An Examination in IFS and IVIFS based on Various Distance Measure
A. Manonmani, M. Suganya

Abstract: There are many distance measure in Fuzzy such as Hamming, Normalized Hamming, Euclidean, Normalized Euclidean, Geometric, Normalized Geometric, Hausdorff’s etc… we discussed, the comparison between IFS and IVIFS based on various distance measure is explored. IFS is converted into IVIFS in various steps. The distance between IFS and IVIFS are also compared.

Index Terms: Hamming distance measure, Normalized Hamming distance measure, Euclidean distance measure, Normalized Euclidean distance measure, Geometric distance measure, Normalized Geometric distance measure, Hausdorff’s distance measure.

I. INTRODUCTION
A fuzzy set is a pair where is a set and a membership function. Fuzzy set is a mathematical model of vague qualitative or quantitative data, frequently generated by means of the natural language. There are many types of fuzzy sets. Particularly here we have taken Intuitionistic fuzzy sets. Intuitionistic fuzzy sets are sets which are represented by its membership and non-membership function. And also here we have converted Intuitionistic fuzzy sets into Interval Valued Intuitionistic fuzzy sets. Since Interval Valued Intuitionistic fuzzy sets is also represented by its membership and non-membership function which lies within an interval. After converting IFS into IVIFS both the sets are compared using various distance measure like Hamming, Normalized Hamming, Euclidean, Normalized Euclidean, Hausdorff, Geometric, Normalized Geometric distance measures.

A Numerical example is also given to explain the above mentioned concept and the distance measures are ranked separately both for IFS and IVIFS and the best distance measure and best fuzzy set among the IFS and IVIFS is detected.

In Chapter 2 we have discussed the different types of distance measure both in IFS and IVIFS. Chapter 3 explains the various steps in conversion of IFS into IVIFS. In Chapter 4 an numerical example is proposed in which IFS and IVIFS is compared based on various distance measures. The Concision is explained in chapter 5.

II. DISTANCE MEASURES IN FUZZY SETS
2.1 Distance measure formulae in IFS:
2.1.1 The Hamming distance measure

\[ d_H (A, B) = \frac{1}{6n} \sum_{i=1}^{n} \left[ |u_A(x_i) - u_B(x_i)| + |v_A(x_i) - v_B(x_i)| \right] \]

2.1.2 The normalized Hamming distance measure

\[ d_{nH} (A, B) = \frac{1}{6n} \sum_{i=1}^{n} \left[ \frac{|u_A(x_i) - u_B(x_i)| + |v_A(x_i) - v_B(x_i)|}{u_A(x_i) + v_A(x_i) + u_B(x_i) + v_B(x_i)} \right] \]

2.1.3 The Euclidean distance measure

\[ d_e (A, B) = \sqrt{\frac{1}{6} \sum_{i=1}^{n} \left[ (u_A(x_i) - u_B(x_i))^2 + (v_A(x_i) - v_B(x_i))^2 \right]} \]

2.1.4 The normalized Euclidean distance measure

\[ d_{ne} (A, B) = \sqrt{\frac{1}{6n} \sum_{i=1}^{n} \left[ \frac{(u_A(x_i) - u_B(x_i))^2 + (v_A(x_i) - v_B(x_i))^2}{u_A(x_i) + v_A(x_i) + u_B(x_i) + v_B(x_i)} \right]} \]

2.1.5 The Geometric distance measure

\[ d_g (A, B) = \sqrt{\frac{1}{6} \sum_{i=1}^{n} \left[ \frac{(u_A(x_i) - u_B(x_i))^2 + (v_A(x_i) - v_B(x_i))^2}{u_A(x_i) + v_A(x_i) + u_B(x_i) + v_B(x_i)} \right]} \]

2.1.6 The normalized Geometric distance measure

\[ d_{ng} (A, B) = \sqrt{\frac{1}{6n} \sum_{i=1}^{n} \left[ \frac{(u_A(x_i) - u_B(x_i))^2 + (v_A(x_i) - v_B(x_i))^2}{u_A(x_i) + v_A(x_i) + u_B(x_i) + v_B(x_i)} \right]} \]

2.1.7 The Hausdorff distance measure

\[ d_H (A, B) = \frac{1}{2} \sum_{j=1}^{n} \left[ \max \left\{ \left| u_A(x_j) - u_B(x_j) \right|, \left| v_A(x_j) - v_B(x_j) \right| \right\} \right] \]

2.2 Distance measure formulae in IVIFS:

2.2.1 The Hamming distance measure

\[ d_H (A, B) = \frac{1}{6n} \sum_{i=1}^{n} \left[ |u_A^1(x_j) - u_B^1(x_j)| + \sum_{j=1}^{m} \left| p_A^1(x_j) - p_B^1(x_j) \right| + |p_A^2(x_j) - p_B^2(x_j)| \right] \]

2.2.2 The normalized Hamming distance measure

\[ d_{nH} (A, B) = \frac{1}{6n} \sum_{i=1}^{n} \left[ \frac{|u_A^1(x_j) - u_B^1(x_j)| + \sum_{j=1}^{m} \left| p_A^1(x_j) - p_B^1(x_j) \right| + |p_A^2(x_j) - p_B^2(x_j)|}{u_A^1(x_j) + v_A^1(x_j) + u_B^1(x_j) + v_B^1(x_j)} \right] \]

2.2.3 The Euclidean distance measure

\[ d_e (A, B) = \sqrt{\frac{1}{6} \sum_{i=1}^{n} \left[ (u_A^1(x_j) - u_B^1(x_j))^2 + (v_A^1(x_j) - v_B^1(x_j))^2 \right]} \]

2.2.4 The normalized Euclidean distance measure

\[ d_{ne} (A, B) = \sqrt{\frac{1}{6n} \sum_{i=1}^{n} \left[ \frac{(u_A^1(x_j) - u_B^1(x_j))^2 + (v_A^1(x_j) - v_B^1(x_j))^2}{u_A^1(x_j) + v_A^1(x_j) + u_B^1(x_j) + v_B^1(x_j)} \right]} \]
2.2.7 The Hausdorff distance measure

\[
d_{H}(A,B) = \frac{1}{2} \sum_{i=1}^{n} \max \left\{ u_{i}^{+}(x_{i}) - u_{i}^{-}(x_{i}), v_{i}^{+}(x_{i}) - v_{i}^{-}(x_{i}) \right\} \]

2.2.6 The normalized Geometric distance measure

\[
d_{g}(A,B) = \frac{1}{4} \sum_{i=1}^{n} \max \left\{ u_{i}^{+}(x_{i}) - u_{i}^{-}(x_{i}), v_{i}^{+}(x_{i}) - v_{i}^{-}(x_{i}) \right\} \]

2.2.5 The Geometric distance measure

\[
d_{g}(A,B) = \frac{1}{6} \sum_{i=1}^{n} \left[ (u_{i}^{+}(x_{i}) - u_{i}^{-}(x_{i}))^{2} + (p_{i}^{+}(x_{i}) - p_{i}^{-}(x_{i}))^{2} \right] +
\left[ (v_{i}^{+}(x_{i}) - v_{i}^{-}(x_{i}))^{2} + (u_{i}^{+}(x_{i}) - u_{i}^{-}(x_{i}))^{2} \right] +
\left[ (p_{i}^{+}(x_{i}) - p_{i}^{-}(x_{i}))^{2} + (v_{i}^{+}(x_{i}) - v_{i}^{-}(x_{i}))^{2} \right] \]

III. CONVERSION OF INTUITIONISTIC FUZZY SETS INTO INTERVAL-VALUED INTUITIONISTIC FUZZY SETS

Step 1: Identify the values of membership, non-membership and hesitation of the IFS.
Step 2: The degree of membership function of IVIFS are calculated using the following formulæ.
\[ \mu_{AI}(x) = \mu_{A} - \alpha_{A} \pi_{A}(x) \]
\[ \mu_{AI}(x) = \mu_{A} - \alpha_{A} \pi_{A}(x) \]
Step 3: The degree of non-membership function of IVIFS are calculated using the following formulæ.
\[ \nu_{AI}(x) = \nu_{A} - \beta_{A} \pi_{A}(x) \]
\[ \nu_{AI}(x) = \nu_{A} - \beta_{A} \pi_{A}(x) \]
Step 4: Finally the IVIFS is arranged as follows

\[ A = \{ x_{i}, [\mu_{AI}(x), \mu_{AI}(x), [\nu_{AI}(x), \nu_{AI}(x), \pi_{A}(x)] \} \} \]

IV. NUMERICAL EXAMPLES

Let consider two IFSs A and B in X as follows:
\[ A = \{ x_{i}, 0.5, 0.2, 0.3 \}, \{ x_{i}, 0.6, 0.2, 0.4 \}, \{ x_{i}, 0.9, 0.0, 0.1 \} \]
4.1 Hamming Distance Measure:
\[ d_{H}(A,B) = \frac{1}{6} \sum_{i=1}^{n} \left[ \mu_{AI}(x_{i}) - \mu_{B}(x_{i}) \right] +
\left[ \nu_{AI}(x_{i}) - \nu_{B}(x_{i}) \right] \]
\[ d_{H}(A,B) = \frac{1}{6} \sum_{i=1}^{n} \left[ 0.2 + 0.2 + 0.4 + 0.2 + 0.4 + 0.2 \right] \]
4.10 Comparative Pictorial Representation between IFS and IVIFS

The distance measure in IFS and IVIFS are listed in the table shown below:

<table>
<thead>
<tr>
<th>Distance Measures</th>
<th>IFS</th>
<th>IVIFS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hamming Distance Measure</td>
<td>0.2333</td>
<td>0.4500</td>
</tr>
<tr>
<td>Normalized Hamming Distance Measure</td>
<td>0.1167</td>
<td>0.2250</td>
</tr>
<tr>
<td>Euclidean Distance Measure</td>
<td>0.2499</td>
<td>0.3317</td>
</tr>
<tr>
<td>Normalized Euclidean Distance Measure</td>
<td>0.1732</td>
<td>0.2466</td>
</tr>
<tr>
<td>Geometric Distance Measure</td>
<td>0.3826</td>
<td>0.4891</td>
</tr>
<tr>
<td>Normalized Geometric Distance Measure</td>
<td>0.0319</td>
<td>0.0480</td>
</tr>
<tr>
<td>Hausdorff’s Distance Measure</td>
<td>0.2383</td>
<td>0.6750</td>
</tr>
</tbody>
</table>

4.8 The table format of A and B according to various distance measure

4.9 Pictorial Representation of the above mentioned various distance measures both in IFS and IVIFS

4.10 Comparative Pictorial Representation between IFS and IVIFS based on above mentioned various distance measures.
V. CONCLUSIONS

Here distance measure is used to find the distance between two fuzzy sets namely A and B both in IFS and IVIFS. These difference in distance is listed in the table and also their pictorial representations are indicated separately. On analyzing the Comparative Pictorial Representation between IFS and IVIFS based on above mentioned various distance measures we conclude that Hausdoff distance measure gives the highest distance between A and B but the distance differs a lot between IFS and IVIFS.

Next to the Hausdoff distance measure Geometric distance measure gives the next highest distance between A and B and also there is only slight difference in distance between IFS and IVIFS. We have ranked the distance measure based on Comparative Pictorial Representation between IFS and IVIFS.

Since in Geometric distance measure there is only slight difference in distance between IFS and IVIFS and also it gives highest distance between A and B we conclude that Geometric distance measure is one of the best distance measure. We also conclude that the difference in distance between IFS and IVIFS is due to the boundary points in IVIFS.

REFERENCES

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