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Implication of Production and Monitoring Techniques in Bayesian Single Sampling Plan using Gamma-Zero Inflated Poisson Distribution

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Abstract*: This article explores the problem of investigate Single Sampling Plan (SSP) by attributes under Bayesian theory and illuminate its importance methodology in manufacturing industries. The modern technological advancements and well monitoring of the production process are facilitate to enhance the standard of product. In such situation products are not meeting the specified quality standards is a rare phenomenon. However, random fluctuations in producing processes might lead some merchandise to an imperfect quality. It has been assumed that the number of defects per unit of product follows a Zero Inflated Poisson distribution (ZIP) and the Gamma distribution is the conjugate prior to the average number of non-conformities per item. This article proposed a new sampling procedure as Bayesian Single Sampling plan (BSSP) using Gamma-Zero Inflated Poisson (G-ZIP) distribution. Necessary tables for the selection of optimal plan parameters and numerical illustrations were made for this sampling plan. Furthermore, the applicability and usefulness of the proposed Bayesian sampling plan under the G-ZIP model have been demonstrated by a few examples and comparisons were made with other sampling plans.

Keywords: Single Sampling Plan by attribute, Bayesian methodology, Gamma prior, Zero Inflated Poisson distribution (ZIP), producer and Consumer risk.

I. INTRODUCTION

Acceptance sampling is the most popularized quality control methodology in the field of Statistical Quality Control (SQC). An important aspects of Acceptance sampling plan is to make a decision either to accept or reject the concerned lots of products dependent on the quality characters determined in the sampling inspection technique. It is also helpful in circumstance where testing is destructive, the expense of 100% inspection is incredibly high, the time taken would be excessively long, the inspection error rate is too high and the product liability risks are serious. Lot by lot sampling inspection by Attribute plan is one of the vital areas of acceptance sampling. In this plan, different types of sampling plans can be studied as Single Sampling Plan (SSP), Double Sampling Plan (DSP), Multiple Sampling Plan, etc. A single sampling attribute plan is a procedure by which a single sample is drawn from a lot and inspected it. The lot is accepted if the number of nonconforming units found in the sample is less than or equal to the acceptance number, or a specified limit, otherwise the lot is rejected. Peach and Littauer [12] proposed to determine SSP under the Poisson distribution and constructed tables of plan parameters based on iterative procedure. Guenther [6] proposed an iterative procedure to determine the SSP by attributes using binomial, hypergeometric and Poisson distributions for specified values of the plan parameters. A detailed discussion on the studies relating to designing of such plans could be found from Hald [7], Duncan [5] and Stephens [16]. Further, Schilling and Neubauer [14] provided details about the applications of unity value approach to determine the sampling plans based on Poisson distribution. In acceptance sampling, the sampling plans are designed under an assumption that error occur may not in sampling inspection, however this assumption is not much potential systematically. That is, the production processes are not continuously stable and also the incoming lots from such processes because of random fluctuation could be occur quality variations. Generally, two types of quality variations perhaps occur in sampling inspection like within-lot variation and between-lot variation. When between-lot variation is will occur more than within-lot variation, the proportion of nonconforming units in the lots will vary frequently. In such circumstances, many studies were made on designing a sampling plan under Bayesian methodology. Additionally, in this methodology can be used when the quality engineer has the prior knowledge on the production process to decide the disposition of the lot. Further, the concepts and models of Bayesian sampling plan and selection of prior distribution for lot fraction nonconforming has been made more details in the literature that include Hald [7], Case and Keats [3] and Calvin [2]. Pandey [13] discussed a Bayesian Single Sampling plan by attributes with three decision criteria for discrete prior distribution. Suresh et.al. [17] investigated the Bayesian Single Sampling Plans for a Gamma Prior distribution. Vijayaraghavan et.al. [18] have analyzed Bayesian Single Sampling Plan using Gamma Poisson distribution and described a method to study the efficiency of their sampling plan compared to Conventional Poisson Single Sampling Plan. Kaviyarasu et.al. [88] designed tables and operating characteristic curve for the selection of parameters of Special Type Double Sampling plan by attributes based on ZIP distribution.

II. STATMENT ABOUT THE PROBLEM

Every producing firms follow strict procedures at each stage of production for developing their own products quality.

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Most of the producing firms are using technological advancement and computer modeling or simulations to detect product defects and handle them within the initial stages of the production. Using such technologies can reduce defective Products early.

However, due to random fluctuations and some inevitable reasons in the manufacturing process, there may lead a few defective products within a batch of perfectly functioning production products. In such a situation, one can perform Zero Inflated Poisson distribution and in these distribution has been applied in many fields.

For example, Lambert [9] reported the application to defects in manufacturing process with Zero Inflated Poisson (ZIP) model. Further Bohning et.al. [1] discussed ZIP distribution is performed better than the Poisson distribution in dental epidemiology research to measure the dental health of individuals. Some of another applications can be find in Sim and Lim [15] and Mussida et.al. [11]. Recently, in acceptance sampling, Loganathan et.al. [10] developed extensive tables and operating procedure for a single sampling attribute plan using the Zero Inflated Poisson Distribution (ZIP). Designing of sampling plans under the conditions of ZIP is desperately important when the occurrence of defects would be a rare event in sampling inspection and additionally once the prior knowledge offered for production process then the Bayesian methodology is more appropriate. In such a way, this article presents Single Sampling plan (SSP) under the conditions of Gamma-Zero Inflated Poisson (G-ZIP) distribution under the Bayesian perspective.

III. AN OVERVIEW OF ZERO INFLATED POISSON DISTIRBUTION

The Zero-Inflated Poisson (ZIP) distribution is employed for count data that exhibit over dispersion and excess zeros. In such circumstance, this model is assumed to be a mixture distribution that includes proportion of extra zeros and a proportion from the Poisson models according to Lambert [99]. Let x be a non-negative integer random variable and the probability mass function of this model is given below,

According to the Lambert [9] the probability distribution of a zero Inflated Poisson random variable X is given by,

$$P(X = x/\omega, \lambda) = \begin{cases} \omega + (1-\omega)e^{-\lambda}, \ when \ \ x = 0 \\ (1-\omega)\frac{e^{-\lambda}\lambda^{x}}{x!}, \ when \ x = 1,2,3\ldots, \end{cases}$$

$$= \omega P_0(x) + (1-\omega)P_1(x), \qquad 0 < \omega < 1$$

$$where, P_0(x) = \begin{cases} 1, & x = 0 \\ 0 & x \neq 0 \end{cases} \text{ and}$$

$$P_0(x) = \frac{e^{-\lambda}\lambda^{x}}{x!}, \qquad x = 0,1,2,\ldots, \ \lambda > 0$$

Here, ω is the weighting parameter and λ is the Poisson parameter. When $\omega = 0$, this model is reduced to the Poisson model. It is termed as the mixing proportion of degenerate of zeros with Poisson model. Furthermore, the mean and variance of the ZIP distribution are established as,

$$\mu = \mathrm{E}(x) = (1-\omega)\lambda$$

$$\sigma^2 = Var(x) = \lambda(1-\omega)(1+\omega\lambda).$$

Moreover, the properties and inference, including maximum likelihood estimation of ω and λ can be obtained in Gupta et al. (1996).

IV. THE DESIGNING OF SSP

The design parameters of a single sampling plan can be implemented and a decision is obtained either to accept or reject a lot based on sampling inspection. This plan requires the specification of three parameters such as Lot size (N), size of the sample (n) and acceptance number for the sample (c). The procedure for implementing the SSP to arrive at a decision about the lot is described in the following steps,

i. Draw a random sample of size (n) from the lot size (N) received from the supplier.
ii. Inspect all units in the sample and count the number of defective units (d).
iii. Compare the number of defective units (d) with the stated acceptance number (c); If $d \leq c$ accept the lot; otherwise, reject the lot.

The operating characteristics (OC) function is an important aspect of an acceptance sampling plan. This function can be determine the discriminatory power of the sampling plan. That is, it may be determine the probability that a lot submitted with an exact fraction defective will be either accept or reject.

According to Loganathan et al [109] the Operating Characteristic function of the SSP under the ZIP distribution is as follows,

$$P_a(p) = [X \leq c]$$

Where, p denotes the lot fraction nonconforming.

$$P_a(p) = \omega + (1-\omega)e^{-\lambda} + \sum_{x=1}^{c}(1-\omega)\frac{e^{-\lambda}(\lambda)^{x}}{x!}$$

Where, $0 < \omega < 1$ and $\lambda > 0$. Here, ω is indicates inflation of zero defects in the sampling inspection and $\lambda = np$.

A. The Bayesian SSP under the condition of Gamma Zero Inflated Poisson distribution

Several works were developed in designing the Bayesian SSP approach for various situations. In production process, the quality variations are must be stable for inspection product units but these conditions are practically not always achieved. That is, the production processes are not constantly stable. In such situations, the Bayesian methodology is more appropriate model to study the sampling plan by attributes. When the number of nonconformities items in the sample is followed by the model of Zero Inflated Poisson distribution with parameter (λ, ω), which varies from a lot to lot, the gamma distribution is assumed to be the conjugate prior to λ and it will be consider as a random variable. Further, the Beta distribution is assumed to be the conjugate prior to ω with parameters a and b.

The probability density function of the independent prior distributions of (λ, ω) is defined as,

$$\omega \sim Beta(a,b) \text{ and } \lambda \sim \mathrm{Gamma}(t,s)$$

$$f(\omega/a,b) = \frac{1}{B(a,b)}\omega^{a-1}(1-\omega)^{b-1}, \qquad a,b > 0$$

In particular, $a = b = 1$ gives the uniform prior on $(0,1)$ and the gamma prior distribution is,

$$f(p/t,s) = \frac{e^{-tp} t^s p^{s-1}}{\Gamma s}, \qquad 0 \le p < \infty, \qquad t,s > 0$$

where $t = s/\mu$, with $E(p) = \bar{p} = \mu$ and t is scale parameter and s is the shape parameter. Here, s is denoted by the prior knowledge and it is estimated from past history of the production process.

Let x be the number of nonconforming units in the sample, and $\bar{p}$ the process average fraction nonconforming prior to sampling are independently distributed. According to Hald [7] the simplest family of prior distributions is studied for the number of nonconforming units x, using Zero Inflated Poisson distribution is given by,

$$P_a(\bar{p}) = \int_0^\infty P_a(p)\, f(p)\, dp$$

So, the posterior joint distribution under the Gamma- ZIP distribution can be given as,

$$P(x;\omega,n\bar{p},s) = \begin{cases} \omega + (1-\omega)(1-\rho)^s, & when\ x = 0 \\ (1-\omega)\binom{x+s-1}{s-1}\rho^x(1-\rho)^s, & when\ x = 1,2,\dots, \end{cases}$$

Where, for convenience $\rho = \left(\frac{n\bar{p}}{n\bar{p}+s}\right)$. $\bar{p}$ is the process average fraction nonconforming. The OC function for the sampling distribution of x under the conditions of gamma prior distribution for λ with shape parameter s and Zero Inflated Poisson sampling distribution for x is given by,

$$P_a(\bar{p}) = \sum_{x=0}^{c} p(x;n\bar{p},\omega,s)$$

The expression of the OC function under the Gamma- ZIP distribution given as,

$$P_a(\bar{p}) = \omega + (1-\omega)(1-\rho)^s + \sum_{x=1}^{c} (1-\omega)\binom{x+s-1}{s-1}\rho^x(1-\rho)^s \quad x = 0,1,2,\dots$$

When $c = 0$, the lot acceptance probability becomes as,

$$P_a(\bar{p}) = \omega + (1-\omega)(1-y)^s$$

Hence, the sample size can be determined for specified plan parameters $(p_1,\alpha,p_2,\beta,\omega)$ and s with a zero acceptance sampling plan as given below,

$$n = \frac{s}{\bar{p}}\left[\left(\frac{1-\omega}{P_a(\bar{p})-\omega}\right)^{\frac{1}{s}} - 1\right]$$

The optimum sample size can be determined with satisfying them fixed $P_a(p_1) \ge 1-\alpha$ and $P_a(p_2) \le \beta$.

B. Determination of Plan Parameters for the proposed sampling plan

The determination of the plan parameters of a sampling plan focus to certain conditions imposed on its measures of performance providing protection for both the producer's and the consumer's. The implication of a single sampling plan is exposed by its Operating Characteristics (OC) curve. Sampling plan usually selected for given two points on the OC curve approach through $(p_1, 1-\alpha)$ and (p_2,β) where p_1 is the Acceptable Quality Level (AQL), α is the producer's risk, p_2 is the Limiting Quality Level (LQL) and β is the consumer's risk.

The parameters (n,c) can be obtained through the two points on the OC curve approach to design the SSP for the specified strength (p_1,α,p_2,β). The plan parameters are determined to protect both the producers and consumers. Hence, the optimum plan parameters are studied by satisfying the conditions as follows, $P_a(p_1) = 1-\alpha$ and $P_a(p_2) = \beta$. Here $P_a(p)$ is the probability of acceptance for given lot or process quality p. Since, due to the discreteness of parameters n and c of the sampling plan have to be integers it is typically not possible to find a plan satisfying the requirements exactly. Therefore reformulate the problem in the following way,

a) $P_a(p_1) \ge 1-\alpha$ and
b) $P_a(p_2) \le \beta$

The above inequalities are in terms of the Operating Characteristics. It should be pointed out the procedure to be described be used to derive sample size (n) and acceptance number (c) for significance tests of Gamma-ZIP distribution. The search values of the parameters can be made satisfying these desired conditions as well as ensuring to reduce both the producer's and consumer's risks simultaneously.

The designing parameters of Bayesian SSP such as (n,c) are found at various values of fixed parameters (p_1,α,p_2,β) and the limiting values of ω and s by using Gunther approach. According to the Hald [7] the values of the parameter s in the prior distribution range over the interval $(0,\infty)$.

According to Gunther [6], using the iterative procedure to determine the design parameters such as the sample size (n) and the acceptance number (c) are determined for various specified parameters p_1, p_2, ω and s with the fixed producer's risk 5% and consumer's risk 10%. The tables of optimal parameters for the SSP under the G-ZIP model given in appendixes *A.2- A.6* for the different values of shape parameter $s = 5,10,25,50,150$ and the zero inflation parameter $\omega = 0.001, 0.01, 0.05, 0.09$ respectively. In this study, one has to assume that the shape parameter s and the proportion of zero parameter ω in Gamma-ZIP distribution are known. The iterative procedure consider the values of n and c were limited values for finding the optimum sampling plan and if the sampling plan are does not exist under these conditions and are denoted by ***.

C. Procedure for Selection of Optimum Sampling Plan (n,c) for Given Parameters

The entries of in Tables *A.2- A.6* are values of n and c for which the proportion of lots expected to be accepted is the stated value. Therefore, the optimum sampling plan parameters (n,c) can be selected corresponding to the specified parameters as stated below;

Step 1 Specify the requirement strength (p_1,α,p_2,β) on the OC curve.

Step 2 Specify the estimated parameters s and ω from the sample data.

Step 3 Find the corresponding value of parameters (n, c) to the given value of p_1 and p_2. These parameters can be consider the required optimum sampling plan.

V. NUMERICAL ILLUSTRATION

A. Illustration 1

For example, all smallest manufacturer food companies have a responsibility to produce consumers with good quality, wholesome foods. Quality is not an option but it is a necessary part of the planning, preparation and production of foods. Any lack of consideration of quality can result in a serious threat to public health. Suppose that the food quality engineer wants to run an experiment to make a decision on a food product whether to accept or reject an item. Assuming that the quality of food products follows the gamma- ZIP distribution with the estimated value of $s = 25$. Suppose one wants to find the strength of the plan parameter of a SSPs for specified values of AQL and LQL say, $p_1 = 0.005$, $p_2 = 0.10$ with producer risk (α) 5% and consumers risk (β) 10% and estimated value of $\omega = 0.09$. From Table *A.3*, one can find the values of the parameters as random sample of $n = 72$ units from the lot with acceptance numbers $c = 1$. Therefore, the food quality engineer drawn 72 samples from a lot with acceptance number one, can be able to make a suitable decision whether to allow food products for shipments or not for the given specified values.

B. Illustration 2

The electronic manufacturing company is striving to be the main dealer of electric cables within the country and consequently around. One of the ways to accomplish the company mission is by ensuring high quality cables produced by them. The manufactured cables need to be inspected in order to ensure only good cables will be delivered to customers. In such a way, the quality engineer wants to run an experiment to make a decision on the electric cables to decide whether the whole lot should be delivered to customers or not based on sampling inspection.

In such a way, the quality engineer wants to run an experiment to make a decision on the electric cables to decide whether the whole lot should be delivered to customers or not based on sampling inspection. Assuming that the life time of the electric cable follows the Gamma Poisson, ZIP and Gamma-ZIP distribution.

The values of $P_a(p)$the SSPs under the conditions of Gamma Poisson, ZIP and Gamma-ZIP are given in Table *A.1* for the various values of p. The quality investigation authorities as consider the lot fraction nonconforming is $p = 0.08$, the consumer risk under the Gamma Poisson SSP has 14.29% and conventional SSP under ZIP distribution has 15.28%, 14.74%, 11.28% risk of accepting the lot for different values of $\omega = 0.01, 0.05, 0.09$. Whereas, under the proposed plan for different values of $\omega = 0.01, 0.05, 0.09$ is given, 14.03%, 12.81%, 10.81% respectively.

Similarly, suppose that the proportion defective is $p = 0.005$ then the SSP under the Gamma Poisson distribution has the producers risk is 0.2% the SSP under ZIP distribution has the producers risk are 0.21%, 0.28% and 0.21% for the different values of $\omega = 0.01, 0.05, 0.09$ are respectively. The SSP under the Gamma-ZIP distribution, the producers risk are 0.22%, 0.16%, 0.04% at the same specified quality levels and values of ω.

The total sum of risk for each sampling plan has been given in Table.1 In this table, can be observed that the total amount of risk of producer as well as consumer under Gamma Poisson SSP has 14.49% and the conventional ZIP SSP when $\omega = 0.01, 0.05, 0.09$ are respectively, 15.49%, 15.03% and 11.49% whereas the total risk under Gamma-ZIP SSP is 14.25%, 12.98% and 10.86% with the different values of $\omega = 0.01, 0.05, 0.09$ are respectively.

Hence, when the value of ω becomes large, the proposed plan was performed better than the classical SSP under the ZIP distribution and the Gamma Poisson SSPs. It should be noted that the proposed plan was significantly reduced the producers risk as well as consumers risk simultaneously.

TABLE.1

The values of Producers Risk, Consumers Risk and Total Risk for the specified strength ($p_1 = 0.08$, $\alpha = 0.05, p_2 = 0.08, \beta = 0.10$ *and* $s = 5$) of the optimum sampling plan.

model	ω	α (%)	β (%)	$\alpha + \beta$ (%)
GP	-	0.20	14.29	14.49
ZIP	0.01	0.21	15.28	15.49
	0.05	0.28	14.74	15.03
	0.09	0.21	11.28	11.49
G-ZIP	0.01	0.22	14.03	14.25
	0.05	0.16	12.81	12.98
	0.09	0.04	10.81	10.86

Notes: α=Producer Risk, β=Consumers Risk, $\alpha + \beta$= Sum of Risk

From this results, the electronic cable company can be used the proposed plan increase the quality and reliable products in production, meet customer expectation and compete in the market.

C. *Comparative study*

The operating characteristic (OC) curve describes the discriminatory power of an acceptance sampling plan. This concept could be a graph of the fraction defective in a lot versus the probability that the sampling plan will accept a lot. In order to study the comparison of OC Curves of SSP under the conditions of G-ZIP with the Conventional SSP under the conditions of ZIP and the SSP under the Gamma Poisson distributions were seen in **Fig.1.** On comparison, it may be observed that the OC curve of the SSP under Gamma-ZIP model has desirable shape as a composite.

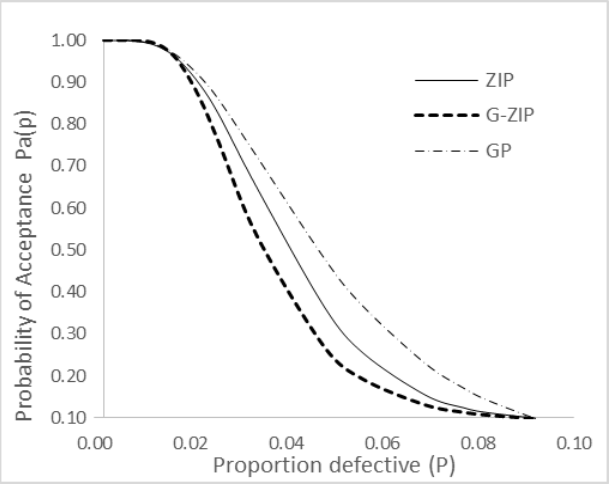

Fig.1 The OC Curve of SSP Plan under the Conditions of distribution are Gamma Poisson, Zero Inflated Poisson and Gamma-Zero Inflated Poisson distribution

Hence, clearly shows that the proposed sampling plan provides additional protection to the producer from the risk of rejecting the lots of good quality compared to the Gamma Poisson and conventional SSP under the ZIP distribution. Moreover, these plans provided more safeguards for consumers and its give more assurance regarding the outgoing quality or the quality of the lot after the inspection.

VI. CONCLUSION

In this paper, designing of Bayesian Single Sampling plan has been developed under Gamma-Zero Inflated Poisson distributions was developed. The optimal parameters of the proposed plan are determined using two point on the OC curve approach. The major advantage of the proposed sampling plan is more appropriate model in manufacturing of product units, when the number of non-defective items are occur frequently and excess number of zeros of defective items are inflated in reality during the inspection period. Hence, the proposed model may simultaneously protect both the producers and consumers risk in manufacturing industries which was given in the OC curve. This plan has illuminated and also shows the effectiveness of this plan when it compared with Conventional ZIP and Gamma Poisson distribution. Further few illustrations are provided and suitable tables are developed for readymade selection of the plan parameters under shop floor conditions.

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APPENDIXES

Table.A.1

Values of OC functions of Bayesian SSP under the Gamma Poisson, ZIP and Gamma-ZIP distribution for the given strength of parameters ($S = 10$, $p_1 = 0.015$, $p_2 = 0.09$, $\alpha = 5\%$ *and* $\beta = 10\%$).

	parameters			Lot fraction defective (p)									
model	**ω**	**n**	**c**	**0.003**	**0.005**	**0.007**	**0.010**	**0.020**	**0.032**	**0.046**	**0.067**	**0.075**	**0.080**
GP	-	108	4	1.0000	0.9995	0.9980	0.9917	0.9134	0.7212	0.4801	0.2307	0.1719	0.1429
ZIP	0.01	76	3	0.9999	0.9994	0.9978	0.9924	0.9325	0.7743	0.5421	0.2598	0.1882	0.1528
	0.05	83	3	0.9999	0.9992	0.9972	0.9902	0.9171	0.7376	0.4963	0.2351	0.1756	0.1474
	0.09	128	4	0.9999	0.9995	0.9979	0.9909	0.8935	0.6452	0.3633	0.1547	0.1244	0.1128
G-ZIP	0.01	111	4	0.9999	0.9995	0.9978	0.9909	0.9073	0.7081	0.4659	0.2234	0.1676	0.1403
	0.05	150	5	0.9999	0.9997	0.9984	0.9918	0.8961	0.6648	0.4089	0.1912	0.1480	0.1281
	0.09	350	10	0.9999	0.9999	0.9996	0.9951	0.8618	0.5327	0.2700	0.1327	0.1151	0.1081

Table.A.2

Optimal Bayesian SSP plan under Gamma-Zero Inflated Poisson for given $p_1, p_2, \alpha = 0.05, \beta = 0.10$ and $s = 5$.

		Consumer Quality Level (p_2)					
	Producer Quality Level (p_1)	**0.05**	**0.06**	**0.07**	**0.08**	**0.09**	**0.10**
ω=0.0001	**0.005**	(147,2)	(124,2)	(106,2)	(65,1)	(58,1)	(52,1)
	0.010	(523,11)	(264,6)	(167,4)	(119,3)	(106,3)	(95,3)
	0.015	***	(1910,54)	(492,15)	(275,9)	(176,6)	(138,5)
	0.020	***	***	***	(1433,54)	(451,18)	(262,11)
	0.025	***	***	***	***	***	(1126,53)
	0.030	***	***	***	***	***	***
ω=0.01	**0.005**	(198,3)	(129,2)	(110,2)	(96,2)	(61,2)	(55,1)
	0.010	(583,12)	(308,7)	(203,5)	(151,4)	(110,3)	(99,3)
	0.015	***	***	(569,17)	(312,10)	(205,7)	(143,5)
	0.020	***	***	***	***	(535,21)	(292,12)
	0.025	***	***	***	***	***	***
ω=0.05	**0.005**	(241,3)	(202,3)	(136,2)	(119,2)	(106,2)	(69,1)
	0.010	(1905,36)	(581,12)	(317,7)	(215,5)	(163,4)	(121,3)
	0.015	***	***	***	(687,20)	(359,11)	(248,8)
	0.020	***	***	***	***	***	(924,35)
	0.025	***	***	***	***	***	***
ω=0.09	**0.005**	(847,9)	(513,6)	(332,4)	(246,3)	(219,3)	(196,3)
	0.010	***	***	***	(1311,25)	(645,13)	(428,9)
	0.015	***	***	***	***	***	***

Table.A.3

Optimal Bayesian SSP plan under Gamma-Zero Inflated Poisson for given $p_1, p_2, \alpha = 0.05, \beta = 0.10$ and $s = 10$.

		Consumer Quality Level (p_2)					
	Producer Quality Level (p_1)	**0.05**	**0.06**	**0.07**	**0.08**	**0.09**	**0.10**
ω=0.0001	**0.005**	(126,2)	(105,2)	(65,1)	(57,1)	(50,1)	(45,1)
	0.010	(227,5)	(162,4)	(115,3)	(100,3)	(70,2)	(63,2)
	0.015	(554,15)	(272,8)	(186,6)	(142,5)	(108,4)	(80,3)
	0.020	***	(731,25)	(350,13)	(204,8)	(145,6)	(114,5)
	0.025	***	***	(1064,44)	(407,18)	(236,11)	(164,8)
	0.030	***	***	***	***	***	(277,15)
ω=0.01	**0.005**	(130,2)	(109,2)	(67,1)	(59,1)	(52,1)	(47,1)
	0.010	(234,5)	(167,4)	(118,3)	(103,3)	(73,2)	(65,2)
	0.015	(600,16)	(306,9)	(191,6)	(146,5)	(111,4)	(83,3)
	0.020	***	(829,28)	(382,14)	(230,9)	(168,7)	(117,5)
	0.025	***	***	(1321,54)	(458,20)	(260,12)	(184,9)
	0.030	***	***	***	***	(552,28)	(317,17)
	0.035	***	***	***	***	***	(744,43)
ω=0.05	**0.005**	(154,2)	(129,2)	(110,2)	(71,1)	(63,1)	(57,1)
	0.010	(346,7)	(225,5)	(166,4)	(122,3)	(108,2)	(77,2)
	0.015	(1154,29)	(474,13)	(273,8)	(193,6)	(150,5)	(116,4)
	0.020	***	***	(640,22)	(356,13)	(234,9)	(174,7)
	0.025	***	***	***	(996,41)	(437,19)	(285,13)
	0.030	***	***	***	***	***	(595,30)
ω=0.09	**0.005**	(330,4)	(231,3)	(201,3)	(143,2)	(128,2)	(114,2)
	0.010	(912,16)	(516,10)	(341,7)	(236,5)	(186,4)	(166,3)
	0.015	***	***	(851,22)	(480,13)	(350,10)	(239,7)
	0.020	***	***	***	***	(764,26)	(456,16)
	0.030	***	***	***	***	***	***

Table.A.4

Optimal Bayesian SSP plan under Gamma-Zero Inflated Poisson for given $p_1, p_2, \alpha = 0.05, \beta = 0.10$ and $s = 25$.

		Consumer Quality Level (p_2)					
	Producer Quality Level (p_1)	**0.05**	**0.06**	**0.07**	**0.08**	**0.09**	**0.10**
ω=0.0001	**0.005**	(114,2)	(69,1)	(59,1)	(52,1)	(46,1)	(42,1)
	0.010	(173,4)	(120,3)	(103,3)	(72,2)	(64,2)	(57,2)
	0.015	(286,8)	(192,6)	(124,4)	(109,4)	(80,3)	(72,3)
	0.020	(559,18)	(285,10)	(185,7)	(144,6)	(113,5)	(87,4)
	0.025	(1735,62)	(534,21)	(283,12)	(197,9)	(144,7)	(115,6)
	0.030	***	(1446,62)	(496,23)	(282,14)	(190,10)	(143,8)
	0.035	***	***	(1239,62	(484,26)	(281,16)	(199,12)

	0.040	***	***	***	(1085,62)	(460,28)	(280,18)
	0.045	***	***	***	***	(964,62)	(441,30)
	0.050	***	***	***	***	***	(868,62)
ω=0.01	**0.005**	(117,2)	(98,2)	(61,1)	(54,1)	(48,1)	(43,1)
	0.010	(178,4)	(123,3)	(106,3)	(74,2)	(65,2)	(59,2)
	0.015	(292,8)	(197,6)	(127,4)	(111,4)	(82,3)	(74,3)
	0.020	(596,19)	(313,11)	(209,8)	(147,6)	(115,5)	(89,4)
	0.025	(2000,71)	(565,22)	(308,13)	(201,9)	(147,7)	(118,6)
	0.030	***	(1667,71)	(542,25)	(304,15)	(209,11)	(160,9)
	0.035	***	***	(1410,70)	(509,27)	(301,17)	(216,13)
	0.040	***	***	***	(1232,70)	(497,30)	(298,19)
	0.045	***	***	***	***	(1097,70)	(474,32)
	0.050	***	***	***	***	***	(1000,71)
ω=0.05	**0.005**	(135,2)	(113,2)	(97,2)	(63,1)	(56,1)	(51,1)
	0.010	(232,5)	(167,4)	(121,3)	(106,3)	(76,2)	(68,2)
	0.015	(384,10)	(245,7)	(166,5)	(126,4)	(112,4)	(85,3)
	0.020	(850,26)	(418,14)	(252,9)	(184,7)	(146,6)	(116,5)
	0.025	***	(829,31)	(422,17)	(258,11)	(196,9)	(147,7)
	0.030	***	***	(814,36)	(424,20)	(279,14)	(207,11)
	0.035	***	***	***	(803,41)	(425,23)	(281,16)
	0.040	***	***	***	***	(794,46)	(425,26)
	0.045	***	***	***	***	***	(773,50)
	0.050	***	***	***	***	***	***
ω=0.09	**0.005**	(226,3)	(156,2)	(133,2)	(118,2)	(105,2)	(72,1)
	0.010	(393,7)	(250,5)	(193,4)	(164,4)	(126,3)	(114,3)
	0.015	(758,18)	(431,11)	(294,8)	(234,7)	(168,5)	(150,5)
	0.020	(2666,75)	(825,25)	(491,16)	(323,11)	(248,9)	(187,7)
	0.025	***	***	(952,35)	(538,21)	(366,15)	(276,12)
	0.030	***	***	***	(1104,48)	(573,26)	(378,18)
	0.035	***	***	***	***	(1201,60)	(612,32)
	0.040	***	***	***	***	***	***

Table.A.5

Optimal Bayesian SSP plan under Gamma-Zero Inflated Poisson for given $p_1, p_2, \alpha = 0.05, \beta = 0.10$ and $s = 50$.

		Consumer Quality Level (p_2)					
	Producer Quality Level (p_1)	**0.05**	**0.06**	**0.07**	**0.08**	**0.09**	**0.10**
ω=0.0001	**0.005**	(111,2)	(67,1)	(58,1)	(50,1)	(45,1)	(41,1)
	0.010	(167, 4)	(116, 3)	(79, 2)	(69, 2)	(62, 2)	(56, 2)
	0.015	(247,7)	(162,5)	(119,4)	(87,3)	(77,3)	(70,3)
	0.020	(402,13)	(250,9)	(158,6)	(121,5)	(93,4)	(84,4)
	0.025	(777,28)	(377,15)	(232,10)	(171,8)	(123,6)	(97,5)
	0.030	***	(648,28)	(341,16)	(219,11)	(167,9)	(124,7)
	0.035	***	***	(555,28)	(315,17)	(209,12)	(163,10)
	0.040	***	***	***	(486,28)	(294,18)	(201,13)

	0.045	***	***	***	***	(432,28)	(277,28)
	0.050	***	***	***	***	***	(389,28)
ω=0.01	**0.005**	(113,2)	(69,1)	(60,1)	(52,1)	(46,1)	(42,1)
	0.010	(171,4)	(119,3)	(81,2)	(71,2)	(63,2)	(57,2)
	0.015	(252,7)	(165,5)	(122,4)	(89,3)	(79,3)	(72,3)
	0.020	(433,14)	(254,9)	(180,7)	(124,5)	(95,4)	(85,4)
	0.025	(836,30)	(383,15)	(236,10)	(174,8)	(126,6)	(99,5)
	0.030	***	(696,30)	(364,17)	(239,12)	(169,9)	(126,7)
	0.035	***	***	(598,30)	(335,18)	(227,13)	(165,10)
	0.040	***	***	(1234,66)	(523,30)	(312,19)	(217,14)
	0.045	***	***	***	(956,58)	(465,30)	(293,20)
	0.050	***	***	***	***	(781,53)	(418,30)
ω=0.05	**0.005**	(130,2)	(109,2)	(70,1)	(61,1)	(55,1)	(49,1)
	0.010	(191,4)	(134,3)	(115,3)	(82,2)	(73,2)	(65,2)
	0.015	(305,8)	(208,6)	(157,5)	(120,4)	(90,3)	(83,3)
	0.020	(549,17)	(323,11)	(218,8)	(156,6)	(122,5)	(96,4)
	0.025	(1124,39)	(503,19)	(296,12)	(209,9)	(154,7)	(125,6)
	0.030	***	(959,40)	(468,21)	(293,14)	(215,11)	(166,9)
	0.035	***	***	(841,41)	(443,23)	(275,15)	(207,12)
	0.040	***	***	***	(703,39)	(408,24)	(275,17)
	0.045	***	***	***	***	(625,39)	(381,25)
	0.050	***	***	***	***	***	(575,40)
ω=0.09	**0.005**	(213,3)	(147,2)	(127,2)	(110,2)	(98,2)	(71,1)
	0.010	(311,6)	(232,5)	(179,4)	(132,3)	(118,3)	(106,3)
	0.015	(531,13)	(339,9)	(246,7)	(195,6)	(156,5)	(124,4)
	0.020	(974,28)	(516,16)	(338,11)	(255,9)	(191,7)	(155,6)
	0.025	***	(911,32)	(507,19)	(351,14)	(261,11)	(204,9)
	0.030	***	***	(822,34)	(498,22)	(346,16)	(266,13)
	0.035	***	***	***	(823,40)	(493,25)	(339,18)
	0.040	***	***	***	***	(783,43)	(488,28)
	0.045	***	***	***	***	***	(750,46)
	0.050	***	***	***	***	***	***

Table.A.6

Optimal Bayesian SSP plan under Gamma-Zero Inflated Poisson for given $p_1, p_2, \alpha = 0.05, \beta = 0.10$ and $s = 150$.

		Consumer Quality Level (p_2)					
	Producer Quality Level (p_1)	**0.05**	**0.06**	**0.07**	**0.08**	**0.09**	**0.10**
ω=0.0001	**0.010**	(163,4)	(113,3)	(77,2)	(68,2)	(60,2)	(54,2)
	0.015	(214,6)	(157,5)	(116,4)	(85,3)	(76,3)	(54,2)
	0.020	(339,11)	(221,8)	(153,6)	(118,5)	(91,4)	(82,4)
	0.025	(577,21)	(303,12)	(207,9)	(150,7)	(119,6)	(95,5)
	0.030	***	(481,21)	(277,13)	(197,10)	(147,8)	(107,6)

	0.035	***	***	(413,21)	(257,14)	(188,11)	(145,9)
	0.040	***	***	***	(361,21)	(242,15)	(170,11)
	0.045	***	***	***	***	(321,21)	(218,15)
	0.050	***	***	***	***	***	(289,21)
ω=0.01	**0.005**	(111,2)	(68,1)	(58,1)	(51,1)	(46,1)	(41,1)
	0.010	(166,4)	(116,3)	(79,2)	(69,2)	(62,2)	(56,2)
	0.015	(244,7)	(160,5)	(119,4)	(87,3)	(77,3)	(70,3)
	0.020	(368,12)	(245,9)	(156,6)	(120,5)	(92,4)	(83,4)
	0.025	(585,21)	(327,13)	(210,9)	(152,7)	(121,6)	(96,5)
	0.030	***	(487,21)	(297,14)	(200,10)	(164,9)	(122,7)
	0.035	***	***	(417,21)	(260,14)	(191,11)	(147,9)
	0.040	***	***	***	(365,21)	(245,15)	(184,12)
	0.045	***	***	***	***	(325,21)	(232,16)
	0.050	***	***	***	***	***	(292,21)
ω=0.05	**0.005**	(126,2)	(105,2)	(68,1)	(59,1)	(53,1)	(48,1)
	0.010	(184,4)	(130,3)	(112,3)	(79,2)	(70,2)	(64,2)
	0.015	(293,8)	(200,6)	(152,5)	(115,4)	(103,4)	(92,4)
	0.020	(446,14)	(287,10)	(191,7)	(150,6)	(118,5)	(93,4)
	0.025	***	(393,15)	(264,11)	(183,8)	(148,7)	(120,6)
	0.030	***	***	(353,16)	(247,12)	(177,9)	(147,8)
	0.035	***	***	***	(326,17)	(234,13)	(211,13)
	0.040	***	***	***	***	(304,18)	(223,14)
	0.045	***	***	***	***	***	***
ω=0.09	**0.005**	(202,3)	(168,3)	(121,2)	(108,2)	(94,2)	(67,1)
	0.010	(294,6)	(198,4)	(168,4)	(126,3)	(113,3)	(101,3)
	0.015	(411,10)	(295,8)	(210,6)	(167,5)	(132,4)	(118,4)
	0.020	(656,19)	(411,13)	(294,10)	(221,8)	(164,6)	(148,6)
	0.025	***	(590,21)	(391,15)	(275,11)	(213,9)	(175,8)
	0.030	***	***	***	(378,17)	(291,14)	(220,11)
	0.035	***	***	***	***	(349,18)	(263,14)
	0.040	***	***	***	***	***	(328,19)
	0.045	***	***	***	***	***	***