Suction Injection Induced MHD Flow through Vertical Narrow Porous Channel with Permeable Properties

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Abstract: The current research problem deals with fluid flows that are electrically conducting known as Magnetohydrodynamic (MHD) flow, viscous oscillatory and stratified fluid in a vertical long small geometry rectangular channel that has permeable property with one side being porous and the other side being nonporous. Corresponding fluid flow equations are simplified and hence solved by applying Lubrication approximation by using similarity transformation. The interpretations of the influences of various quantities that are involved to the problem on velocity profiles, pressure and density distributions are explained in detail. The results of the research problem shows that the Magnetohydrodynamic parameter encourages backflow nearer to the boundaries of the channel while permeability parameter influences the flow differently for axial and transverse velocity profiles. The results for \( \kappa = 0 \) reduces to the results that are already available in the literature.

Index Terms: Magnetohydrodynamics, Oscillatory Viscous Flow, Stratification Parameter, Similarity Transformation, Permeability Parameter, Narrow Channel Approximation.

I. INTRODUCTION

Fluids with density or viscosity not being constant are known as stratified flow. In reality the stratification often depends significantly only on density. Such density stratification are mostly affected by gravity phenomena. The stratified property of sea water and similar nature of air throws light on research of density stratified flow of fluids. The flow of electrically conducting MHD fluids between parallel porous plates referred as the Hartmann flow. There are numerous applications of plane MHD oscillatory stratified flow in narrow channels with one permeable boundary in many areas of Engineering such as transmission parts, bearing, Cams and followers, seal faces, any situations involving metal and metal contacts and Lubrication approximation problems. Hartmann [7] examined the characteristics of perpendicular magnetic field on the fluid flow with conducting properties through two parallel infinite insulated plates. By obtaining perturbation solution Berman[3] was the first to report on the steady flow of an non compressible stokes fluid passing into a permeable channel with rectangular cross-section, with a low Reynolds number with normal wall velocities being equal. Further studies on the same problem was reported by Sellars [19] with a high Reynolds number, Tertil [20] with many values of Reynolds numbers for suction and injection and Tertil & Shrestha [21] assuming different velocities normal to the walls.

Variable separable similarity solution to the Navier-Stokes equation was reported by Rajagopal [18]. On the similar lines the unsteady magnetohydrodynamic flow along a porous medium between two parallel infinite plates by Hassanien & Mansour [8] and the two-dimensional viscous flow of fluid in a channel that contains porous walls by Cox[5] were also discussed. The effects of MHD flow through parallel plates with viscoelastic effects and heat transfer was reported by Attia H.A [1,2]. There are many reports on Forced Oscillation in an inviscid stratified fluid [4, 9, 12,13] but only few reports were recorded on the oscillations in stratified viscous fluid. Small geometry parameter flow was reported by Panton [14] where the fluid flow geometry is small perpendicular to the direction of the transport with time independence and a suitable Poiseuilli flow as velocity at the entering point. By considering the withdrawal velocity through porous wall as function of \( x \) alone, an exact solution was derived using narrow channel approximation. Magnetohydrodynamic stokes flow through narrow rectangular channel was discussed by Prasanna Venkatesh and Surya Prabha [17] by presenting analytical solution using similarity transformation. Narrow channel approximation solution to flows of multidirectional coating with surfactant and clean interfaces was obtained by R. Krehchevtikov[11]. Gupta and Goyal [6] discussed the stratified MHD flow in porous medium. Unsteady viscosity stratified MHD fluid flow in medium which is porous in slip flow regime over moving plate was reported by Khandelwal and Jain [10]. A class of solution was derived by assuming the pressure as a prescribed function of spatial variables and a variable quantity with respect to time. OM Prakash, Kumar and Dwivedi [15] discussed MHD dusty viscoelastic stratified fluid flow with heat transfer in porous medium with variable viscosity by presenting the analytical results to velocity function by taking them as dependent on \( y \) and \( t \) only in flow direction and in the direction perpendicular to it as a function of \( y \) and \( t \) only. The current discussion focuses on fluid flow that is MHD viscous oscillatory and having density variations in vertical direction flowing through a small geometry permeable channel having one of its wall as porous, the fluid is removed with a velocity which is constant through the porous wall. The related equations governing the motion of the fluid are manipulated and then transformed into a total differential equations by using variable separable substitution. Pictorially the velocity, pressure and density distributions are interpreted.
For \( B_0 = 0, k=0 \& N = 0 \) and a prescribed initial velocity the problem is well supported by already existing literature.

II. PROBLEM FORMULATION AND SOLUTION

The research mainly focuses on the flow where fluid is removed though suction along a porous side plate in a permeable channel of width \( h_0 \). The coordinate system origin is assumed to be placed at the lower end. The non porous plate is positioned on Y-axis and the pervious plate is considered to be placed at a distance of \( h_0 \) from y-axis. Through the porous side, the fluid is separated with a non varying removal velocity \( u_1 \) at a length of \( L \). There is no flow prior to this end as the channel boundaries are concrete ahead to this end and the movement of fluid is considered to be fully started. If the flow geometry is so narrow such that the ratio \( h_0/L \) approaches to nullity then the terms of resistance to velocity in the associated equations of fluid flow could be considered to vanish with a small withdrawal velocity in comparison with mean velocity along the length of the channel. The fluid taken here is assumed to be electrically non insulating and also to be linearly distributed density stratified in the direction of gravity. The motionless state of density is considered to be related with height linearly and in the disturbed condition, it is taken to be dependent of spatial and time variables. The associated equations governing the problem are as follows.

\[
\frac{\partial \rho}{\partial t} + \rho \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) = 0
\]

(1)

\[
\frac{dp}{dt} + \rho \frac{\partial u}{\partial x} + u \frac{\partial \rho}{\partial x} + v \frac{\partial \rho}{\partial y} = 0
\]

(2)

Equation of Motion

\[
\rho \left( \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = -\frac{\partial p}{\partial x} + \mu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) - \frac{\mu}{\kappa} u
\]

(3)

\[
\rho \left( \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right) = -\frac{\partial p}{\partial y} + \mu \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) - \sigma_e B_0^2 \varepsilon - \frac{\mu}{\kappa} \varepsilon - \rho g
\]

(4)

\[
\rho \frac{\partial \psi}{\partial t} = -\frac{\partial p}{\partial x} + \mu \left( \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} \right) - \frac{\mu}{\kappa} \varepsilon - \rho g
\]

(5)

\[
\rho_0 (y) = \rho_0^1 \left( 1 - \beta y \right)
\]

(6)

\[
\rho_0 (y) = \rho_0^1 \left( 1 - \beta y \right)
\]

(7)

For Stokes’ flow (3) and (4) becomes

\[
\rho \frac{\partial u}{\partial t} = -\frac{\partial p}{\partial x} + \mu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) - \frac{\mu}{\kappa} u
\]

(8)

\[
\rho \frac{\partial v}{\partial t} = -\frac{\partial p}{\partial y} + \mu \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) - \sigma_e B_0^2 \varepsilon - \frac{\mu}{\kappa} \varepsilon - \rho g
\]

(9)

\[
X = x/h_0, \; Y = y/L, \; U = Lu/h_0v_0, \; V = v/v_0, \; P = (p - p_{ref})/h_0^2/L \mu u_0, \; Re = h_0v_0/v
\]

(10)

As \( \frac{h_0}{L} \to 0 \)

\[
\rho_0 \frac{\partial U}{\partial t} = -\frac{\partial P}{\partial X} - \frac{\mu}{\kappa} U
\]

(11)

\[
\rho_0 \frac{\partial V}{\partial t} = -\frac{\partial P}{\partial Y} + \mu \frac{\partial^2 V}{\partial X^2} - \left( \sigma_e B_0^2 + \frac{\mu}{\kappa} \right) V - \rho g
\]

(12)

Simplifying the above set of equations, we get

\[
\rho_0 \frac{\partial}{\partial t} \left( \frac{\partial U}{\partial Y} - \frac{\partial V}{\partial X} \right) = -\mu \frac{\partial^3 V}{\partial X^3} + \sigma_e B_0^2 \frac{\partial V}{\partial X} + \rho_0 N^2 \frac{\partial V}{\partial X}
\]

(13)

\[
U = U(X,Y) e^{i \omega t}, \; V = V(X,Y) e^{i \omega t}, \; \text{and} \; P = P(X,Y) e^{i \omega t}
\]

(14)

\[
\frac{\partial \psi}{\partial Y} = 0, \; U(0, Y) = 0, \; U(1, Y) = u_1
\]

(15)

\[
\frac{\partial \psi}{\partial X} = 0, \; V(0, Y) = 0, \; V(1, Y) = 0
\]

(16)

\[
\Psi = (v_0 - u_1 Y)f(X)
\]

(17)
The impact of various quantities on velocity in u and v direction, Density function and Pressure drop is discussed here. The spatial variables x and y is measured to be in the range of 0 to 1 while that in each of others are considered varying. Figure 1 explains the relation with velocity in flow direction for various time parameter of (0 to π) which helps us to observe that there is a decrease in values of flow direction velocity with increase in values of time parameter. Figure 2 explains about the velocity in transverse direction when the time parameter is zero and for different depth values varying from 0 to 0.6. It is noted from the above mentioned graphical representations that vertical velocity has opposite effects at centre compared with that of the boundaries. Figure 3, 4 and 5 show the influence of $B_0$ (0 to 50), N (5 to 25) and $\alpha$ (0.2 to 1) on velocity in the transverse direction. The solutions interpretation confirms that velocity reduces with an increase in electromagnetic induction and density parameter but speeds up with permeability around the centre of the channel. Figure 6, 7, 8 and 9 explains the effects in velocity profiles perpendicular to flow direction with reference to time (0 to π), density parameter (0 to 5), electromagnetic induction (0 to 10) and permeability parameter (0.5 to 2.5) respectively. It is very clear from the graphs that velocity normal to flow direction increases with time, symmetric about point placed at the center point of the channel for varied height y, $B_0$ and N values. The influences on pressure drop due to various quantities, time (0 to π), electromagnetic induction (0 to 1) and Permeability parameter (1 to 5) on pressure drop are depicted in Figures 10, 11 and 12 respectively. It is very clear from the graphical representations that pressure drop decreases as time increases and it increases as height and electromagnetic induction values increases. On the other hand there is no clear relationship with respect to the Permeability parameter $\alpha$. Density distribution for different time (0 to π) and height (0 to 1) are explained by Figure 13 and 14. Clear indications are there from figure 14 that about central axis density distribution is symmetric. The problems coincide with that of viscous oscillatory flow through narrow channel for the values N=0 and $B_0=0$ with a known entering velocity.

**III. RESULTS AND DISCUSSIONS**

Pressure drop is as follows

$$P(x, y) = \frac{\rho_0}{\omega} \left[ (1 - \beta y) + \frac{\rho_0}{\omega} \left( \frac{1 - e^{i\alpha \omega}}{i} \right) \right](x, y)$$  \hspace{1cm} (31)

**IV. CONCLUSIONS**

A particular type of variable separable exact solution to the fluid flow through vertically permeable small geometry channel flow with a permeable wall having the property of MHD, density stratification, oscillation and Stokes flow is analyzed in this research work. Small geometry channel approximation is applied in the equation governing the fluid flow and the system of partial differential equations are converted to ordinary differential equation by using similarity variable separable transformation. The following observations are obtained. As time increases, transverse velocity decreases and vice versa. Mixed effects are noted in case of height (y). Reverse flows are observed at the centre of the channel due to stratification while similar effects are observed nearer to the plates due to electromagnetic induction and permeability parameter $\alpha$. Symmetric nature of velocity perpendicular to flow direction are witnessed about the middle point of the small geometry channel because of the variations in electromagnetic induction.
permeability parameter and stratification. For higher values of electromagnetic induction and for stratification parameter N, Pressure drop is non linear. Decrement in density distribution due to increase in time were shown from the figures and density distributions are symmetric about a vertical line for different values of height.

REFERENCES
Fig. 5 Vertical velocity for $\kappa = 2$ to 10

Fig. 6 Horizontal velocity for $\omega t = 0$ to $\pi$

Fig. 7 Horizontal velocity for $N = 0$ to 5

Fig. 8 Horizontal velocity for $B_0 = 2$ to 10

Fig. 9 Horizontal velocity for $\kappa = 0.5$ to 2.5

Fig. 10 Pressure Distribution for $\omega t = 0$ to $\pi$

Fig. 11 Pressure Distribution for $B_0 = 0.2$ to 1.0

Fig. 12 Pressure Distribution for $\kappa = 1$ to 5
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Fig. 13 Density Distribution for $\omega t = 0$ to $\pi$

Fig. 14 Density Distribution for $y = 0$ to 1

AUTHORS PROFILE

Mr. L. Prasanna Venkatesh, born in 1979 graduated in Mathematics from A.M.Jain College, Meenambakkam, Chennai-600 027, affiliated to University of Madras in the year 1999. He completed his post graduation and Master of Philosophy from the same college in the years 2001 and 2003 respectively. He has completed his Ph.D in the year 2016. Starting his teaching career from 2003 onwards, he is having a teaching experience 12 years at both undergraduate & post graduate level. He also trains students for various competitive examinations at school level and college level. He is at present working as Assistant Professor, Department of Mathematics, Sathyabama Institute of Science and Technology. Having research interest in the field of Fluid dynamics and Graph theory, he has published many research articles and oral presentations inclusive of both international and national conferences. His core research area is in Stratified flows. Apart from his academic interests, he is proficient in Hatha Yoga and Raja Yoga and is associated with Sathyananda Yoga Center, West Mambalam, a branch of Bihar School of Yoga, Munger. He is also a FIDE rated chess player with a rating of 1286.