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# Estimation of the Catastrophe Bonds Price by using Risk Neutral Measurement

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***Abstract*****:** ***Catastrophe Bonds (CAT bonds) or disaster bonds are securities products that work by transferring risk in the form of natural disaster losses to the capital market, so that it is necessary to estimate CAT bond prices. This study intends to discuss the model to determine the price of CAT bonds in arbitrage-free scope based on the approach of risk neutral measurement. The data used is extreme data in the form of losses due to natural disasters in the range of 2000-2019. There are several stages carried out in this study. The first step is to calculate descriptive data statistics. Then, estimating the data parameters using the Maximum Likelihood Estimation (MLE) method assuming the data distributed Generalized Extreme Value (GEV). Next, determine the price of CAT bond using the formula of risk neutral measurement. From the results of the analysis carried out, the CAT bond price is USD98.63, and there is the effect of risk neutral measurement on the price of CAT bonds paid by investors.***

***Index Terms*****:** ***CAT bond, arbitrage-free, Generalized Extreme Value, risk neutral measurement.***

## I. INTRODUCTION

Disasters are extraordinary events or series of events that pose threats and disturbances to life, as well as people's livelihoods caused by natural factors or non-natural factors, as well as humans. Thus resulting in the emergence of human casualties, environmental damage, property losses, and psychological impacts [1; 2]. Meanwhile, disaster risk is a potential occurrence of losses that occur in an area within a certain period of time, can be: death, injury, illness, life threatening, the emergence of insecurity, displacement, destruction or loss of property, and disruption of social activities [3; 4]. The implementation of disaster mitigation carried out by the government and Indonesian social institutions by providing compensation assistance in the form of places to evacuate, drugs, food ingredients and building materials and equipment for victims whose homes have been damaged [2]. However, the compensation still cannot cover the overall loss of natural disasters. Therefore, according to Sukono et al. [1], insurance can be a solution to minimize losses due to natural disasters. In insurance companies, disaster risk is delegated by buying reinsurance contract. However, this can be a financial pressure on the reinsurance company due unpredictable nature of the loss [5; 6]. Therefore, insurance solutions are issued in the form of disaster bonds. Catastrophe bonds (CAT bonds) or disaster bonds are securities that work by transferring risk to the capital market [7; 8]. In previous studies regarding the calculation of CAT bond prices, as Shao at al. [9], Lai et al. [10], and Mariani & Amoruso [7], using quantitative methods to estimate the factors needed in calculating CAT bond prices. Zimbidis et al. [11] uses Extreme Value Theory to obtain a numerical result in the form of CAT bond prices, based on the stochastic interest rate in an incomplete market scope. The research conducted by Gang & Qun [5], developed a formula based on stochastic interest rates with losses following a non-homogenic compound Poisson process and obtained a numerical solution for CAT bond prices. Nowak and Romaniuk [12], in his research, the Monte Carlo simulation method was used to determine the price of CAT bonds using different payment functions.

Based on the description above, this research intends to use the theory of risk neutral measure to estimate the price of CAT bonds. The purpose of this study was to obtain and determine the effect of risk neutral measurement on CAT bond prices. Calculate the cash value of CAT bonds and estimate to get CAT bond prices using the approach and the theory of risk neutral measurement. Some of the benefits expected from the results of this study include: Obtaining opportunities for Generalized Extreme Value and models to calculate CAT bond prices; and provide convenience with the results obtained to investors who are interested in investing in CAT bonds.

## II. RESEARCH METHODOLOGY

### A. *Distribusi Loss Function*

*Loss function* $I_i$ is the maximum loss from natural disasters over a period of time $i$ and $I_i = \max\{X_{1i}, X_{2i}, \ldots, X_{mi}\}$, with

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$m$ is the day of each period and $X_{1i}, X_{2i}, \dots, X_{mi}$ is a line of losses for each period. Where $\{X_{1i}, X_{2i}, \dots, X_{mi}\}$ is a set of independent independent variables with unknown distribution functions. If there is a constant line $\{a_k\ ;\ a_k > 0, \forall\ k\ \in\ \mathbb{N}\}$, $\{b_k\} k \in \mathbb{N}$ and distribution function $G(z)$ non degenerate as follows [13]:

$$P\left(\frac{I_k - b_k}{a_k} \le z\right) \to G(z), \qquad k \to \infty, z \in \mathbb{R}$$

$G$ is a member of the Generalized Extreme Value (GEV) distribution set, and the distribution can be stated as follows:

$$G(z) = exp\left\{-\left[1 + \xi\left(\frac{z-\mu}{\sigma}\right)\right]^{-1/\xi}\right\} \qquad (1)$$

where $\left\{z\ ;\ 1 + \xi\left(\frac{z-\mu}{\sigma}\right) > 0\right\}$, with parameter scale fulfilling $\sigma > 0$, and form parameters $-\infty < \xi < \infty$ and location parameters meet $-\infty < \mu < \infty$. If a value estimator has been obtained from three parameters, then the large distribution function is the maximum loss $I_i$ in the event of natural disasters can also be obtained.

### B. *GEV Distribution Parameter Estimation by Maximum Likelihood Estimator*

In this study the MLE method is used to determine the parameter estimator of $\{\sigma, \xi, \mu\}$. Suppose the process the loss is an independent variable from the GEV distribution, with a density function as in equation (1).

If it is assumed that the process of large losses with data is as much as the independent variable $I_1, I_2, \dots, I_n$ following the GEV distribution, the likelihood function obtained from the GEV distribution is [14]:

$$L(\mu,\sigma,\xi) = \prod_{i=1}^{n} g(I_i)$$

$$= exp\left\{-\sum_{i=1}^{n}\left[1+\xi\left(\frac{z_i-\mu}{\sigma}\right)\right]^{-1/\xi}\right\}\frac{1}{\sigma^n}\left\{\sum_{i=1}^{n}\left[1+\xi\left(\frac{z_i-\mu}{\sigma}\right)\right]^{-1/\xi^{-1}}\right\} \qquad (2)$$

Next the original logarithmic equation is determined from equation (2), as follows:

$$l = \ln(L(\mu,\sigma,\xi))$$

$$= n\ln\sigma - \left(\frac{1}{\xi}+1\right)\left\{\ln\sum_{i=1}^{n}\left[1+\xi\left(\frac{z_i-\mu}{\sigma}\right)\right]\right\} - \sum_{i=1}^{n}\left[1+\xi\left(\frac{z_i-\mu}{\sigma}\right)\right]^{-1/\xi} \qquad (3)$$

Furthermore, from equation (3) it will be lowered to $\mu, \sigma, \xi$ and together with it $0$ as follows:

Derivative against $\mu$:

$$\frac{\partial \ln(L(\mu,\sigma,\xi))}{\partial \mu} =$$

$$\left(\frac{\xi+1}{\sigma}\right)\sum_{i=1}^{n}\left[1+\xi\left(\frac{z_i-\mu}{\sigma}\right)\right]^{-1} - \frac{1}{\sigma} - \sum_{i=1}^{n}\left[1+\xi\left(\frac{z_i-\mu}{\sigma}\right)\right]^{-1/\xi^{-1}} = 0$$

Derivative against $\sigma$:

$$\frac{\partial \ln(L(\mu,\sigma,\xi))}{\partial \sigma} =$$

$$\frac{n}{\sigma} + (\xi+1)\sum_{i=1}^{n}\left(\frac{z_i-\mu}{\sigma}\right)\left[1+\xi\left(\frac{z_i-\mu}{\sigma}\right)\right]^{-1} - \sum_{i=1}^{n}\left(\frac{z_i-\mu}{\sigma}\right)\left[1+\xi\left(\frac{z_i-\mu}{\sigma}\right)\right]^{-1/\xi^{-1}} = 0$$

Derivative against $\xi$:

$$\frac{\partial \ln(L(\mu,\sigma,\xi))}{\partial \xi} =$$

$$\frac{1}{\xi^2}\left(\ln\sum_{i=1}^{n}\left[1+\xi\left(\frac{z_i-\mu}{\sigma}\right)\right]\right) - \left(1+\frac{1}{\xi}\right)\sum_{i=1}^{n}\left(\frac{z_i-\mu}{\sigma}\right)\left[1+\xi\left(\frac{z_i-\mu}{\sigma}\right)\right]^{-1}$$

$$-\sum_{i=1}^{n}\left[1+\xi\left(\frac{z_i-\mu}{\sigma}\right)\right]^{-1/\xi}\left\{\frac{1}{\xi^2}\ln\left[1+\xi\left(\frac{z_i-\mu}{\sigma}\right)\right] - \frac{1}{\xi}\sum_{i=1}^{n}\frac{\left(\frac{z_i-\mu}{\sigma}\right)}{\left[1+\xi\left(\frac{z_i-\mu}{\sigma}\right)\right]}\right\} = 0$$

Because the completion of the first derivative of the likelihood logarithm function for each parameter is not closed form, the solution is done using the help of the *extRemes* package in software R.

### C. *Cash Value Model for CAT Bond*

The cash value for CAT bond modeled in this study is a model for one CAT bond period. Using the analytical framework for discrete time, the following is the notation used in the cash value CAT bond:

$C$ = Cash value CAT bond at maturity,
$F$ = principal value (principal),
$R$ = coupon rate,
$I$ = probability of a random variable that has a loss function distribution.

Usually, the coupon payment function is designed according to company policy. So, the model for calculating the CAT bond cash value at maturity with possible coupon payments is as follows:

$$C = \begin{cases} F \cdot (1+2R), & I \in [\alpha_1, \alpha_2] \\ F \cdot (1+R), & I \in [\alpha_2, \alpha_3] \\ F, & I \in [\alpha_3, \alpha_4] \\ \frac{1}{2}F, & I \in [\alpha_4, \alpha_5] \\ 0, & I \in [\alpha_5, \infty] \end{cases} \qquad (4)$$

where $\alpha_1, \dots, \alpha_5$ is a trigger point which is the interval of data, which follows the loss function distribution and affects the level of securitization of the bonds. Where each company must balance profits and selling power by analyzing historically the loss of natural disasters [9].

## III. CAT BOND PRICE ESTIMATION MODEL

Before determining the model to estimate CAT bond prices, assume that the CAT bond investment market is arbitrage-free. Where arbitrage-free states that it does not allow the possibility to invest as much as 0 today. But get a number of non-negatives with positive opportunities tomorrow. In other words, if there are two portfolios that have the same value, in the future they must be equal to today's prices.

Therefore, if a capital market is in an arbitrage-free scope that has a probability $\mathbb{P}$, if and only if there is another probability equal to $\mathbb{Q}$, then:

a. For an event A, $P(A) = 0$ if and only if $Q(A) = 0$, case on $P$ and $Q$ called equivalent probability measurement.
b. Discounted price process is martingale at $\mathbb{Q}$. Measurement $\mathbb{Q}$ which fulfills the points a. and b. this is what is called the risk neutral measurement [15].

Risk neutral measurement is a probability measure where the share price is equal to the expected value of a stock discount. Each stock asset can be priced only by calculating the present value of the expected result. Therefore, when calculating CAT bonds by using risk neutral measurement at the time $t = 0$, must calculate the expectation value from the discounted future cash value.

So, the equation for estimating the price of CAT bonds is as follows:

$$P = E_Q(e^{-rt} \cdot C) \qquad (5)$$

with expectation calculations under probability $\mathbb{Q}$. $e^{-rt}$ is a discounted price with $r$ is risk-free rate and $t$ is time period as well $C$ (4) is the CAT bond cash value at maturity [16; 17].

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## IV. RESULTS AND DISCUSSION

### A. *Statistical Data*

Events of natural disasters are extreme events where the occurrence is rare and cannot be predicted when they occur. But it has a big impact, so the data from natural disaster losses used in this study are extreme data. This extreme data is data on insurance losses due to natural disasters occurred in 2000 to 2019 (February), which were taken as many as 20 of them.

Descriptive statistical analysis is intended to do an analysis by describing a set of data as it is, and does not intend to make generally accepted conclusions [1; 3]. Descriptive statistics in this study are used to obtain location measurements, such as: mean; measures of variability, such as: variance, standard deviation; and the size of the shape, such as: skewness and kurtosis. Calculation of descriptive data statistics was performed using SPSS software. The following are the results of the descriptive statistical calculations of the largest natural disaster loss data that occurred from 2000-2019, which are presented in Table 1.

**Table 1. Results of descriptive data statistics**

| Number of Data | 20 |
|---|---|
| Mean | 23.826 |
| Standard Deviation | 16.38081 |
| Variance | 268.331 |
| Skewness | 2.519 |
| Kurtosis | 8.523 |

Observing Table 1, it can be seen that the value of losses in this study has 20 data with an average of USD23.826, a standard deviation of USD16.38081, and a variance of USD268.331. In Table 1, it can also be explained that the loss data in this study has a skewness of 2.519 where the data is right-skewed as shown in Figure 1, and has a kurtosis of 8.523. As explained earlier, the data has a relatively large variance, so the data used is heavy-tailed.

The histogram graph of the data is given in Figure 1 as follows.

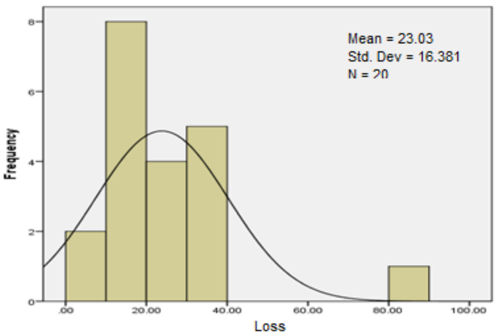


**Figure 1. Histogram and polygon of data loss**

From Figure 1, it is seen that the stem histogram does not have the same shape as the normal curve. This explains that the distribution of data is not a normal distribution.

### B. *Parameters Estimation of GEV Distribution Using MLE*

In actuarial literature, heavy-tailed data is usually modeled by theoretically heavy-tailed distribution [5]. One distribution for modeling heavy-tailed data is using the Generalized Extreme Value (GEV) model approach. To estimate parameters $(\mu, \sigma, \xi)$ GEV distribution as described in Part II, in this study carried out using the Maximum Likelihood Estimation (MLE) method. Data assumed to be an independent random variable. Estimates are made using the *extRemes* program package in software R.

The following are the results of parameter estimates $(\mu, \sigma, \xi)$ obtained from the steps described in Section II:

$$(\mu, \sigma, \xi) = (15.8892609\,; 7.6786421; 0.3029915)$$

From the parameter estimation process also obtained the variance covariance matrix as follows:

$$V = \begin{bmatrix} 1.380108 \times 10^{-5} & 2.087614 \times 10^{-5} & -0.0002379158 \\ 2.087614 \times 10^{-5} & 2.369233 \times 10^{-4} & 0.002793512 \\ -2.379158 \times 10^{-4} & 2.729351 \times 10^{-3} & 0.0516561947 \end{bmatrix}$$

Values that are in the diagonal of the variance covariance matrix show the variance of each parameter $(\mu, \sigma, \xi)$, so the distribution of each parameter in this GEV distribution is respectively large $1.380108 \times 10^{-5}$, $2.369233 \times 10^{-4}$, and $0.051656194$. Meanwhile, the value that is outside the diagonal is the covariance of each parameter $(\mu, \sigma, \xi)$.

Because of the covariance between parameters $\mu$ and $\sigma$ positive value (i.e. $2.087614 \times 10^{-5}$), both of them move in the same direction. So is the covariance between parameters $\sigma$ and $\xi$ positive value (i.e. $2.729351 \times 10^{-3}$), so the two move in the same direction. While the covariance between parameters $\mu$ and $\xi$ negative (i.e. $-2.379158 \times 10^{-4}$), means both of them move in opposite directions. Using the same package in the R program, also obtained the standard error of the parameter $(\mu, \sigma, \xi)$ each is $0.003714981$, $0.015392313$, and $0.227279992$.

Using the same package, a plot is obtained to estimate the accuracy of the GEV model presented in Figure 2.

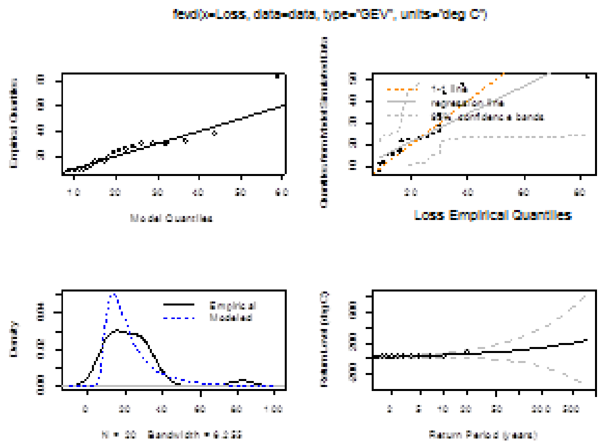


**Figure 2. Plot Accuracy of the GEV distribution model**

Graph Q-Q plot is a method for assessing the data analyzed from which distribution. How to judge by analyzing whether the Q-Q plot is around the line $y = x$ or not. If the plot is around the line $y = x$, then the analyzed data comes from a distribution, for example D. distribution. Conversely, if the data spreads away from the line $y = x$, then the data does not come from distribution D.

In Figure 2, it shows that each set of points in the Q-Q plot between the quantiles model and the empirical quantiles approaches the line $y = x$ with a confidence level of 95%. The plot of the density between empirical and modeled corresponds, and in the return level plot each set of points gathers around a straight line. From Figure 2, the model used is valid and the data analyzed is true from the GEV distribution. Therefore, the GEV distribution model matches the data analyzed.

**C.** *Calculating the price of catastrophe bonds*

Before calculating CAT bond prices, first make a probability interval that is exceeded for the GEV distribution using equation (1) and the data analyzed, the probability intervals generated are presented in Table 2.

**Table 2. GEV distribution probability interval from data**

| Opportunity Intervals ($\alpha_1, \ldots, \alpha_5$) | Large Probability ($I$) |
|---|---|
| Pr(8.99<$I$<23.99) | 0.6124878 |
| Pr(23.99<$I$<38.99) | 0.2187449 |
| Pr(38.99<$I$<53.99) | 0.0639400 |
| Pr(53.99<$I$<68.99) | 0.0235010 |
| Pr($I$>68.99) | 0.9762720 |

Furthermore, to estimate CAT bond prices using the one period model, assume the principal ($F$) is USD100.00, with a coupon rate ($R$) of 9%, and a risk free rate ($r$) of 5%. Because this model is for one period, then the time period t is 1. Then, to calculate CAT bonds is done using equations (4) and (5). Using the help of MS Excel software. The results obtained at 98.63. That is, investors pay USD98.63 to get USD109.00 at maturity if natural disasters do not occur. If a natural disaster occurs, the principal and coupon are forfeited. The greater the value of the coupon obtained, the more expensive the price.

## V. CONCLUSION

This paper has discussed the risk neutral measurement to determine the CAT bond prices. The results of the analysis can conclude as follows: Estimation of parameters for GEV distribution with the MLE method is done by using the *extRemes* package in software R. Significant estimation results for each parameter are obtained, namely: location parameters ($\mu$) as big as 15.8892609, scale parameter ($\sigma$) as big as 7.6786421, and form parameters ($\xi$) as big as 0.3029915. The Q-Q plot results show that the GEV distribution matches the data analyzed. After the parameters are obtained, each parameter is used to get the probability of the data interval. Obtained opportunities for interval classes [8.99; 23.99] as big as 0.6124878, for [23.99; 38.99] as big as 0.2187449, [38.99; 53.99] as big as 0.06394, for [53.99; 68.99] as big as 0.023501, and interval [68.99; ∞] as big as 0.976272. These opportunities are then used to calculate the cash value for CAT bonds. The results of the cash value calculation then used determine the value of CAT bond prices for today and in the future. The CAT bond price obtained is USD98.63, where the investor pays to get the principal that has already been added the coupon at maturity. The influence of the risk neutral measurement is seen, namely the CAT bond prices obtained are cheaper than the principal. The principal that has been added to the coupon can be received by the investor if there is no natural disaster during the investment period. If a natural disaster occurs, the principal and coupon are forfeited.

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