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Radio Reciprocal Membership Function on Cycle Related Graphs

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***Abstract*: Radio labeling is graph labeling which deals with nodes of a graph. A new approach fuzzy radio reciprocal labeling proposed. Fuzzy radio reciprocal labeling deals with membership function [0,1] for every vertex and edge for making flexible which is stand for by FR_L^{-1} .Fuzzy radio reciprocal labeling is determined for fan graph and wheel graph**

***Index Terms*: fuzzy graph, μ-length, μ -distance, diameter, fuzzy labeling graph**

I. INTRODUCTION

The concept of uncertainty of fuzzy set was briefed by Zadeh (1965).The fuzzy graph was defined by Rosenfield by fuzzy relation which represents the relation between the objects by previously indicating the level of relationship between the objects of the function sets [2].

II. PRELIMINARIES

A fuzzy graph $G = (V, \sigma, \mu)$ of a nonempty set V together with a pair of function $\sigma : V \to [0,1]$ and $\mu : E \to [0,1]$ it satisfy $\mu(v,u) \le \sigma(u) \wedge \sigma(v)$ for all $u,\ v \in V$. A path P in a fuzzy graph is a sequence of specific nodes $v_1, v_2 ... v_n$ such that $\mu(v_1, v_{i+1}) > 0; 1 \le i \le n$; here $n \ge 1$ is called the length of the path P. The successive pairs (v_1, v_{i+1}) are called the edge of the path.

A graph $G = (V, \sigma, \mu)$ is said to be a fuzzy labelling graph, if $\sigma : V \to [0,1]$ and $\mu : E \to [0,1]$ is bijective such that the membership value of lines and nodes are specific and $\mu(v,u) \le \sigma(u) \wedge \sigma(v)$ for all $u,v \in V$ [1] [2].

If ρ is the path consisting of the vertices $x_1, x_2 ... x_n$ in a fuzzy graph $G = (V, \sigma, \mu)$, the μ- length fuzzy graph is defined by $l(\rho)$ where $l(\rho) = \sum_{i=}^{n} \mu(x_{i-1}, x_i)^{-1}$ .For two vertices x, y in G, the μ - length of all paths joining X and Y. The μ - distance $\delta(u , v)$ is the smallest μ -length of any u- v path and δ is metric.

$G = (V, \sigma, \mu)$ is a fuzzy graph with the set of nodes V. Then $e(v)$ = {the maximum of all the μ –distance $\delta(u , v)$, for $u ,v \in V$. where radius = minimum of $e(v)$ and the v is the diametrical node of $e(v) = diam(G)$ [3], [6].

III. METHODOLOGY

A *radio labeling c* of G is an assignment of positive integers to the nodes of G satisfying

$$d(u,v) + |f(u) - f(v)| \ge 1 + diam(G),$$

for every two specific nodes u and v. The maximum integer in the range of the labeling is its span. The *radio number* of G, $rn(G)$, is the minimum possible span of any radio labeling for G. Where $diam(G)$ is the diameter of the graph G and $d(u,v)$ is the distance between the vertices u and v [4].

In this paper, a new fuzzy labelling of a graph is converted as *a fuzzy radio reciprocal labeling*. A fuzzy graph $G = (V, \tau, \gamma)$ of a non-empty set V together with a pair of functions $\tau : V \to [0,1]$ and $\gamma : E \to [0,1]$ said to be a fuzzy radio reciprocal labeling for all, then the following condition satisfy

$$\delta(v,u) + \frac{1}{|\tau(u) - \tau(v)|} > diam(G)$$

Where $\delta(v,u)$ is μ -distance of a fuzzy graph, $\tau(u)$ and $\tau(v)$ are membership function of nodes of u and v , $\gamma(u,v)$ is membership function of edges and $diam(G)$ is diameter of a fuzzy graph G. Fuzzy radio reciprocal labeling is denoted by FR_L^{-1} .

Example

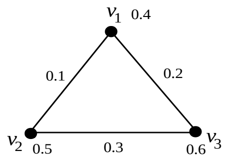

Fig.1: Fuzzy radio reciprocal labeling graph

$$\delta(v_1, v_2) = 8.3, \delta(v_1, v_3) = 5$$
$$e(v_1) = 8.3$$
$$\delta(v_2, v_1) = 8.3, \delta(v_2, v_3) = 3.3$$
$$e(v_2) = 8.3$$
$$\delta(v_3, v_1) = 5, \delta(v_3, v_2) = 3.3$$
$$e(v_3) = 5$$
$$diam(G) = 8.3$$

When i = 1, *j*= 2

$18.3 > 8.3$

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IV. RESULTS

1. Fuzzy Radio Reciprocal labelling of Fan graph

A fan graph denoted by F_n is the path P_{n-1} plus and extra vertex connected to all vertices of the path P_{n-1} [7].A fuzzy fan graph $G_f = (V, \tau, \gamma)$ of a nonempty set V together with a pair of functions $\tau : V \to [0,1]$ and $\gamma : E \to [0,1]$ it satisfy $\gamma(v_i, v_j) \le \tau(v_i) \wedge \tau(v_j)$ for all $v_i, v_j \in V$ where i =1,2…n *and* j =1,2,3…n

It is denoted by G_f .

Lemma1.1: If the nodes has the maximum member function of any fuzzy fan graph, then it has maximum incident line but not in P_{n-1} path of graph G_f, where $n \le 3$.

Proof: let G_f be a fuzzy fan graph, *if* $n \le 3$, then more than one vertices of graph G_f has maximum number of incident lines. Select all the nodes of G_f which has equal number of incident lines. Fix any one of the node, say v_0, of graph G_f which has the maximum membership of the fuzzy set, if it does not belong to P_{n-1} path of a graph G_f .

Lemma1.2: In any fuzzy fan graph of G_f , the node has the maximum member function of G_f if it has maximum number of incident lines of graph where $n \ge 4$.

Theorem1: Every Fan graph admits FR_L^{-1} .

Proof: Let G be a Fan graph, it has lines stand for by G_f . Define a fuzzy fan graph $G = (V, \tau, \gamma)$ is a non-empty set V together with a pair function $\tau : V \to [0,1]$ and $\gamma : E \to [0,1]$ for all $v_i, v_j \in V$

$$\gamma(v_i, v_j) \le \tau(v_i) \wedge \tau(v_j) \qquad (1)$$

and denoted by G_f .

Let $\phi(v_i, v_j)$ be μ-distance of fuzzy fan graph and ϕ has the minimum μ - length of G_f . Consider,

$$l(\rho) = \sum_{i=1}^{n} \left(\frac{1}{\gamma(v_i, v_j)} \right),$$

where $l(\rho)$ is μ- length of G_f and $diam(G_f)$ is maximum value of μ - distance of graph G_f .

Let G_f be fuzzy fan graph. Fix v_0, the node of a graph G_f with maximum membership function of node fuzzy set, i.e., $\tau(v_0) = 1$. Let P be the *consecutive path* of the graph set of nodes $v_1, v_2 ... v_{n-1}$ and adjacent with v_0 . Construct $\tau(v_0)$, where i varies from 1 to n-1. Every membership function of nodes are specific and $\gamma(v_i, v_j)$ are specific, and if satisfies the following condition (i.e) every membership function of lines always less than minimum of every pair of the nodes which adjacent to that lines.

$$min(\tau(v_i), \tau(v_j)) > \gamma(v_i, v_j) \qquad (2)$$

Where i= from 1 to n-1, j=from 1 to n-1 and above proof of fuzzy fan graph admits fuzzy labeling of a graph in equation (2) and fuzzy fan graph of labeling admits fuzzy reciprocal radio labeling of graph G_f , satisfies the following condition

$$\delta(v_i, v_j) + \frac{1}{|\tau(v_i) - \tau(v_j)|} > diam(G_f) \qquad (3)$$

Therefore fan graph satisfies all the above equation (1), (2) and (3).

Hence fan admits Fuzzy Radio Reciprocal labeling FR_L^{-1} .

2. Fuzzy Radio Reciprocal labeling of Wheel graph

Wheel graph is attain from a cycle C_n by adding a new vertex and edges joining it, to all the nodes of the cycle [7]. A wheel graph is called fuzzy wheel graph in which all the vertices and edges has membership function and said to satisfy

$$\gamma(v_i, v_j) \le \tau(v_i) \wedge \tau(v_j) \text{ for all } v_i, v_j \in V$$

Fuzzy wheel graph is denoted by FW_n .

Lemma2.1: In any fuzzy wheel graph of FW_n , the vertex has the maximum member function of FW_n if it has maximum number of incident edges of graph where $n \ge 4$.

Theorem2: If any fuzzy wheel graph has n nodes then it satisfies fuzzy radio reciprocal labeling of a graph.

Proof: Let G be wheel graph, Define a fuzzy wheel graph $FW_n = (V, \tau, \gamma)$, has nodes and lines then $\tau : V \to [0,1]$ and $\gamma : E \to [0,1]$ for all $v_i, v_j \in V$,it satisfy

$$\gamma(v_i, v_j) \le \tau(v_i) \wedge \tau(v_j) \qquad (4)$$

Let $\phi(FW_n)$ be distance of fuzzy fan graph is the minimum μ- length of FW_n . Considerμ- length of FW_n ,

$$l(\rho) = \sum_{i=1}^{n} \left(\frac{1}{\gamma(v_i, v_j)} \right)$$

Let FW_n be fuzzy wheel graph. Fix v_0, the node adjacent to all other remaining nodes of a graph which has maximum membership function that is $\tau(v_0) = 1$. Let P be the consecutive path of a cycle with nodes and adjacent with v_0.

In a *path of the cycle*, the start and end nodes are same and path P contains n-1 nodes which satisfies nodes $v_1, v_2 ... v_{n-1}$ and adjacent with v_0 .Construct $\tau(v_i)$, where i varies from 1 to n-1. Every membership function of nodes are specific and $\gamma(v_i, v_j)$ are specific, and it satisfies the following condition

$$min(\tau(v_i), \tau(v_j)) > \gamma(v_i, v_j) \qquad (5)$$

Where i from 1 to n-1, j from 1 to n-1. The above proof of fuzzy wheel graph admits fuzzy labeling of a graph in equation (5) and fuzzy wheel graph of labelling admits fuzzy reciprocal radio labeling of graph, satisfies the following condition

$$\delta(v_i, v_j) + \frac{1}{|\tau(v_i) - \tau(v_j)|} > diam(FW_n) \qquad (6)$$

Therefore wheel graph satisfies all the above equation (4), (5) and (6).

Hence any wheel graph has n nodes that satisfies fuzzy radio reciprocal labeling of a graph.

Algorithm of fuzzy radio reciprocal labeling of fan graph and wheel graph

Step 1: Fix $\tau(v_0)=1$, v_0 has maximum number of adjacent node.

Step 2: Fix the membership function to all other remaining nodes of a graph, i,e $\tau(v_i)$,where $i = 1,2,3\ldots n$-1

Step 3: Fix the membership function to lines of a graph, i,e $\gamma(v_i,v_j)$,where i varies from 1 to n-1 and j from 1 to n-1, $(i \neq j)$.

Step 4: If the following condition must satisfies

$$\gamma(v_i,v_j) \leq \tau(v_i) \wedge \tau(v_j) \text{ and}$$

$$\delta\left(v_i,v_j\right)+\frac{1}{\left|\tau(v_i)-\tau(v_j)\right|} > diam(G)$$

If not, repeat from step 2.

Verification of fan graph

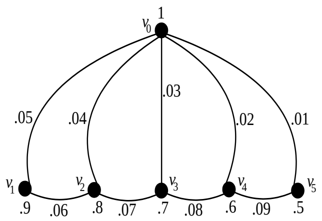

Fig.2: Fuzzy radio reciprocal labeling fan graph

Verification of wheel graph

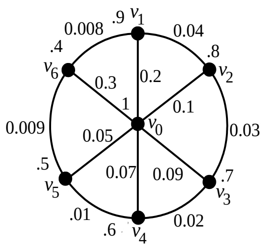

Figure 3: Fuzzy radio reciprocal labeling wheel graph

V. CONCLUSION

Fuzzy radio reciprocal has been introduced. Fuzzy radio reciprocal labeling for cycle related graphs have been discussed. We further extend the study on interconnection networks and cycle free graphs.

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