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Support Neighbourly Edge Irregular Graphs

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***Abstract*: In this paper, we introduce a new family of irregular graphs called support neighbourly edge irregular graphs based on degree in edge sense. In any graph, the support of an edge is the sum of the edge degrees of its neighbours. A graph is said to be support neighbourly edge irregular(or simply SNEI), if any two adjacent edges have different support. Basic properties of theses graphs are studied. A necessary and sufficient condition for a graph to be SNEI has been obtained and several methods to construct SNEI graph from other graphs have been discussed.**

Key words: Neighbourly irregular graphs, Neighbourly edge irregular fuzzy graphs, support neighbourly irregular graphs, Support neighbourly edge irregular graphs.

AMS subject classification :Primary: 05C12, Secondary: 03E72, 05C72.

I. INTRODUCTION

Throughout this paper we consider finite, simple connected graphs. Let G be a graph with n vertices and m edges. The vertex set and edge set of Gare denoted by V (G) and E(G) respectively. The degree of a vertex v ∈ V (G) is the number of vertices adjacent to v and is denoted by dG(v) or simply d(v).

The concept of Neighbourly irregular graphs was introduced and studied by S. GnaanaBhragsam and S.K.Ayyaswamy[2]. N.R. SanthiMaheswari and C.Sekar introduced the concept of Neighbourly edge irregular fuzzy graphs[9]. The concept of support of a vertex and and support neighbourly irregular graphs has been introduced and studied by Selvam Avadayappan, M.Bhuvaneshwari and R.Sinthu[11]. The degree of an edge e = (u,v) as the number of edges which have a common vertex with the edge e.(i.e)deg(e) = deg(u) + deg(v) − 2[5]. The distance between two edges e1 = (u1,v1) and e2 = (u2,v2) is defined as ed(e1,e2)=0 min{d(u1,u2),d(u1,v2),d(v1,u2),d(v1,v2)}. If ed(e1,e2) = 0, these edges are neighbour edges[4]. The purpose of this paper is to introduce a new family of irregular graphs based on distance property in edge sense.

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This is the background to introduce support neighbourly edge irregular graphs and we have discussed some of its properties.

II. PRELIMINARIES

We present some known definitions and results for ready reference to go through the work presented in the paper.

Definition 2.1. A graph G is said to be neighbourly irregular if no two adjacent vertices of G have the same degree.

Definition 2.2. Let G : (σ,μ) be a connected fuzzy graph on G∗(V,E). Then G is said to be neighbourly edge irregular fuzzy graph if every pair of adjacent edges having distinct degrees.

Definition 2.3. The support sG(v) or simply s(v) of a vertex v is the sum of degrees of its neighbours. That is, s(v) = Pu∈N(v) d(u).

Definition 2.4. *A* connected graph is said to be support neighbourly irregular (or simply SNI), if no two vertices having same support are adjacent.

Definition 2.5. Let be G be a graph. For any two distinct vertices u and v in G, u is pairable with v if N[u] = N[v] in G. A vertex in G is called a pairable vertex if it is pairable with a vertex in G.

Definition 2.6. Let G be a graph. A full vertex of G is a vertex in G which is adjacent to all other vertices of G.

Definition 2.7. A simple graph G(V,E) is neighbourly edge irregular if no two adjacent edges of G have the same edge degree.

III. REVIEW CRITERIA

In this section, we introduce Support Neighbourly edge irregular graphs and study some properties of these graphs.

Definition 3.1. *The support* $s_G(e)$ *or simply* $s(e)$ *of an edge e is the sum of edge degrees of its neighbour edges. That is,* $s(e) = P_{e_i \in N(e)} d(e_i)$.

Definition 3.2. *A simple graph G(V,E) is Support Neighbourly edge irregular graph(or simply SNEI) if no two edges of G having same support are adjacent. A graph H proving the existence of SNEI graphs is shown in Figure* 1.

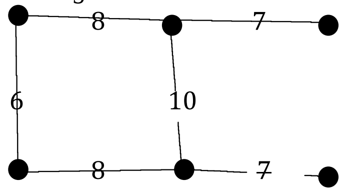

Figure 1

Fact 3.3. *Regular graphs, edge regular graphs, pairable graphs and paths* ($P_n, n \neq 6$) *are not SNEI graphs.*
Fact 3.4. *Not all Support neighbourly edge irregular graphs are NEI graphs. Not all NEI graphs are SNEI. For example,* P_3 *is NEI but not SNEI and* P_6 *is SNEI but not NEI.*
Fact 3.5. *Let G be a SNEI graph. Then for any two vertices u and v in V (G),* $N(u) \neq N(v)$.
Fact 3.6. *Any graph with more than one full vertex is not a SNEI graph.*
Fact 3.7. *If G is a SNEI graph, then the set of all extreme vertices is independent in G.* **Fact 3.8.** *If G is a SNEI graph, there is no* P_4 *such that external vertices of degree* 1 *and one internal vertex of degree* 2*, For if there is a* P_4 *(say uvwx) such that* $d(u) = d(x) = 1$ *and* $d(w) = 2$*, then* $s(uv) = s(vw)$.
Result 3.9. *If G is a SNEI graph, then there is no* P_5 *with external vertices of same degree and internal vertices of degree* 2.
Note 3.10. *For any edge* $e \in E(G)$*, support of* $e=s(e)=s(uv)=d(u)^2 + d(v)^2 + s(u) + s(v) - 4d(u) - 4d(v) + 4$.
The following theorem proves a necessary and sufficient condition for a graph to be SNEI graph.
Theorem 3.11. *A graph G is a SNEI graph if and only if for any two adjacent edges uv and vw , then* $d(u)^2 - d(w)^2 - 4(d(u) - d(w)) \neq s(w) - s(u)$.
Proof. Let *G* be a SNEI graph. Then no two adjacent edges have same support. If possible $d(u)^2 - d(w)^2 - 4(d(u) - d(w)) = s(w) - s(u)$ for some adjacent edges *uv*and *vw*, then $d(u)^2 - 4d(u) + s(u) = d(w)^2 - 4d(w) + s(w) \Longrightarrow s(uv) = s(vw)$, which is a contradiction.
Conversely, suppose,

$d(u)^2 - d(w)^2 - 4(d(u) - d(w)) \neq s(w) - s(u$
and vw. Then $d(u)^2$ $4d(u) + s(u) \neq d($)for any two adjacent edges *uv*–*w*$)^2 -$
$4d(w) + s(w) \Longrightarrow s(uv) \neq s(vw)$, for any two adjacent edges *uv*and *vw*. Hence *G* is a SNEI graph.
Corollary 3.12. *Let G be a SNEI graph. If there are two adjacentedges(say uv and vw) with same edge degree, then* $s(u) \neq s(w)$.
Proof. Let *G* be a SNEI graph. Let *uv*and *vw*be two adjacent edges with $ed(uv) = ed(vw)$. Then $d(u) = d(w)$. By above theo 3.10, $d(u)^2 - 4d(u) + s(u) \neq d(w)^2 - 4d(w) + s(w)$, which implies $s(u) \neq s(w)$. □
Corollary 3.13. *Let G be a SNEI graph. If there are two adjacent edges (say uv and vw) with* $s(u) = s(w)$ *in G, then* $d(u)^2 - d(w)^2 = 4(d(u) - d(w))$.
Corollary 3.14. *Let G be a SNEI graph. If there are no two adjacent edges (say uv and vw) with* $s(u) = s(w)$ *and* $d(u) = d(w)$ *in G.*
Corollary 3.15. *Let G be a SNEI graph. Then there is no* P_3 *(say uvw) such that* $d(u) = d(w)$ *and* $s(u) = s(w)$.
Corollary 3.16. *Let G be a SNEI graph. Let v be any vertex. Then for any two vertices* v_i *and* v_j*such that* $d(v_i) + k = d(v_j)$, $s(v_i) + m \neq s(v_j)$ *where*

$$m = \begin{cases} 2kd(vi) - 3k + k2 \text{ if } vi \text{ and } vj \text{are not adjacent} \\ 2kd(vi) - 4k + k2 \text{ if } vi \text{ and } vj \text{ are adjacent} \end{cases}$$

Yousef alavi[1] *proved that for every positive integer n* = 36 *,*5,7*, there exists a highly irregular graph of order n.*
Theorem 3.17. *For every positive even integer* $n = 2k, k \geq 3$*, there exists a SNEI graph of order n and it is denoted by* $SNEI_{(n)}$.

Theorem 3.18. *There exist SNEI graph of order n, except* 3,5 *and* 7.

Proof. It is enough to prove there is a SNEI graph of order *n* where $n \geq 8$.
$(n-2)$ i i For $(n-2)$ *n*V(*SNEI* ,$_{i\,j}$,
$j \leq k\}$. In $SNEI_{(n-2)}$, u and v introduce two new verticesand join the edges $uu_j$and $vv_j$for $1 \leq j \leq k$. We obtain a *SNEI* graph of order *n*. Note further that introduce a pendent edge at either *u* or *v*, we will get a SNEI graph of order $n+1 = 2k+2+1 \geq 9$. Therefore there exists a SNEI graph of order $n \geq 8$.

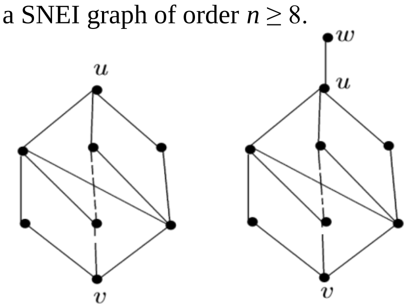

Figure 2 illustrates theorem 3.18 for *n* = 8 and *n* = 9

is even, let us first construct $SNEI_{(n-2)}, n - 2 = 2k, k \geq 3$. Let
$)=\{u\ ,v\ \ 1 \leq i \leq k\}$ and E($SNEI$ $)=\{u\ v\ \ 1 \leq i \leq k, i \leq$

This journal uses double-blind review process, which means that both the reviewer (s) and author (s) identities concealed from the reviewers, and vice versa, throughout the review process. All submitted manuscripts are reviewed by three reviewer one from India and rest two from overseas. There should be proper comments of the reviewers for the purpose of acceptance/ rejection. There should be minimum 01 to 02 week time window for it. **Result 3.19.** *We can construct a SNEI graph* G_1^* *from* $SNEI_{(n)}$ *, Suppose n* = 2*d. Let* $V(SNEI_{(n)}) = U \cup V$ *where* $U = u_i$*and* $V = v_i, 1 \leq i \leq d$.
By introducing a new vertex u and joining the edges $uu_1, uu_2, \ldots, uu_d$*. The resulting graph* G_1^* *is also SNEI graph. Further we can construct a SNEI graph* G_2^* *from* G_1^* *by introducing a pendant edge at u.*
Result 3.20. *We can construct a SNEI graph* G_3^* *from two copies of* $SNEI_{(n)}$ *, say* G_1 *and* G_2 *. Let* $V(G_1) = U_1 \cup V_1$ *where* $U_1 = u_{1i}$ *and* $V_1 = v_{1i}, 1 \leq i \leq d$ *and* $V(G_2) = U_2 \cup V_2$ *where* $U_2 = u_{2i}$ *and* $V_2 = v_{2i}, 1 \leq i \leq d$*. By introducing two new vertices u and v and joining the edges* $u_{1j}u, u_{2j}v$ *and uv. The resulting graph* G_3^* *is also SNEI graph.*

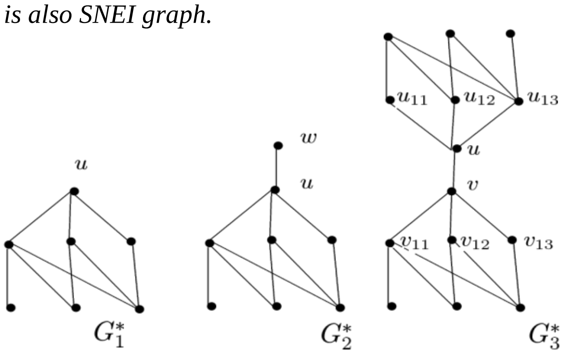

Figure 3

Result 3.21. *We can construct a SNEI graph G^*_4 from $SNEI_{(n1)}$ and $SNEI_{(n2)}$ where $n_1 \neq n_2$ of order $n_1 + n_2 + 3$. Suppose $n_1 = 2d$ and $n_2 = 2k$. Let $V(SNEI_{(n1)}) = U_1 \cup V_1$ where $U_1 = u_{1i}$ and $V_1 = v_{1i}, 1 \leq i \leq d$ and $V(SNEI_{(n2)}) = U_2 \cup V_2$ where $U_2 = u_{2j}$ and $V_2 = v_{2j}, 1 \leq j \leq k$. By introducing three new vertices u,v and w and joining the edges $u_{1i}u$, $u_{2j}w$, uv and vw , $1 \leq i \leq d$ and $1 \leq j \leq k$. The resulting graph G^*_4 is also a SNEI graph.*

Result 3.22. *We can construct a SNEI graph G^*_5 from $SNEI_{(n)}$ by introducing x_j and y_j for each u_j and $v_j, 1 \leq j \leq k$ respectively and for each $x_j, 1 \leq j \leq k-1$, s_{ji} and t_{ji} where $x_j s_{ji}, y_j p_{ji}, s_{ji} t_{jd}$ $1 \leq i \leq k-(j-1)$ and for each y_j, and joining the edges $u_j x_j, v_j y_j$, and $i \leq$, $1 \leq \leq -(x-1)$ $d \leq k$ and attach y . The resulting graph G^*_5 pendant vertices at $_d$ and $_d$ is also a SNEI graph.*

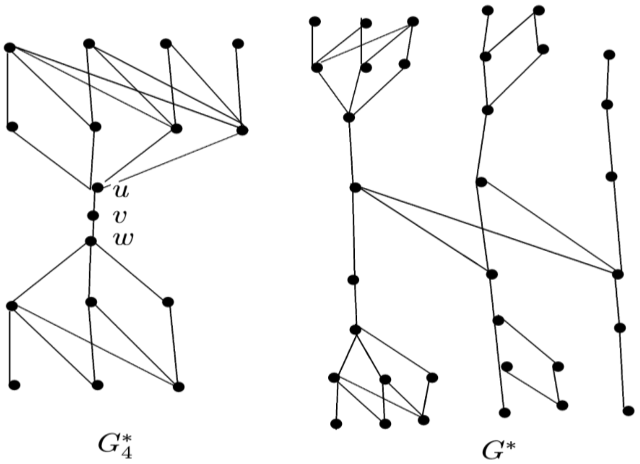

Figure 4

Result 3.23. *Let G be a triangle free SNEI graph with diameter* 3. *Let v be any vertex of the graph G having degree d, adjacent to the vertices $v_1,v_2,...,v_d$ and u_{ji} be the vertices adjacent to each v_j, $1 \leq j \leq d$. If* (*a*) *$s(v_j u_{ij}) \neq s(vv_j)+2(d(v)-1)$ and* (*b*) *$Ped(v_j u_{ij}) \neq ed(vv_j)-(d(v)-1)$ for each i and j , then we can construct a SNEI graph G^* from G by introducing a pendant edge at v . For, $s(v_j u_{ij})$ in $G^* = s(v_j u_{ij})$ in $G+1$ and $s(vv_j)$ in $G^* = s(vv_j)$* in G $+2d(v)-1$ and by (a) $\implies s(v_j u_{ij})$ in $G^* \neq s(vv_j)$ in G^*. $s(vv_j)$ in $G^* = \sum ed(v_j u_{ij})$ in $G + \sum ed(vv_k) + 2d(v) - 1$ and $s(vw) = \sum ed(vv_j)$ in $G + d(v) = ed(vv_j) + \sum_{k \neq j} ed(vv_k) + d(v) - 1$ and by (b), $s(vv_j) \neq s(vw)$.

Theorem 3.24. *Every cycle C_n of order $n \geq 4$ is an induced subgraph of a SNEI graph of order at most $n + 2\lfloor \frac{n}{4} \rfloor + 2$*

Proof. Let C_n be a cycle of order n, $n \geq 4$. Let $v_1,v_2,...,v_n$ be the vertices of C_n . Suppose n is even, introduce new pendant vertices v_{i1},v_{i2} at u_{4i+1},u_{4i+2} respectively, $0 \leq i \leq \lceil \frac{n}{4} \rceil - 1$. The resulting graph is a SNEI graph of order $n + 2\lceil \frac{n}{4} \rceil$. Suppose n is odd, introduce new pendent vertices v_{i1},v_{i2} at u_1,u_2 and P_3 at u_3 and pendant vertices $v_{(i+1)1},v_{(i+2)2}$ at u_{4i+2} , u_{4i+3} respectively, $1 \leq i \leq \lfloor \frac{n}{4} \rfloor - 1$. The resulting graph is also a SNEI graph of order $n + 2\lfloor \frac{n}{4} \rfloor + 2$.

Figure 5 illustrates theorem 3.24 for $n = 12$ and $n = 11$

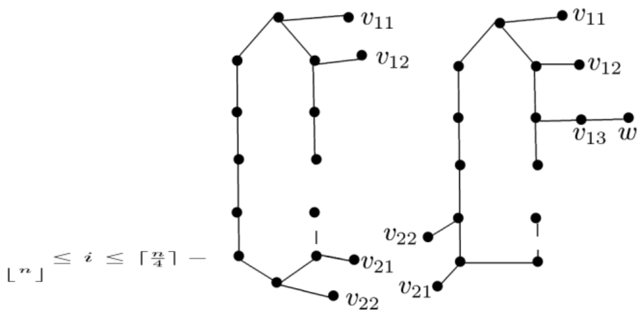

Figure 5

Theorem 3.25. *Every complete graph of order $n \geq 3$ is an induced subgraph of SNEI graph of order $n^2 + 2n$.*

Proof. Let G be a complete graph of order $n \geq 3$. Let $u_1,u_2,...,u_n$ be the vertices of G. For each u_i, $1 \leq i \leq n$, we introduce new vertices v_{ij} and w_{ij}, $1 \leq i \leq n, 1 \leq j \leq i$. The vertices u_i,v_{ij},w_{ij}, $1 \leq i \leq n, 1 \leq j \leq i$ constitute the vertex set for the desire graph H . For the edge set, along with the edges of G, join the edges (*a*) $u_i v_{ij}$, $1 \leq i \leq n, 1 \leq j \leq i$ and (*b*) $w_{ij} v_{ik}$, $1 \leq i \leq n, 1 \leq j \leq i, 1 \leq k \leq j$ .The resulting graph H is a SNEI graph and it contains G as an induced subgraph.

Figure 6 illustrates theorem 3.25 for $n = 3$

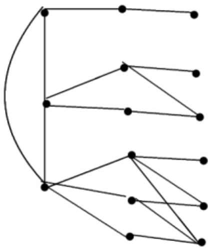

Figure 6

Theorem 3.26. *Every complete bipartite graph $K_{r,r}$ is an induced subgraph of a SNEI graph of order 4r.*

Proof. Let $u_1,u_2,...,u_r$ and $v_1,v_2,...,v_r$ be two partites of $K_{r,r}$. Let $u'_1, u'_2, ..., u'_r$ and $v'_1, v'_2, ..., v'_r$ be the newly added vertices. Construct the graph with vertex set $V(H) = \{u_i,v_i,u^0_i,v_i^0\}$ and edge set $E(H)=E(K_{r,r}) \cup \{u_i u^0_j$ and $v_i v_j^0$, $1 \leq i \leq r$, $i \leq j \leq r\}$. It is obvious that G is an induced subgraph of H .From our construction of H it is clear that H is a SNEI graph of order $4r$.

Figure 7 illustrates theorem 3.26 for $r = 3$

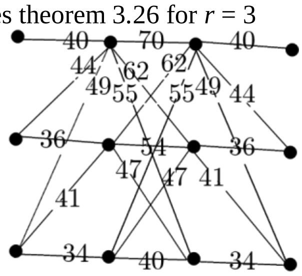

Figure 7

j in

Theorem 3.27. *Any graph G of order $n \geq 3$ is an induced subgraph of a SNEI graph.*

Proof. Let G be a graph of order $n \geq 3$. Let G^0 be another copy of G where $V(G)=\{u_1,u_2,...,u_n\}$ and $V(G^0)=\{v_1,v_2,...,v_n\}$, u_i corresponds to v_i, $1 \leq i \leq n$. Let $u_1,u_2,...,u_n$ and $v'_1, v'_2, ..., v'_n$ be the newly added vertices. Construct a graph H with the vertex set $V(H) = \{u_i, v_i, u'_i, v'_i\} (1 \leq i \leq n)$ and

$E(G) = E(G) \cup E(G^0) \cup \{u_i v_j : u_i u_j \notin E(G), 1 \le i \le n, 1 \le i \le n$ and $u_i v_i, 1 \le i \le n\} \cup \{u_i u'_j, v_i v'_j, 1 \le i \le n, 1 \le i \le n\}$. It is obvious that G is an induced subgraph of H . From our construction of Hit is clear that H is a SNEI graph of order $4n$.

Figure 8 illustrates theorem 3.27 for $n = 3$

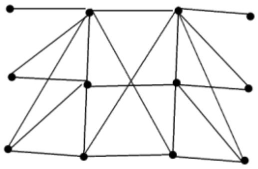

Figure 8

Result 3.28. *For any positive integer $d \ge 3$, we can construct a SNEI graph with maximum degree $2d$ and order $3d + 4$.*

Proof. Let $d \ge 3$ be any positive integer. Let $V(G) = \{v_i, u_i, w_i, u, v, w, x, 1 \le i \le d\}$, be the vertex set of the required graph G and the $E(G) = \{v_i w_j\} \cup$ edge set, $u_i w_j, 1 \le i \le d, \le j \le d\} \cup \{v_i v, 1 \le i \le d\} \cup \{u_i u, 1 \le i \le d - 1$ $\{ux, vx, wx, u_d w\}$. The resulting graph is a SNEI graph of order $3d + 4$ and maximum degree d.

Figure 9 illustrates theorem 3.28 for $d = 6$

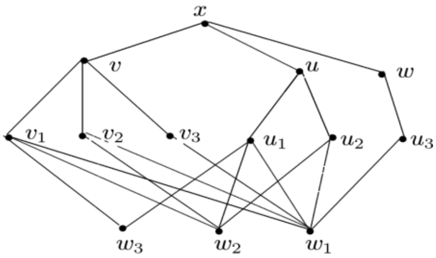

Figure 9

IV. CONCLUSION

In this paper we deals irregular graphs called support neighbourly edge irregular graphs based on degree in edge sense. A necessary and sufficient condition for a graph to be SNEI has been obtained and several methods to construct SNEI graph from other graphs have been discussed.

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