=v_2, u_1 v_1 \in E_1} \sigma_2^{G_2}(u_1) \\ &= d_2^{G_2}(u_2) + d^{G_1^*}(u_1) \sigma_2^{G_2}(u_1) \\ &= q + r_1 k_2 \\ d^{G_1 \times G_2}(u_1, u_2) &= (p + r k_1, q + r k_2) \text{ is constant for all} \\ &\text{vertices } (u_1, u_2) \in V_1 \times V_2. \end{aligned}$$

Hence  $G_1 \times_m G_2$  is a regular intuitionistic fuzzy graph.

Conversely,

Let  $G_1 \times_m G_2$  is a regular intuitionistic fuzzy graph.

Then, for any two vertices  $(u_1, u_2)$  and  $(v_1, v_2)$  in  $V_1 \times V_2$ .

$$\begin{aligned} d_1^{G_1 \times_m G_2}(u_1, u_2) &= d_1^{G_1 \times_m G_2}(v_1, v_2) \\ d_1^{G_2}(u_2) + d^{G_1^*}(u_1) \sigma_1^{G_2}(u_1) &= d_1^{G_2}(v_2) + d^{G_1^*}(v_1) \sigma_1^{G_2}(v_1) \\ r_1 k_1 + d_1^{G_2}(u_2) &= r_1 k_1 + d_1^{G_2}(v_2) \\ d_1^{G_2}(u_2) &= d_1^{G_2}(v_2) \end{aligned}$$

Similarly,

$$d_2^{G_1 \times_m G_2}(u_1, u_2) = d_2^{G_1 \times_m G_2}(v_1, v_2)$$

$$\begin{aligned} d_2^{G_2}(u_2) + d^{G_2^*}(u_1) \sigma_2^{G_2}(u_1) &= d_2^{G_2}(v_2) + d^{G_2^*}(v_1) \sigma_2^{G_2}(v_1) \\ r_1 k_2 + d_2^{G_2}(u_2) &= r_1 k_2 + d_2^{G_2}(v_2) \\ d_2^{G_2}(u_2) &= d_2^{G_2}(v_2) \end{aligned}$$

This condition is true for all vertices  $(u_1, u_2) \in V_1 \times V_2$ .

Hence  $G_2$  is a regular intuitionistic fuzzy graph.

**Theorem 4.2**

Let  $G_1:((\sigma_1^{G_1}, \sigma_2^{G_1}), (\mu_1^{G_1}, \mu_2^{G_1}))$  and  $G_2:((\sigma_1^{G_2}, \sigma_2^{G_2}), (\mu_1^{G_2}, \mu_2^{G_2}))$  be two intuitionistic fuzzy graphs with underlying crisp graphs  $G_1^*$  and  $G_2^*$  are regular. If  $\sigma_1^{G_1} \leq \mu_1^{G_2}$ ,  $\sigma_2^{G_1} \geq \mu_2^{G_2}$ ;  $\sigma_1^{G_2} \geq \mu_1^{G_1}$ ,  $\sigma_2^{G_2} \leq \mu_2^{G_1}$  and  $\sigma_1^{G_2}, \sigma_2^{G_2}$  are constant functions, then  $G_1 \times_m G_2$  is regular if and only if  $\sigma_1^{G_1}$  and  $\sigma_2^{G_1}$  are constant functions.

**Proof:**

Let  $G_1:((\sigma_1^{G_1}, \sigma_2^{G_1}), (\mu_1^{G_1}, \mu_2^{G_1}))$  and  $G_2:((\sigma_1^{G_2}, \sigma_2^{G_2}), (\mu_1^{G_2}, \mu_2^{G_2}))$  be two intuitionistic fuzzy graphs with underlying crisp graphs  $G_1^*$  and  $G_2^*$  are  $r$ -regular and  $\sigma_1^{G_1} \leq \mu_1^{G_2}$ ,  $\sigma_2^{G_1} \geq \mu_2^{G_2}$ ;  $\sigma_1^{G_2} \geq \mu_1^{G_1}$ ,  $\sigma_2^{G_2} \leq \mu_2^{G_1}$  and  $\sigma_1^{G_2}, \sigma_2^{G_2}$  are constant functions of values  $k_1$  and  $k_2$  respectively.

Now assume that  $\sigma_1^{G_1}$  and  $\sigma_2^{G_1}$  are constant functions of values  $k_3$  and  $k_4$  respectively.

Then the degree of any vertex in max-product of two intuitionistic fuzzy graphs is given by

$$\begin{aligned} d_1^{G_1 \times G_2}(u_1, u_2) &= \sum_{(u_1, u_2), (v_1, v_2) \in E_1 \times E_2} \mu_1^{G_1 \times G_2}((u_1, v_1)(u_2, v_2)) \\ &= \sum_{u_1=v_1, u_2 v_2 \in E_2} \sigma_1^{G_1}(u_1) \vee \mu_1^{G_2}(u_2 v_2) \\ &+ \sum_{u_2=v_2, u_1 v_1 \in E_1} \mu_1^{G_1}(u_1 v_1) \vee \sigma_1^{G_2}(u_2) \\ &= \sum_{u_1=v_1, u_2 v_2 \in E_2} \sigma_1^{G_1}(u_1) + \sum_{u_2=v_2, u_1 v_1 \in E_1} \sigma_1^{G_2}(u_1) \\ &= d^{G_2^*}(u_2) \sigma_1^{G_1}(u_1) + d^{G_1^*}(u_1) \sigma_1^{G_2}(u_1) \\ &= r_2 k_3 + r_1 k_1 \end{aligned}$$

$$\begin{aligned} d_2^{G_1 \times G_2}(u_1, u_2) &= \sum_{(u_1, u_2), (v_1, v_2) \in E_1 \times E_2} \mu_2^{G_1 \times G_2}((u_1, v_1)(u_2, v_2)) \\ &= \sum_{u_1=v_1, u_2 v_2 \in E_2} \sigma_2^{G_1}(u_1) \wedge \mu_2^{G_2}(u_2 v_2) \\ &+ \sum_{u_2=v_2, u_1 v_1 \in E_1} \mu_2^{G_1}(u_1 v_1) \wedge \sigma_2^{G_2}(u_2) \end{aligned}$$



## Degree of a Vertex in Max-Product of Intuitionistic Fuzzy Graph

$$\begin{aligned}
 &= \sum_{u_1=v_1, u_2, v_2 \in E_2} \sigma_2^{G_1}(u_1) + \sum_{u_2=v_2, u_1, v_1 \in E_1} \sigma_2^{G_2}(u_1) \\
 &= d^{G_2^*}(u_2) \sigma_2^{G_1}(u_1) + d^{G_1^*}(u_1) \sigma_2^{G_2}(u_1) \\
 &= rk_4 + rk_2
 \end{aligned}$$

This is true for all vertices in  $V_1 \times V_2$ . Hence  $G_1 \times_m G_2$  is regular intuitionistic fuzzy graph.

Conversely, assume that  $G_1 \times_m G_2$  is a regular intuitionistic fuzzy graph. Then for any two vertices  $(u_1, u_2)$  and  $(v_1, v_2)$  in  $V_1 \times V_2$ .

$$\begin{aligned}
 d_1^{G_1 \times_m G_2}(u_1, u_2) &= d_1^{G_1 \times_m G_2}(v_1, v_2) \\
 d^{G_2^*}(u_2) \sigma_1^{G_1}(u_1) + d^{G_1^*}(u_1) \sigma_1^{G_2}(u_1) \\
 &= d^{G_2^*}(v_2) \sigma_1^{G_1}(v_1) + d^{G_1^*}(v_1) \sigma_1^{G_2}(v_1) \\
 r_2 \sigma_1^{G_1}(u_1) + r_1 k_1 &= r_2 \sigma_1^{G_1}(v_1) + r_1 k_1 \\
 r_2 \sigma_1^{G_1}(u_1) &= r_2 \sigma_1^{G_1}(v_1) \\
 \sigma_1^{G_1}(u_1) &= \sigma_1^{G_1}(v_1)
 \end{aligned}$$

Similarly,

$$\begin{aligned}
 d_2^{G_1 \times_m G_2}(u_1, u_2) &= d_2^{G_1 \times_m G_2}(v_1, v_2) \\
 d^{G_2^*}(u_2) \sigma_2^{G_1}(u_1) + d^{G_1^*}(u_1) \sigma_2^{G_2}(u_1) \\
 &= d^{G_2^*}(v_2) \sigma_2^{G_1}(v_1) + d^{G_1^*}(v_1) \sigma_2^{G_2}(v_1) \\
 r_2 \sigma_2^{G_1}(u_1) + r_1 k_2 &= r_2 \sigma_2^{G_1}(v_1) + r_1 k_2 \\
 \sigma_2^{G_1}(u_1) &= \sigma_2^{G_1}(v_1)
 \end{aligned}$$

This is true for all vertices in  $V_1$ . Hence  $\sigma_1^{G_1}$  and  $\sigma_2^{G_2}$  are constant functions.

### V. CONCLUSION

Intuitionistic fuzzy graphs play an important role in uncertainty quantification in networking, route mapping, shortest path problem, etc... Intuitionistic fuzzy model are more realistic compared to other fuzzy and interval-valued fuzzy models. In this paper, the degree of a vertex in max-product of two given intuitionistic fuzzy graph is studied. The necessary and sufficient condition for the max-product of two given intuitionistic fuzzy graph to be regular are stated and proved. In future, we will extend these results in some new operations on intuitionistic fuzzy graph and interval-valued intuitionistic fuzzy graph.

### ACKNOWLEDGMENT

The authors extend their sincere thanks to the anonymous reviewers and Editor-in-Chief for their valuable comments and suggestion to improve the quality of the research paper.

### REFERENCES

1. Akram M and Davvaz B, Strong intuitionistic fuzzy graphs, Filomat, 26(1) (2012) 177- 196
2. Atanassov K. T, Intuitionistic fuzzy sets, Fuzzy Sets and Systems 20(1) (1986) 87-96.
3. Atanassov K.T, Intuitionistic Fuzzy Sets: Theory and Applications, Studies in Fuzziness and Soft Computing, Physica-Verlag GmbH Heidelberg, Germany, 1999.
4. Parvathi R and Karunambigai M.G, Intuitionistic fuzzy graphs, Comput. Intell., International Conference in Germany, Sept (2006) 18-20.
5. Shannon and K. Atanassov, A first step to a theory of the intuitionistic fuzzy graphs, Proceedings of the First Workshop on Fuzzy Based Expert Systems, D. Lakov.ed., Sofia, Sept 28-30 (1994) 59-61.

6. Yahya Mohamed S, Mohamed Ali A, Modular product on intuitionistic fuzzy graphs, International Journal of Innovative Research in Science, Engineering and Technology, Vol. 6, Issue 9, 19258-19263 September (2017).
7. Yahya Mohamed S, Mohamed Ali A., Max-product on intuitionistic fuzzy graph", Proceeding of 1<sup>st</sup> International Conference on Collaborative Research in Mathematical Sciences (ICCRMS'17), Page No, 181-185, September (2017).
8. Yahya Mohamed S, Mohamed Ali A, Degree of a Vertex in Complement of Modular Product of Intuitionistic Fuzzy Graphs, Journal of Physical Sciences, Vol.24, 2019, 115-124.
9. Yahya Mohamed S, Mohamed Ali A, Interval-Valued Pythagorean Fuzzy Graph", Journal of Computer and Mathematical Sciences, Vol.9 (10), 2018, pp.1497-1511.
10. Yahya Mohamed S, Mohamed Ali A, Intuitionistic Fuzzy Graph Metric Space, International Journal of Pure and Applied Mathematics, Vol.118, No.6, 2018, pp.67-74.
11. Yahya Mohamed S, Mohamed Ali A, Fixed Point Theorems In Intuitionistic Fuzzy Graph Metric Space, Annals of Pure and Applied Mathematics Vol. 17, No. 1, 2018, pp.57-65.
12. Yahya Mohamed S, Mohamed Ali A, On Strong Interval-Valued Pythagorean Fuzzy Graph, Journal of Applied Science and Computations, Vol.5, Issue 10, 2018, pp.699-713.
13. L.A. Zadeh, Fuzzy sets, Information and Control 8 (1965), 338-353.

### AUTHORS PROFILE



**Dr. S. Yahya Mohamed** is working as Assistant Professor in P .G. and Research Department of Mathematics, Government Arts College, Trichy-22, Tamil nadu, India, since the year 2009. He has more than 25 years teaching experience at collegiate level. His research areas are fuzzy Mathematics, Intuitionistic fuzzy graphs, Fuzzy Algebras and Fuzzy Topology. He has published nearly 50 Research papers in the International journals.



**A. Mohamed Ali** is working as Assistant Professor in Department of Mathematics, G.T.N. Arts College (Autonomous), Dindigul- 05, Tamilnadu, India, since the year 2016. He has published nearly 20 research papers in national and international journals.