Magnetohydro Dynamic Steady Flow Between Two Parallel Porous Plates of a Viscous Fluid Under Angular Velocity with Inclined Magnetic Field

R. Delhi Babu, S. Ganesh, V. Yuvaraj

***Abstract**: The Model is made as the Steady Magnetohydro dynamic streams with an exact speed between parallel penetrable plates are considered. The issue is seen methodically by using comparability change, whose game plan oversees growing fluid stream with a dashing velocity. The Major Applications of Magnetohydro dynamic (MHD) are the controller of generators, the system containing Cooling and thermal structures, improvement of polymer, Fuel industries etc. The objective of this paper is to look at the Steady Magnetohydro dynamic stream of thick fluid with a saucy speed between parallel porous plates when the fluid forced to their back position by the way of the dividers of each partition at a comparative rate. The issue is decreased to a third solicitation direct differential condition which depends upon a Suction Reynolds number R and M1 for which a right course of action is gotten.*

***Keywords* :** Magnetohydrodynamic flow, fluid flow, parallel plates, angular velocity.

I. INTRODUCTION

Magneto hydro elements are the examination of the association connecting alluring fields and motion fluids. The effect of MHD and Hall current on gooey streams has unprecedented vitality for real time Engineering and related fields. Accordingly, this concept presents the practice of Engineering concepts in early 1960’s. In astrophysical fluid and geophysical components many comparable and relevance wide region alluring field are implemented in electrically driving concept and the surge of a fluid. MHD accept colossal employment in many domains for instance, sun-based material science, sun-controlled cycle and turning alluring stars.

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Using rectangular channel, the weight inclined viscoelastic Maxwell fluid with issue of precarious stream Bagchi [1]. Attia and Kotb [2] the temperature dependent thickness between two parallel plates by the concept of MHD stream and Warmth trade.Attia [3] cleared the transient state issues in MHD. Ezzat, Othman and Helmy [4] Micropolar Magnetohydrodynamics highlighted in issues of breaking point the stream layers. Aboul-Hassan and Attia [5] concentrated on the progression of transverse appealing field between the penetrable plates at two levels progression of the main viscoelastic fluid. Nabil, Eldabe, Galal, Moatimid and HodaSm Ali [6] experimented the visco-adaptablefluid of Non-Newtonian MHD stream of animated plate by orous medium.Attia [7] determined the viscoelastic fluid of Precarious Hartmann Stream with the Corridor sway. S. Krishnambal and S.Ganesh [8] researched the Temperamental blends streams in-between two parallel and penetrable plates whose fluid considered as thick fluid.R. Delhi Babu and S. Ganesh [9] given the rakish speed and their impact in magnetohydrodynamic steam experiment in parallel penetrable plates. R. Delhi Babu and S. Ganesh [10] highlighted the angular velocity of the Magneto HF in unsteady manner in a platform of porous plates in parallel view.

II. MATHEMATICAL FORMULATION OF THE MODEL

The estimation of the Crossway attractive field into dividers in the vertical direction applying the steady laminar progression in a liquid as incompressible gooey in main interface platform of Permeable plates which are aligned as parallel plates. In the beginning the channels are analysed and verified by considering the two major axes named as parallel and inverse axis for the two divider channels for tomahawks simulation.

L named as the Channel length and the distance between the two plates in parallel conditions are given as 2h. The velocity segment in the x direction named as u and in the y direction the velocity is named as v, Ω is the rakish speed.

The equation of continuity is $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$ (1)

Equations of momentum are

$$\rho\frac{\partial u}{\partial t} = -\frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right) + 2\Omega u - \sigma_e B_0{}^2 u \sin^2\alpha - \frac{\mu u}{k} \quad (2)$$

$$\rho\frac{\partial v}{\partial t} = -\frac{\partial p}{\partial y} + \mu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right) - 2\Omega v \quad (3)$$

Angular velocity is denoted as Ω and the electrical conductivity is mentioned as σ.B_0 represents induction of Electromagnetic.B_0 Defined as $\mu_e H_0$.

The magnetic permeability is denoted by μ_e and also the transverse of the magnetic field is denoted by H_0

Under Steady State conditions, $\frac{\partial u}{\partial t}= 0$ and $\frac{\partial v}{\partial t}= 0$, equation (2) and (3) are simplified as

$$0 = -\frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right) + 2\Omega u - \sigma_e B_0{}^2 u \sin^2\alpha - \frac{\mu u}{k} \quad (4)$$

$$0 = -\frac{\partial p}{\partial y} + \mu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right) - 2\Omega v \quad (5)$$

The values of y take h and -h in the initial conditions and at the same time v takes the value of v_0

The Speed in the wall channels due to suction is v_0

Considering $\eta = \frac{y}{h}$, u[x,y] =u, v[x,y]=v, p[x,y]= p and frequency is denoted by ω.

After the simplification of (4) and (5) using equation (1) then the equations (1), (4) and (5) converted

$$\frac{\partial u}{\partial x}+\frac{1}{h}\frac{\partial v}{\partial \eta}=0 \quad (6)$$

$$-\frac{\partial p}{\partial x}+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{1}{h^2}\frac{\partial^2 u}{\partial \eta^2}\right)+2\Omega u-\sigma_e B_0^2 u\sin^2\alpha-\frac{\mu u}{k}=0 \quad (7)$$

$$0=-\frac{\partial p}{\partial y}+\mu\left(\frac{\partial^2 v}{\partial x^2}+\frac{1}{h^2}\frac{\partial^2 v}{\partial \eta^2}\right)-2\Omega v \quad (8)$$

Let $v = \frac{\mu}{\rho}$ denotes the kinematic viscosity, μ is the coefficient of viscosity, Pressure is named as p and ρ represents the fluid density.

Simplified conditions based on boundary defined are

$$u[x,1] = 0 \text{ and } u[x,-1] = 0 \quad (9)$$

$$v[x,-1] = 0 \text{ and } v[x,1] = 0 \quad (10)$$

ψ is the function established the stream equation with the continuity concept (1) given as

$$u = \frac{1}{h}\frac{\partial\psi}{\partial\eta} \quad (11)$$

$$v = -\frac{\partial\psi}{\partial x} \quad (12)$$

$$f(\eta)(hU(0) - v_0 x) = \psi[x,\eta] \quad (13)$$

The entrance velocity's average represented by U(0) when x=0.

Considering the equations (12) and (11) in the equation (13)

$$\frac{f'(\eta)}{h}(U(0)(h) - v_0 x) = u \quad (14)$$

$$v = f(\eta) v_0 \quad (15)$$

While considering dimensionless variable which gives the common difference of the prime and is denoted by $\eta = \frac{y}{h}$. The fixed rate of flow of fluid is withdrawn from the two walls, then so v_0 is not a function of x.

In the equation (8) and (7) is simplified using the equations (14) and (15)

$$\left(U[0]\frac{v_0 x}{h}\right)\cdot\left(-\frac{v}{h^2}f'''(\eta) + \left(\frac{\sigma_e B_0^2\sin^2\alpha}{\rho}+\frac{\mu}{k\rho}-\frac{2\Omega}{\rho}\right)f'(\eta)\right) = -\frac{1}{\rho}\frac{\partial p}{\partial x} \quad (16)$$

$$-\frac{1}{h\rho}\frac{\partial p}{\partial\eta} = -\frac{v\cdot v_0}{h^2}f''(\eta) + \frac{2\Omega v_0}{\rho}f(\eta) \quad (17)$$

(17) equation, differentiated with respect to x

$$\frac{\partial^2 p}{\partial\eta\partial x} = 0 \quad (18)$$

(16) equation, differentiated with respect to 'η' which gives

$$\frac{-1}{\rho}\frac{\partial^2 p}{\partial x\partial\eta} = \left(U(0) - \frac{v_0 x}{h}\right)\cdot\frac{d}{d\eta}\left(-\frac{v}{h^2}f'''(\eta) + \left(\frac{\sigma_e B_0^2\sin^2\alpha}{\rho} + \frac{\mu}{k\rho} - \frac{2\Omega}{\rho}\right)f'(\eta)\right) \quad (19)$$

from equations (18) and (19) we get,

$$\frac{d}{d\eta}\left(-\frac{v}{h^2}f'''(\eta) + \left(\frac{\sigma_e B_0^2\sin^2\alpha}{\rho}+\frac{\mu}{k\rho}-\frac{2\Omega}{\rho}\right)f'(\eta)\right) = 0 \quad (20)$$

Let Suction Reynolds number (R) = $\frac{hv_0}{v}$

and $M_1 = B_0 h\sin\alpha\left(\frac{\sigma_e}{v\rho}+\frac{\mu}{kB_0{}^2 v\rho}-\frac{2\Omega}{B_0{}^2 v\rho}\right)^{\frac{1}{2}}$

(20) equation, integrated with respect to η, simplified equation is determined as

$$f'''(\eta) - a_1 R f'(\eta) = C \quad (21)$$

Considering, $R = \frac{hv_0}{v}$, $a_1 = \frac{\sigma_e h B_0^2\sin^2\alpha}{\rho v_0}+\frac{\mu h}{k\rho v_0}-\frac{2\Omega h}{\rho v_0}$ constant which is arbitrary is K.

f(η) conditions which defined using boundary are

$$f'(-1) = f'(1) = 0 \text{ and } f(-1) = f(1) = 1 \quad (22)$$

Linear third order differential equations give the solution of the continuity and equation of the motion (21) with respect to the conditions defined using the boundary (22)

∴The simplified equation (21) given as

$$C = -M_1{}^2 f'(\eta) + f'''(\eta) a_{1R=} M_1{}^2$$

$$(D^2 - M_1{}^2)(D)(f(\eta)) = C \text{ where } D = \frac{d}{d\eta} \,\&\, D^2 = \frac{d^2}{d\eta^2}$$

$$C_1 + C_2 e^{M_1\eta} + C_3 e^{-M_1\eta} - \frac{C\eta}{M_1{}^2} = f(\eta) \quad (23)$$

Using the boundary conditions,

$$f(\eta) = \frac{1}{(1 - M_1 CothM_1)}\cdot\left(\frac{SinhM_1\eta}{SinhM_1} - \eta M_1 CothM_1\right) \quad (24)$$

So, the velocity of their components figured as

$$u = \frac{f'(\eta)}{h}[hU(0) - v_0 x]$$

$$u = \left(U(0) - \frac{v_0 x}{h}\right)\cdot\left(\frac{M_1}{(1 - M_1 CothM_1)}\right)\cdot$$

$$\left(\frac{Cosh\, M_1\eta}{Sinh M_1} - Coth M_1\right) \quad (25)$$

Since v =v_0f(η)

$$v = v_0\left(\frac{1}{1-M_1\, Coth\, M_1}\right)\cdot\left(\frac{Sinh\, M_1\eta}{Sinh\, M_1} - \eta M_1 Coth M_1\right)$$

(26).

A. Distribution of the Pressure

From the (16) equation if simplified as,

$$-\frac{1}{\rho}\frac{\partial p}{\partial x} = \left(U[0] - \frac{v_0 x}{h}\right)\cdot$$

$$\left(-\frac{v}{h^2}f'''(\eta) + \left(\frac{\sigma_e B_0^2 \sin^2\alpha}{\rho} + \frac{\mu}{k\rho} - \frac{2\Omega}{\rho}\right)f'(\eta)\right)$$

Multiplying by Dividing by $-\frac{h^2}{v}$, it gives the equation as,

$$\frac{(h)^2}{(\rho v)}\frac{\partial p}{\partial x} = \left(U[0] - \frac{v_0 x}{h}\right)$$

$$\cdot\left(f'''(\eta) - \frac{h^2}{v}\left(\frac{\sigma_e B_0^2 \sin^2\alpha}{\rho} + \frac{\mu}{k\rho} - \frac{2\Omega}{\rho}\right)f'(\eta)\right)$$

since $f'''(\eta) - a_1 R f'(\eta) = C$[from equation (21)]

$$f'''(\eta) - {M_1}^2 f'(\eta) = C$$

$$\frac{h^2}{\rho v}\frac{\partial p}{\partial x} = C\left(U(0) - \frac{v_0 x}{h}\right)(from(21))$$

$$\therefore \frac{\partial p}{\partial x} = \frac{C\rho v}{h^2}\left(U(0) - \frac{v_0 x}{h}\right) \quad (27)$$

from equation (17) we get,

$$-\frac{1}{h\rho}\frac{\partial p}{\partial \eta} = -\frac{v v_0}{h^2}f''(\eta)$$

$$\frac{\partial p}{\partial \eta} = \frac{v\rho v_0}{h}f''(\eta) \quad (28)$$

Since $dp = dx\frac{\partial p}{\partial x} + dy\frac{\partial p}{\partial y}$

$$= dx\frac{\partial p}{\partial x} + d\eta\frac{\partial p}{\partial \eta} \because \left(\eta = \frac{y}{h}\right)$$

$$\Rightarrow dp = \frac{Cv\rho}{h^2}\left(U(0) - \frac{v_0 x}{h}\right)dx + \left(\frac{v\rho v_0}{h}f''(\eta)\right)d\eta$$

(29)

(42) equation is integrated, the simplified version is

$$p(x,\eta) = \frac{Cv\rho}{h^2}\left(U(0)x - \frac{v_0 x^2}{2h}\right) + \left(\frac{v\rho v_0}{h}f'(\eta)d\eta\right) + K_1$$

(30)

∴ Pressure drop is

$$\Rightarrow p(x,\eta) - p(0,0) = \frac{Cv\rho}{h^2}\left(U(0)x - \frac{v_0 x^2}{2h}\right) + \frac{v\rho v_0}{h}\left(f'(\eta) - f(0)\right)$$

(31)

III. GRAPHICAL REPRESENTATION OF MODEL

The graphs of the axial velocity and radial velocity profiles have been drawn for different values of M_1. Figures 2.1, 2.2, 2.3 represents the axial velocity profiles at different values of M_1 namely M_1=1, M_1=2, and M_1=5 when the average entrance velocity is u_o = 0.5 and h = 1.0, x = -3 to 3. Figures 2.4, 2.5, 2.6 represents the axial velocity profiles at different values of M_1 namely M_1=1, M_1=2, and M_1=5 when the average entrance velocity u_o is increased from u_o = 0.5 to u_o = 1.0 and h = 1.0, x = -3 to 3. From the figures 2.1 to 2.6, it is observed that the magnitudes of the axial velocity u increase as x varies from -3 to 0 and decreases for x varying from 2 to 3. For x = 1, the axial velocity profile is found to be linear. The Figures 2.7 represent the radial velocity profiles of v at v_0 =0.5, h=1.0, α=1.0 and for different values of M_1 namely M_1=1, M_1=2, and M_1=5. As M_1 increases from M_1=1 to M_1=5, it is seen that there is a marginal increase in the magnitude of the radial velocity.

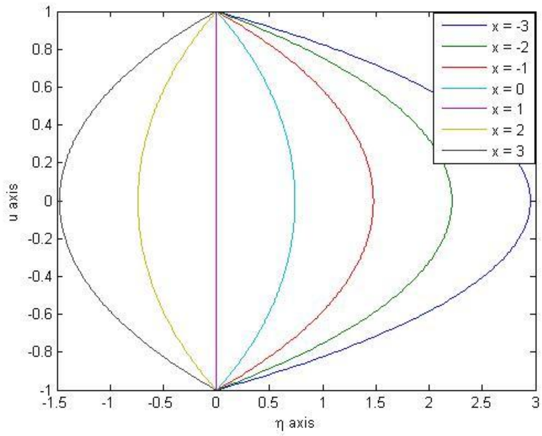

Fig. 2.1.Axial velocity profiles for x=-3 to 3 increase by 1 as u_0 = 0.5, v_0 = 0.5, α = 1.0, h = 1.0, M_1 = 1

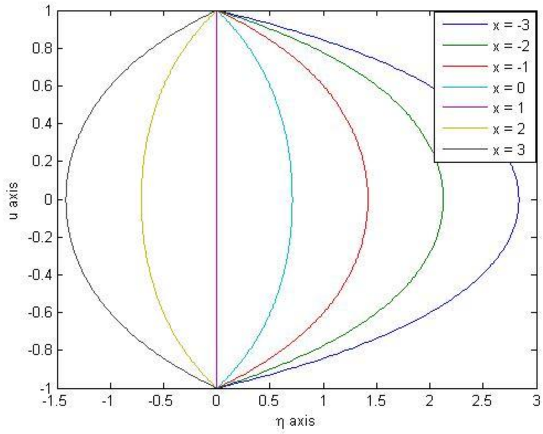

Fig. 2.2.Axial velocity profiles for x = -3 to 3 increase by 1 as u_0 = 0.5, v_0 = 0.5, α = 1.0, h = 1.0, M_1 = 2

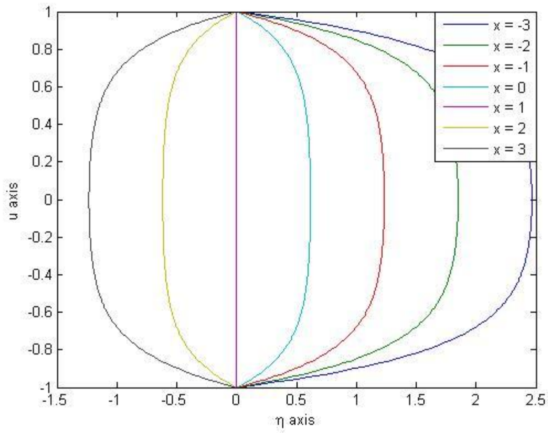

Fig. 2.3.Axial velocity profiles for x = -3 to 3 increase by 1 as u_0 = 0.5, v_0 = 0.5, α = 1.0, h = 1.0, M_1 = 5

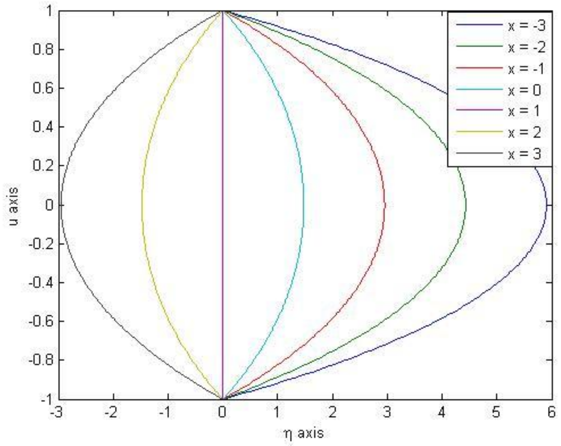

Fig. 2.4.Axial velocity profiles for x = -3 to 3 increase by 1 as $u_0 = 1$, $v_0 = 1$, $\alpha = 1.0$, $h = 1.0$, $M_1 = 1$

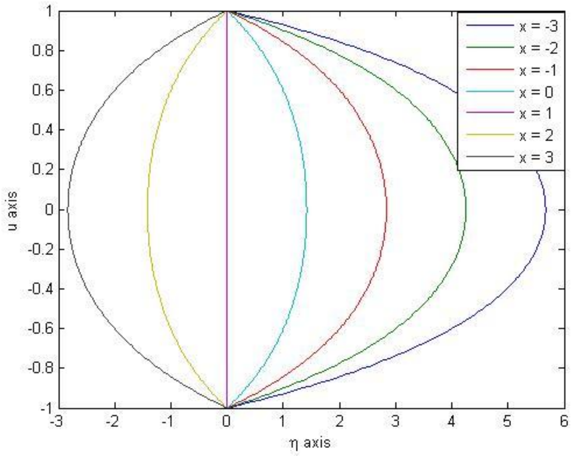

Fig. 2.5.Axial velocity profiles for x = -3 to 3 increase by 1 as $u_0 = 1$, $v_0 = 1$, $\alpha = 1.0$, $h = 1.0$, $M_1 = 2$

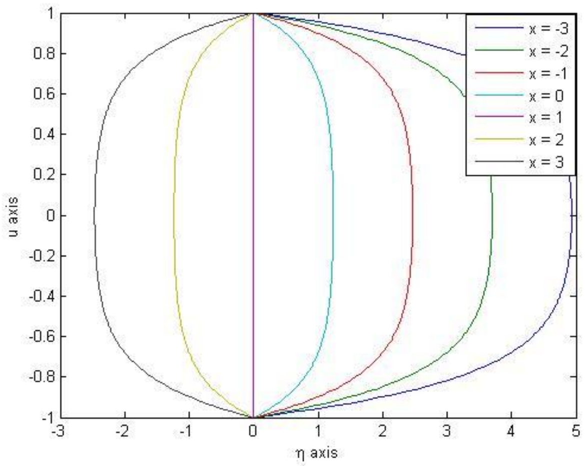

Fig. 2.6.Axial velocity profiles for x = -3 to 3 increase by 1 as $u_0 = 1$, $v_0 = 1$, $\alpha = 1.0$, $h = 1.0$, $M_1 = 5$

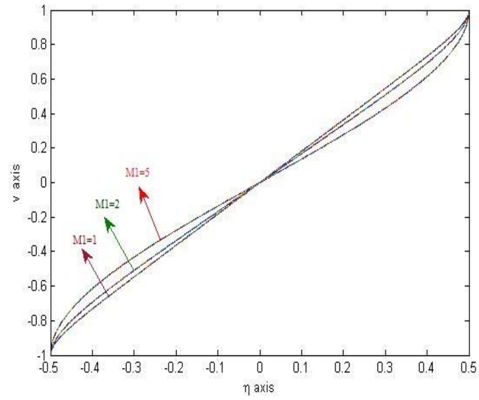

Figure 2.7.Radial velocity profiles for M_1=1, M_1=2, M_1=5

IV. CONCLUSION

The velocity between the permeable plates aligned as parallel porous plates under angular velocity with inclined magnetic field are identified through the thick liquid stream implemented using the steady Magnetohydrodynamic. This work identifies the fluids are pulled back opposite to their channel dividers by the parallel alignment in the equivalent rate in the attractive filed. This article establishes the alternative M_1 and for the suction Reynolds number R.

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