OPEN ACCESS

International Journal of Recent Technology and Engineering (IJRTE)
ISSN: 2277-3878, Volume-8 Issue-3, September 2019

Graceful and Magic Labeling in Special Fuzzy Graphs

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***Abstract**: In this paper, we present an algorithm to find fuzzy labeling of a star graph $K_{1,n}$, bi star graph $B_{n,n}$ and double star graph $K_{1,n,n}$. We prove that star graph $K_{1,n}$ at most 89 edges are fuzzy graceful iff it admits fuzzy magic graph. Also we prove that bi star graph $B_{n,n}$ having at most 59 edges are fuzzy graceful iff it is a fuzzy bi magic graph .We prove that fuzzy labeled double star graph $K_{1,n,n}$ at most 30 edges is fuzzy graceful.*

***Keywords**: Fuzzy Labeling, Fuzzy Graceful Labeling, Fuzzy Magic Labeling, Fuzzy Bi magic Labeling.*

I. INTRODUCTION

In 1965, fuzzy sets were introduced by Lofti.A. Zadeh[1] which had a greater improvement in mathematical modeling in case of uncertainty. Narsingh deo[2] extended the concepts of graph theory which are models of relations between vertices and edges . A. Solairaju and S. Ambika [3] proved some results on gracefulness of a new class of stars merged with trees. Fuzzy relations on fuzzy sets had been leading an excellent way to make a fuzzy graph model when there is an ambiguity in vertices and edges.

Fuzzy graph model has been developed by K.R.Bhutani, J.N.Moderson and A.Rosenfield[4]. S.Mathew and M.Sunitha[5] proved many standard results on fuzzy graphs.

A. Nagoorgani, Muhammed Akram and D.Rajalakshmi(a)Subhasini [6,7] introduced the concepts of labeling in fuzzy graph and proved some properties in fuzzy graph labeling . Existence of fuzzy edge vertex graceful labeling in some special graphs has been proved by S.Vimala and R.Jebesty Shajila [8].An algorithmic approach on fuzzy graceful labeling technique for extended duplicate graph discussed by S.Bala, M.L.MorslinLifin Lee, K.Thirusangu [9].Some results on fuzzy bimagic and anti magic labeling in star graphs has been proved by K. Thirusangu and D. Jeevitha[10]. Fuzzy vertex graceful labeling on bi star graph discussed by K. Ameenal Bibi, M. Devi[11].

In this paper, we present an algorithm to find fuzzy labeling of a star graph $K_{1,n}$, bistar graph $B_{n,n}$ and double star graph $K_{1,n,n}$. We prove that star graph $K_{1,n}$ atmost 89 edges are fuzzy graceful iff it admits fuzzy magic graph. Also we prove that bistar graph $B_{n,n}$ having atmost 59 edges are fuzzy graceful iff it is a fuzzy bi magic graph. We prove that fuzzy labeled double star graph $K_{1,n,n}$ at most 30 edges is fuzzy graceful.

Manuscript published on 30 September 2019
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II. PRELIMINARIES

Definition [1] 2.1. Let X be the space of points(objects),with a generic element of X denoted by x. Thus X={x}. A fuzzy set (class) A in X is characterized by a membership (characteristic) function $f_A(x)$ which associates with each point in X a real number in the
interval [0,1], with the value of $f_A(x)$ at x , representing the "grade of membership" of x in A.

Definition [2] 2.2. A Linear graph (or simply a graph) G = (V, E) consists of set of objects $V=\{v_1,v_2,\ldots v_n\}$ called vertices, and another set $E=\{e_1,e_2,\ldots e_m\}$ whose elements are called edges such that each edge ek is identified with an unordered pair of vertices (v_i,v_j).

Definition [3] 2.3. The star graph S_n of order 'n' is a tree on 'n' nodes with one node having vertex degree (n-1) and the other (n-1) node having vertex degree 1. Star is a complete bi graph $K_{1,n}$.

Definition [10] 2.4.The bi star $B_{n,n}$ is a graph obtained by joining the center(apex) vertices of two copies of $K_{1,n}$ by an edge.

Definition[3] 2.5 The double star $K_{1,n,n}$ is a tree obtained from the star $K_{1,n}$ by adding a new pendant edge to each of the existing 'n'pendant vertices.

Definition [5] 2.6. Let U and V be two sets. Then ρ is said to be a fuzzy relation from U into V if ρ is a fuzzy set of $U \times V$.A fuzzy graph $G = (\sigma, \mu)$ is a pair of functions $\sigma : V \rightarrow [0,1]$and $\mu : V \times V \rightarrow [0, 1]$, where for all $u, v \in V$, we have $\mu(u, v) \leq \sigma(u) \wedge \sigma(v)$.

Definition [6] 2.7 A graph $G = (\sigma, \mu)$ is said to be a fuzzy labeling graph if $\sigma : V \rightarrow [0,1]$ and $\mu : V \times V \rightarrow [0,1]$ is bijective such that the membership value of edges and vertices are distinct and $\mu(u, v) < \sigma(u) \wedge \sigma(v)$ for all $u, v \in V$.

Definition 2.8. . A fuzzy labeling graph $G = (\sigma, \mu)$ is said to be a fuzzy graceful graph if $\sigma : V \rightarrow [0,1]$, $\mu : V \times V \rightarrow [0,1]$ such that $\mu(u,v) = |\sigma(u) - \sigma(v)|$ and $\mu(u,v)$ are distinct for all $u, v \in V$.

Definition[6] 2.9. A fuzzy labeling graph is said to be a fuzzy magic graph if $\sigma(u) + \mu(u, v) + \sigma(v)$ has a same magic value for all $u, v \in V$ which is denoted as $m_0(G)$.

Definition[9] 3.0 .A fuzzy graph is said to admit bi-magic labeling if the sum of membership values of vertices and edges incident at the vertices are k_1 and k_2 where $k_1$and k_2 are constants and denoted by $\tilde{B}m_0G$.

A fuzzy labeling graph which admits a bi-magic labeling is called a fuzzy bi-magic labeling graph.

III. MAIN RESULTS

In this section , we present an algorithm to find fuzzy labeling of a star graph $K_{1,n}$, bistar graph $B_{n,n}$ and double star graph $K_{1,n,n}$ .Also derived the results on fuzzy graceful labeling ,fuzzy magic labeling and fuzzy bi magic labeling for the above mentioned graphs.

3.1 FUZZY LABELING ON STAR GRAPH $K_{1,n}$

In this section ,we develop an algorithm for fuzzy labeling of vertices and edges of a star graph $K_{1,n}$, at most 89 edges such as the apex vertex v_0, pendent vertices v_i and pendent edges $e_i=(v_0,v_i)$ whose membership functions $\sigma(v_0)$, $\sigma(v_i)$ and $\mu(v_0,v_i)$.

Algorithm 3.1.1

Input : Star Graph

Procedure {fuzzy Labeling of vertices and edges for a star graph ($K_{1,n}$) ,$n \leq 89$}

$v_0 \leftarrow$ Apex vertex of $K_{1,n}$

$\sigma(v_0) = 0.9$

for i = 1 to n

{ $v_i \leftarrow$ Pendent vertices of $K_{1,n}$

$\sigma(v_i) = \left[\sigma(v_0) - \left(\frac{i}{200}\right)\right]$

$v_i \leftarrow$ Pendent edges of $K_{1,n}$

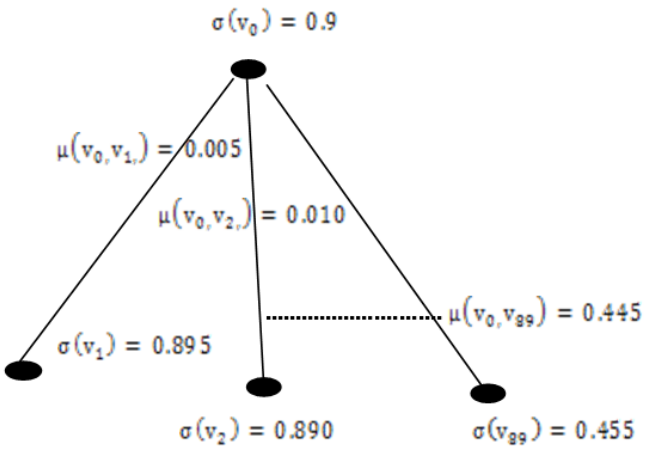
$\mu(v_0, v_i) = \left(\frac{i}{200}\right)$

}

end procedure.

Example 3.1.2

The fuzzy labeled star graph $K_{1,n}$ at most 89 edges is shown in the following Figure(1).



Figure(1) Fuzzy labeled star graph $K_{1,n}$

Theorem3.1.3:

The fuzzy labeled star graph $K_{1,n}$ with ($n \leq 89$) admits fuzzy graceful labeling.

Proof:

Let $K_{1,n}$ with ($n \leq 89$) be the star graph with 'n+1' vertices and 'n' edges.

To prove $K_{1,n}$ admits fuzzy graceful labeling. That is to prove that fuzzy labeled star graph satisfies distinct membership values of edges with $\mu(v_0, v_i) = |\sigma(v_0) - \sigma(v_i)|$, i =1 to n...........(1)

From Algorithm 3.1.1, Pendent vertices 'v_i' defined as

$$\sigma : v \rightarrow [0,1] \ni$$

$$\sigma(v_i) = \left[\sigma(v_0) - \left(\frac{i}{200}\right)\right] \quad , \quad i = 1 \ to \ n$$

and the pendent edges $e_i=(v_0,v_i)$ which have the membership function defined as

$$\mu(v_0, v_i) = \left[\frac{i}{200}\right] \quad , \quad i = 1 \ to \ n$$

..........(2)

Now, $\mu(v_0, v_i) = |\sigma(v_0) - \sigma(v_i)|$,

$$= \left|\sigma(v_0) - \left(\sigma(v_0) - \left(\frac{i}{200}\right)\right)\right|$$

$$= \left| \left(\frac{i}{200}\right) \right| \quad , \quad i = 1 \ \text{to} \ n$$

and also we have $\mu(v_0, v_l) \neq \mu(v_0, v_m)$ for any l ,m ϵ i with $l \neq m$....................................(3)

Hence by (1) ,(2) and (3) fuzzy labeled star graph $K_{1,n}$ with 'n+1' vertices and 'n' edges admits fuzzy graceful labeling.

Theorem 3.1.4:

The fuzzy labeled star graph $K_{1,n}$ with ($n \leq 89$) admits fuzzy magic labeling.

Proof:

Consider a fuzzy labeled star graph $K_{1,n}$ with ($n \leq 89$) with 'n+1' vertices and 'n' edges.

To prove $K_{1,n}$ admits fuzzy magic labeling.

That is to prove that $\sigma(v_0) + \mu(v_0, v_i) + \sigma(v_i)$ has the same magic value.

Using algorithm 3.1.1,we have

$$\sigma(v_0) + \sigma(v_i) + \mu(v_0, v_i)$$

$$= \sigma(v_0) + \sigma(v_i) + \left(\frac{i}{200}\right)$$

$$= \sigma(v_0) + \left(\sigma(v_0) - \left(\frac{i}{200}\right)\right) + \left(\frac{i}{200}\right)$$

$$= 2 \ \sigma(v_0) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

Clearly by (4) , $\sigma(v_0) + \mu(v_0, v_i) + \sigma(v_i)$ has same magic value. Hence $K_{1,n}$ admits fuzzy magic labeling.

Theorem3.1.5:

A fuzzy labeled star graph $K_{1,n}$ at most 89 edges admits fuzzy graceful labeling iff it admits fuzzy magic labeling.

Proof:

Suppose $K_{1,n}$ with at most 89 edges admits fuzzy graceful labeling. Hence from theorem 3.1.3, we have

$\mu(v_0, v_i) = |\sigma(v_0) - \sigma(v_i)|$ ....(5).

Now $\sigma(v_0) + \mu(v_0, v_i) + \sigma(v_i)$
$= \sigma(v_0) + |\sigma(v_0) - \sigma(v_i)| + \sigma(v_i)$

$= 2\ \sigma(v_o)$ ....(6)

Clearly by (6), $\sigma(v_0) + \mu(v_0, v_i) + \sigma(v_i)$ has the same magic value. Hence $K_{1,n}$ admits fuzzy magic labeling.Now, suppose $K_{1,n}$ at most 89 edges admits fuzzy magic labeling.

3.2 FUZZY LABELING ON BI STAR GRAPH $B_{n,n}$

In this section ,we develop an algorithm for fuzzy labeling of vertices and edges of a bistar graph $B_{n,n}$, at most 59 edges such as apex vertices v_1, v_2, apex edge e_0, pendent vertices u_i of v_1 , w_i of v_2 and pendent edges $e_i=(v_1,u_i)$, $e_i^*=(v_2,w_i)$ whose membership functions $\sigma(v_1)$, $\sigma(v_2)$, $\mu(v_1,v_2)$ $\sigma(u_i)$, $\sigma(w_i)$, $\mu(v_1,u_i)$ and $\mu(v_2,w_i)$.

Algorithm 3.2.1

Input : Bi star Graph

Procedure {fuzzy Labeling of vertices and edges for a bistar graph $(B_{n,n})$,n $\leq$ 59}

$\{v_1, v_2\} \leftarrow$ Apex vertices of of $B_{n,n}$

$\sigma(v_1) = 0.8$

$\sigma(v_2) = 0.9$

$\mu(v_1, v_2) = 0.1$

for i = 1 to n

{ $u_i \leftarrow$ Pendent vertices of v_1 of $B_{n,n}$

$\sigma(u_i) = \left[\frac{280-i}{400}\right]$

$\mu(v_1, u_i) = \left[\frac{40+i}{400}\right]$

$w_i \leftarrow$ Pendent vertices of v_2 of $B_{n,n}$

$\sigma(w_i) = \left[\frac{540-i}{600}\right]$

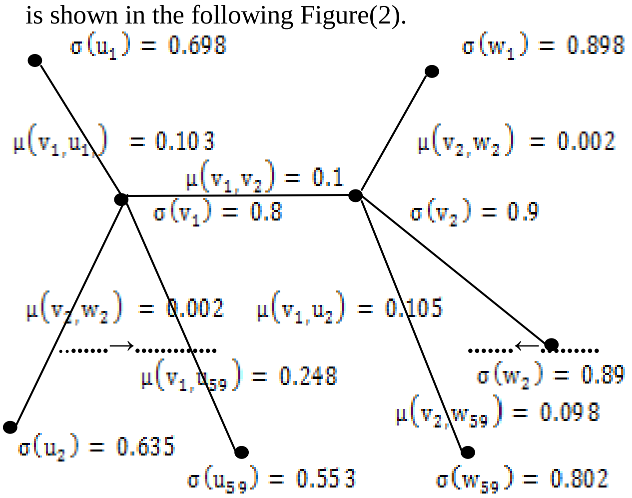
$\mu(v_2, w_i) = \left[\frac{i}{600}\right]$

}

end procedure.

Example 3.2.2

The fuzzy labeled bi star graph Bn,n at most 59 edges is shown in the following Figure(2).



Figure(2) Fuzzy labeled bi star graph $B_{n,n}$

Theorem3.2.3:

The fuzzy labeled bi star graph $B_{n,n}$ with (n $\leq$ 59) admits fuzzy graceful labeling.

Proof:

Let $B_{n,n}$ with (n $\leq$ 59) be the bi star graph with '2n+2' vertices and '2n+1' edges.

To prove $B_{n,n}$ admits fuzzy graceful labeling. we can prove in three cases.

Case (i):

To prove apex vertices v_1,v_2 and an apex edge e_0 = (v_1,v_2) of $B_{n,n}$ admits fuzzy graceful labeling. That is to prove that $\mu(v_1, v_2) = |\sigma(v_1) - \sigma(v_2)|$ with distinct membership values.

Consider membership value of apex vertices v_1,v_2 and apex edge

$e_0 = (v_1,v_2)$ using algorithm 3.2.1, we have

$\sigma : v_1 \rightarrow [0,1] \ni \sigma(v_1) = 0.8$

$\sigma : v_2 \rightarrow [0,1] \ni \sigma(v_2) = 0.9$

$\mu : v_1 \times v_2 \rightarrow [0,1] \ni \mu(v_1, v_2) = 0.1$

Now,

$$\mu(v_1, v_2) = |\sigma(v_1) - \sigma(v_2)|$$
$$= |(0.8 - 0.9)|$$
$$= |0.1| = 0.1 \text{..........(8)}$$

Hence by (8) ,the apex vertices v_1,v_2 and an apex edge $e_0 = (v_1,v_2)$ of $B_{n,n}$ admits fuzzy graceful labeling.

Case (ii):

To prove pendent vertices u_i of v_1 and pendent edges $e_i = (v_1,u_i)$ of $B_{n,n}$ admits fuzzy graceful labeling. That is to prove that $\mu(v_1, u_i) = |\sigma(v_1) - \sigma(u_i)|$ with distinct membership values.

Consider membership function of pendent vertices u_i of v_1 and pendent edges $e_i = (v_1,u_i)$ using algorithm 3.2.1, we have

$\sigma : u_i \rightarrow [0,1] \ni \sigma(u_i) = \left[\frac{280-i}{400}\right]$

$\mu : v_1 \times u_i \rightarrow [0,1] \ni \mu(v_1, u_i) = \left[\frac{40+i}{400}\right]$

Now, $\mu(v_1, u_i) = |\sigma(v_1) - \sigma(u_i)|$

$$= \left|0.8 - \left(\frac{280-i}{400}\right)\right|$$

$$= \left|\left(\frac{320-280+i}{400}\right)\right|$$

$$= \left|\left(\frac{40+i}{400}\right)\right|$$

....................(9)

and also we have $\mu(v_1, v_l) \neq \mu(v_1, v_m)$ for any l ,m є i with l≠m...............(10)

Hence by (9) and (10) pendent vertices u_i of v_1 and pendent edges $e_i = (v_1,u_i)$ of $B_{n,n}$ admits fuzzy graceful labeling.

Case (iii):

To prove pendent vertices w_i of v_2 and pendent edges $e_i^* = (v_2,w_i)$ of $B_{n,n}$ admits fuzzy graceful labeling. That is to prove that $\mu(v_2, w_i) = |\sigma(v_2) - \sigma(w_i)|$ with distinct membership values.

Consider membership function of pendent vertices w_i of v_2 and pendent edges $e_i^* = (v_2,w_i)$ using algorithm 3.2.1, we have

$$\sigma : \quad w_i \to [0,1] \ni \sigma(w_i) = \left[\frac{540-i}{600}\right]$$

$$\mu : \quad v_2 \times w_i \to [0,1] \ni \mu(v_2, w_i) = \left[\frac{i}{600}\right]$$

Now, $$\mu\ (v_2\ ,w_i\ ) = \left|\sigma(v_2) - \sigma(w_i)\right|$$

$$= \left|0.9 - \left(\frac{540-i}{600}\right)\right|$$

$$= \left|\left(\frac{540-540+i}{600}\right)\right|$$

$$= \left|\left(\frac{i}{600}\right)\right| \text{...............(11)}$$

and also we have for any l ,m ϵ i $\mu(v_2 ,w_l) \neq \mu(v_2 ,w_m\ )$ with $l \neq m$................(12)

Hence by (11) and (12) pendent vertices w_i of v_2 and pendent edges $e_i^* = (v_2,w_i)$ of $B_{n,n}$ admits fuzzy graceful labeling.

Hence by ,Case(i) ,Case(ii) and Case (iii) Bi star graph $B_{n,n}$ admits fuzzy graceful labeling.

Theorem3.2.4:

The fuzzy labeled bi star graph $B_{n,n}$ with ($n \leq 59$) admits fuzzy bi magic labeling.

Proof:

Consider a fuzzy labeled bi star graph $B_{n,n}$ with ($n \leq 59$) with '2n+2' vertices and '2n+1' edges.

To prove $B_{n,n}$ admits fuzzy bi magic labeling. That is to prove that the sum of membership values of vertices and edges incident at the vertices are $k_1$and k_2 where $k_1$and k_2 are constants .

we can prove in three cases.

Case (i):

To prove sum of membership values of internal vertices v_i and internal edges e_i attain magic value. That is to prove that $\sigma(v_1)+\sigma(u_i)+\mu\ (v_1\ ,u_i\ ) = k_1$

Using algorithm 3.2.1, we have

$$\sigma(v_1)+\sigma(u_i)+\mu\ (v_1\ ,u_i\ )$$

$$= (0.8) + \left(\frac{280-i}{400}\right) + \left(\frac{40+i}{400}\right),\ i = 1\ to\ n$$

$$= \left(\frac{320}{400}\right) + \left(\frac{280-i}{400}\right) + \left(\frac{40+i}{400}\right)$$

$$= \left(\frac{320+280-i+40+i}{400}\right)$$

$$= \left(\frac{640}{400}\right) = 1.6 = k_1 \text{..............(13)}$$

Case (ii):

=

To prove sum of membership values of pendent vertices "v_i" and pendent edges "e_i" attain magic value. That is to prove that

$$\sigma(v_2)+\sigma(w_i)+\mu\ (v_2\ ,w_i\ ) = k_2$$

Using algorithm 3.2.1, we have

$$\sigma(v_2)+\sigma(w_i)+\mu\ (v_2\ ,w_i\ )$$

$$= (0.9) + \left(\frac{540-i}{600}\right) + \left(\frac{i}{600}\right),\ i = 1\ to\ n$$

$$= \left(\frac{540}{600}\right) + \left(\frac{540-i}{600}\right) + \left(\frac{i}{600}\right)$$

$$= \left(\frac{540+540-i+i}{600}\right)$$

$$= \left(\frac{1080}{600}\right) = 1.8 = k_2 \text{ ....(14)}$$

Case (iii):

To prove sum of membership values of apex vertices v_1,v_2 and apex edge e_0 attain magic value. That is to prove that $\sigma(v_1)+\sigma(v_2)+\mu\ (v_1\ ,v_2\ ) = k_1\ or\ \ k_2$.

Using algorithm 3.2.1, we have

$$\sigma(v_1)+\sigma(v_2)+\mu\ (v_1\ ,v_2\ ) = 0.8+0.9+0.1$$

$= k_2$

Hence by (13),(14) and(15), bi star graph $B_{n,n}$ admits bi magic labeling with two magic constants $k_1$and k_2.

Theorem 3.2.5:

A fuzzy labeled bi star graph $B_{n,n}$ at most 59 edges be the fuzzy graceful graph iff it is a fuzzy bi magic graph.

Proof:

Given $B_{n,n}$ at most 59 edges be fuzzy graceful bi star graph. To prove $B_{n,n}$ be the fuzzy magic graph. we can prove in three cases.

Case (i):

To prove that $\sigma(v_1)+\sigma(u_i)+\mu\ (v_1\ ,u_i\ ) = k_1$

By Theorem 3.2.3 and algorithm 3.2.1 we have

$$\sigma(v_1)+\sigma(u_i)+\mu\ (v_1\ ,u_i\ )$$

$$= \sigma(v_1)+\sigma(u_i)+\left|\sigma(v_1)-\sigma(u_i)\right|$$

$$= 2\ \sigma(v_1) = 1.6 = k_1 \text{ ....(16)}$$

Case (ii):

To provet that $\sigma(v_2)+\sigma(w_i)+\mu\ (v_2\ ,w_i\ ) = k_2$

By Theorem 3.2.3 and algorithm 3.2.1 we have

$$\sigma(v_2)+\sigma(w_i)+\mu\ (v_2\ ,w_i\ )$$

$$= \sigma(v_2)+\sigma(w_i)+\left|\sigma(v_2)-\sigma(w_i)\right|$$

$$= \sigma(v_2)+\sigma(w_i)+\sigma(v_2)-\sigma(w_i)$$

$$= 2\ \sigma(v_2) = 1.8 = k_{\ 2} \text{ ....(17)}$$

Case (iii):

To prove that $\sigma(v_1)+\sigma(v_2)+\mu\ (v_1\ ,v_2\ ) = k_1\ or\ \ k_2$.

By Theorem 3.2.3 and algorithm 3.2.1 we have

$$\sigma(v_1)+\sigma(v_2)+\mu\ (v_1\ ,v_2\ )$$

$$= \sigma(v_1)+\sigma(v_2)+\left|\sigma(v_2)-\sigma(v_1)\right|$$

$$= \sigma(v_1)+\sigma(v_2)+\sigma(v_2)-\sigma(v_1)$$

$$= 2\ \sigma(v_2) = 1.8 = k_2 \text{ ....(18)}$$

Hence by (16),(17) and(18), bi star graph $B_{n,n}$ admits bi magic labeling with two magic constants $k_1$and k_2.

Now Suppose $B_{n,n}$ with at most 59 edges be fuzzy bi magic bi star graph. To prove $B_{n,n}$ be the fuzzy graceful graph. we can prove in three cases.

Case (i): Given

$$\sigma(v_1)+\sigma(u_i)+\mu(v_1, u_i)=k_1$$

That is $\sigma(v_1)+\sigma(u_i)+\mu(v_1, u_i)=2\,\sigma(v_1)$ [using (16)]

Hence clearly

$\mu(v_1, u_i) = |\sigma(v_1)-\sigma(u_i)|$....................(19)

Case (ii): Given

$$\sigma(v_2)+\sigma(w_i)+\mu(v_2, w_i)=k_2$$

That is

$$\sigma(v_2)+\sigma(w_i)+\mu(v_2, w_i),$$
$$=2\,\sigma(v_2) \quad [using\,(17)]$$

Hence clearly

$\mu(v_2, w_i) = |\sigma(v_2)-\sigma(w_i)|$......................(20)

Case (iii): Given

$$\sigma(v_1)+\sigma(v_2)+\mu(v_1, v_2)=k_2.$$

Now

$$\sigma(v_1)+\sigma(v_2)+\mu(v_1, v_2),$$
$$=2\,\sigma(v_2) \quad [using\,(18)]$$

Hence clearly

$\mu(v_1, v_2) = |\sigma(v_1)-\sigma(v_2)|$.........................(21)

Hence by (19),(20) and(21), bi star graph $B_{n,n}$ admits bi magic labeling with two magic constants $k_1$and k_2.

3.3 FUZZY LABELING ON DOUBLE STAR GRAPH $K_{1,n,n}$

In this section ,we develop an algorithm for fuzzy labeling of vertices and edges of a double star graph $K_{1,n,n}$, at most 30 edges such as apex vertex v_0, internal vertices v_i of v_0 , internal edges $e_i=(v_0,v_i)$, pendent vertices w_i of v_i and pendent edges $e_i^*=(v_i,w_i)$ whose membership functions $\boldsymbol{\sigma}(v_0)$, $\boldsymbol{\sigma}(v_i)$, $\mu(v_0,v_i)$,$\boldsymbol{\sigma}(w_i)$and$\mu(v_i,w_i)$ for i =1 to n.

Algorithm 3.3.1

Input : Double Star Graph

Procedure {fuzzy Labeling of vertices and edges for a double star graph ($K_{1,n,n}$) ,n $\leq$ 30}

$\{v_0\} \leftarrow$ Apex vertex of $K_{1,n,n}$

$\sigma(v_0) = 0.8$

for i = 1 to n

{ $v_i \leftarrow$ internal vertices of v_0 of $K_{1,n,n}$

$$\sigma(v_i) = \left(\frac{200+i}{400}\right)$$

$$\mu(v_0, v_i) = \left(\frac{120-i}{400}\right)$$

$w_i \leftarrow$ pendent vertices of v_i of $K_{1,n,n}$

$$\sigma(w_i) = \left(\frac{400+(3*(i+1))}{500}\right)$$

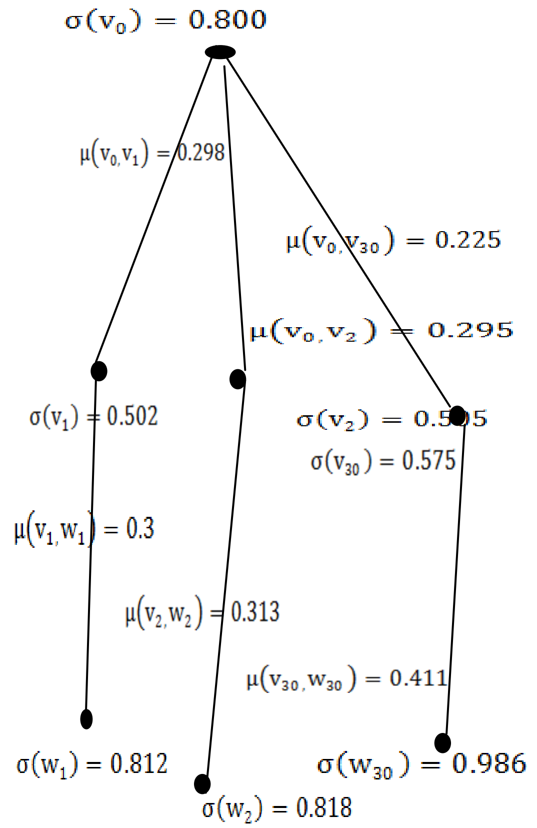
$$\mu(v_i, w_i) = \left(\frac{61.2+(0.7*i)}{200}\right)$$

}

end procedure.

Example 3.3.2:

The fuzzy labeled double star graph $K_{1,n,n}$ at most 30 edges is shown in the following Figure(3).



Figure(3) Fuzzy labeled double star graph $K_{1,n,n}$

Theorem3.3.3:

The fuzzy labeled double star graph $K_{1,n,n}$ with (n $\leq$ 30) admits fuzzy graceful labeling.

Proof:

Let $K_{1,n,n}$ with (n $\leq$ 30) be the double star graph with '2n+1' vertices and '2n' edges.

To prove $K_{1,n,n}$ admits fuzzy graceful labeling. we can prove in two cases.

Case (i):

Consider apex vertex v_0 ,the internal vertices u_i and the internal edges $e_i=(v_0,u_i)$ of v_0.

Using algorithm 3.3.1, we have

$$\sigma : v_0 \rightarrow [0,1] \quad \ni \quad \sigma(v_0) = 0.8$$

$$\sigma : v_i \rightarrow [0,1] \quad \ni \quad \sigma(v_i) = \left(\frac{200+i}{400}\right)$$

$$\mu : v_0 \times v_i \rightarrow [0,1] \ni \mu(v_0, v_i) = \left(\frac{120-i}{400}\right)$$

Now,

$$\mu\ (v_0\ ,\ v_i\ ) = |\sigma(v_0) - \sigma(v_i)|$$

$$= \left[(0.8) - \left(\frac{200+i}{400}\right)\right]$$

$$= \left(\frac{(320-200-i}{400}\right)$$

$$\mu\ (v_0\ ,\ v_i\ ) = \left(\frac{(120-i)}{400}\right) \quad ....(22$$

)

and also we have $\mu(v_0, v_l) \neq \mu(v_0, v_m)$ for any l ,m ϵ i with l≠m.............(23)

Hence by (22) and (23) pendent edges $e_i=(v_1,u_i)$ for i = 1 to n , admits fuzzy graceful labeling.

Case (ii):

Consider the internal vertices v_i , pendent vertices w_i and the pendent edges $e_i^*=(v_i,w_i)$ of v_i , Using algorithm 3.3.1, we have

$$\sigma:\ v_i \to [0,1] \ni \sigma(v_i) = \left(\frac{200+i}{400}\right)$$

$$\sigma:\ w_i \to [0,1] \ni \sigma(w_i) = \left(\frac{400+(3*(i+1))}{500}\right)$$

$$\mu:\ v_i \times w_i \to [0,1] \ni \mu(v_i, w_i) = \left(\frac{61.2+(0.7*i)}{200}\right)$$

Now,

$$\mu\ (v_i\ ,\ w_i\ ) = |\sigma(v_i) - \sigma(w_i)|$$

$$= \left(\frac{(400+3+3i)}{500} - \frac{(200+i)}{400}\right)$$

$$= \left(\frac{400(400+3+3i)}{500*400} - \frac{500(200+i}{500*400}\right)$$

$$= \left(\frac{400*400+3*400+3*400i-200*500-500i}{500*400}\right)$$

$$= \left(\frac{160000+1200-1200i-100000-500i}{500*400}\right)$$

$$= \left(\frac{61200+700i}{500*400}\right)$$

$$\mu\ (v_i\ ,\ w_i\ ) = \left(\frac{61.2+0.7i}{200}\right) \quad ....(24)$$

and also we have have $\mu(v_i, w_l) \neq \mu(v_i, w_m)$

for any l ,m ϵ i with l≠m.............................(25)

Hence by (24) and (25) pendent edges $e_i^*=(v_i,w_i)$ of v_2 for i = 1 to n , admits fuzzy graceful labeling.

Hence by Case(i) and Case (ii) double star graph $K_{1,n,n}$ admits fuzzy graceful labeling.

IV. APPENDIX

The following be the program on fuzzy labeled star graph $K_{1,n}$ at most 89 edges which has the apex vertex v_0, pendent vertices v_i and pendent edges $e_i=(v_0,v_i)$ whose membership functions □(v_0), □(v_i) and μ(v_0,v_i) represented as (lv_0), (lv_i) and (le_i).

Program 4.1 :

```
#include<stdio.h>
#include<conio.h>
void main()
{
    float i, j, v[500], e[500], lv[500], le[500];
    printf(" Fuzzy Labeled Star Graph at most 89
edges \n");
    printf("-------------------------------------------\n");
    printf("lv[0]   lv[i]     le[i] \n");
    lv[0]=0.9;
   for(i=1;i<=89; i++)
    {
     lv[i]=( lv[0]-(i/200));
     le[i]= (i/200);
     printf(" % 1.3f  %1.3f  %1.3f ",lv[0],lv[i],
     le[i]\n");
    }
   getch();
  }
```

```
C:\TURBOC3\BIN>TC
  Fuzzy Labeled Star Graph  atmost  89 edges
------------------------------------------------------------
lv[0]    lv[i]    le[i]
  0.900   0.795   0.105
  0.900   0.790   0.110
  0.900   0.785   0.115
  0.900   0.780   0.120
  0.900   0.775   0.125
  0.900   0.770   0.130
  0.900   0.765   0.135
  0.900   0.760   0.140
  0.900   0.755   0.145
  0.900   0.750   0.150
  0.900   0.745   0.155
  0.900   0.740   0.160
  0.900   0.735   0.165
  0.900   0.730   0.170
  0.900   0.725   0.175
  0.900   0.720   0.180
  0.900   0.715   0.185
  0.900   0.710   0.190
  0.900   0.705   0.195
  0.900   0.700   0.200
_
```

Retrieval Number: C6877098319/2019©BEIESP
DOI:10.35940/ijrte.C6877.098319
Journal Website: www.ijrte.org

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Blue Eyes Intelligence Engineering
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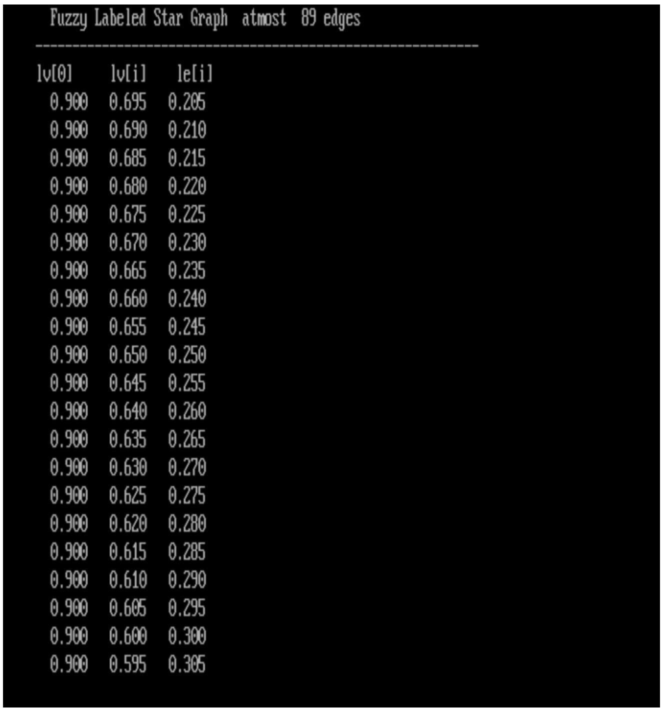
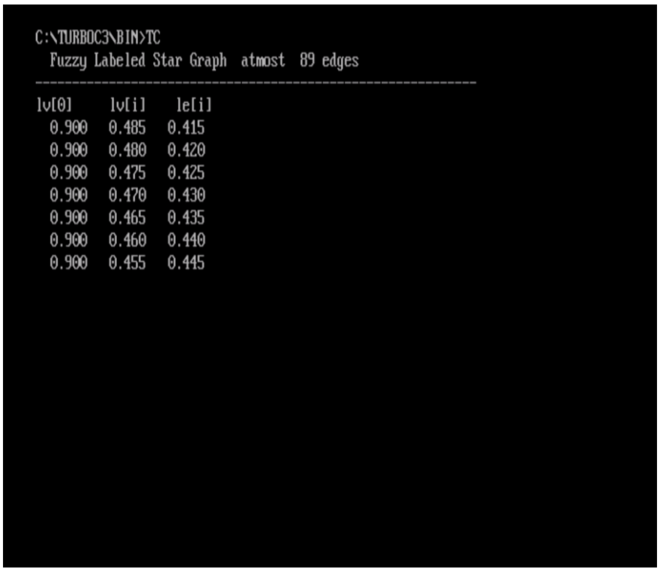


Fuzzy Labeled Star Graph atmost 89 edges

lv[0]	lv[i]	le[i]
0.900	0.695	0.205
0.900	0.690	0.210
0.900	0.685	0.215
0.900	0.680	0.220
0.900	0.675	0.225
0.900	0.670	0.230
0.900	0.665	0.235
0.900	0.660	0.240
0.900	0.655	0.245
0.900	0.650	0.250
0.900	0.645	0.255
0.900	0.640	0.260
0.900	0.635	0.265
0.900	0.630	0.270
0.900	0.625	0.275
0.900	0.620	0.280
0.900	0.615	0.285
0.900	0.610	0.290
0.900	0.605	0.295
0.900	0.600	0.300
0.900	0.595	0.305

Fuzzy Labeled Star Graph atmost 89 edges

lv[0]	lv[i]	le[i]
0.900	0.590	0.310
0.900	0.585	0.315
0.900	0.580	0.320
0.900	0.575	0.325
0.900	0.570	0.330
0.900	0.565	0.335
0.900	0.560	0.340
0.900	0.555	0.345
0.900	0.550	0.350
0.900	0.545	0.355
0.900	0.540	0.360
0.900	0.535	0.365
0.900	0.530	0.370
0.900	0.525	0.375
0.900	0.520	0.380
0.900	0.515	0.385
0.900	0.510	0.390
0.900	0.505	0.395
0.900	0.500	0.400
0.900	0.495	0.405
0.900	0.490	0.410



C:\TURBOC3\BIN>TC
Fuzzy Labeled Star Graph atmost 89 edges

lv[0]	lv[i]	le[i]
0.900	0.485	0.415
0.900	0.480	0.420
0.900	0.475	0.425
0.900	0.470	0.430
0.900	0.465	0.435
0.900	0.460	0.440
0.900	0.455	0.445

The following be the program on fuzzy labeled bi star graph $B_{n,n}$ at most 59 edges which has the apex vertices v_1, v_2 ,apex edge $e_0=(v_1,v_2)$ pendent vertices u_i of v_1, pendent edges $e_i=(v_1,u_i)$, pendent vertices w_i of v_2, pendent edges $e_i^*=(v_2,w_i)$ whose membership functions □(v_1), □(v_2), $\mu(v_1,v_2)$,□(u_i), $\mu(v_i,u_j)$, □(w_i) and $\mu(v_2,w_i)$ represented as lv_1, lv_2, le_0, lu[i], le[i], lw[i], and le_1[i] for i=1 to n.

Program 4.2:

```
#include<stdio.h>
#include<conio.h>
void main()
{
    float i,lv[500], le[500], lu[500],lv2[500], lw[500],
le1[500];

    clrscr();
    printf(" Fuzzy Labeled Bi star Graph At most 59
Edges \n");
printf("_______________________________________________
__________\n");
    printf("lv[1]   lv[2]   le[0]    lu[i]   le[i]    lw[i]
le1[i]\n");
    lv[1]=0.8;
    lv[2]=0.9;
    le[0]=0.1;
    for(i=1;i<=59;i++)
    {
      lu[i]=((280-i)/(400));
      le[i]=((40+i)/(400));
      lw[i]=((540-i)/(600));
      le1[i]=(i/600);
      printf("% 1.3f\t % 1.3f\t %1.3f\t %1.3f\t %1.3f
\t%1.3f\t %1.3f",lv[1], lv[2], le[0],
```

```
                lu[i], le[i], lw[i],le1[i]);
          printf("\n");
        }
        getch();
    }
```

Fuzzy Labeled Bistar Graph Atmost 59 Edges

lv[1]	lv[2]	le[0]	lu[i]	le[i]	lw[i]	le1[i]
0.800	0.900	0.100	0.697	0.102	0.898	0.002
0.800	0.900	0.100	0.695	0.105	0.897	0.003
0.800	0.900	0.100	0.692	0.108	0.895	0.005
0.800	0.900	0.100	0.690	0.110	0.893	0.007
0.800	0.900	0.100	0.688	0.112	0.892	0.008
0.800	0.900	0.100	0.685	0.115	0.890	0.010
0.800	0.900	0.100	0.683	0.117	0.888	0.012
0.800	0.900	0.100	0.680	0.120	0.887	0.013
0.800	0.900	0.100	0.678	0.123	0.885	0.015
0.800	0.900	0.100	0.675	0.125	0.883	0.017
0.800	0.900	0.100	0.673	0.127	0.882	0.018
0.800	0.900	0.100	0.670	0.130	0.880	0.020
0.800	0.900	0.100	0.668	0.132	0.878	0.022
0.800	0.900	0.100	0.665	0.135	0.877	0.023
0.800	0.900	0.100	0.663	0.138	0.875	0.025
0.800	0.900	0.100	0.660	0.140	0.873	0.027
0.800	0.900	0.100	0.658	0.142	0.872	0.028
0.800	0.900	0.100	0.655	0.145	0.870	0.030
0.800	0.900	0.100	0.652	0.147	0.868	0.032
0.800	0.900	0.100	0.650	0.150	0.867	0.033

Fuzzy Labeled Bistar Graph Atmost 59 Edges

lv[1]	lv[2]	le[0]	lu[i]	le[i]	lw[i]	le1[i]
0.800	0.900	0.100	0.647	0.153	0.865	0.035
0.800	0.900	0.100	0.645	0.155	0.863	0.037
0.800	0.900	0.100	0.642	0.157	0.862	0.038
0.800	0.900	0.100	0.640	0.160	0.860	0.040
0.800	0.900	0.100	0.637	0.162	0.858	0.042
0.800	0.900	0.100	0.635	0.165	0.857	0.043
0.800	0.900	0.100	0.632	0.168	0.855	0.045
0.800	0.900	0.100	0.630	0.170	0.853	0.047
0.800	0.900	0.100	0.627	0.172	0.852	0.048
0.800	0.900	0.100	0.625	0.175	0.850	0.050
0.800	0.900	0.100	0.623	0.177	0.848	0.052
0.800	0.900	0.100	0.620	0.180	0.847	0.053
0.800	0.900	0.100	0.618	0.183	0.845	0.055
0.800	0.900	0.100	0.615	0.185	0.843	0.057
0.800	0.900	0.100	0.613	0.188	0.842	0.058
0.800	0.900	0.100	0.610	0.190	0.840	0.060
0.800	0.900	0.100	0.608	0.192	0.838	0.062
0.800	0.900	0.100	0.605	0.195	0.837	0.063
0.800	0.900	0.100	0.603	0.198	0.835	0.065
0.800	0.900	0.100	0.600	0.200	0.833	0.067

Fuzzy Labeled Bistar Graph Atmost 59 Edges

lv[1]	lv[2]	le[0]	lu[i]	le[i]	lw[i]	le1[i]
0.800	0.900	0.100	0.598	0.203	0.832	0.068
0.800	0.900	0.100	0.595	0.205	0.830	0.070
0.800	0.900	0.100	0.592	0.207	0.828	0.072
0.800	0.900	0.100	0.590	0.210	0.827	0.073
0.800	0.900	0.100	0.587	0.213	0.825	0.075
0.800	0.900	0.100	0.585	0.215	0.823	0.077
0.800	0.900	0.100	0.582	0.218	0.822	0.078
0.800	0.900	0.100	0.580	0.220	0.820	0.080
0.800	0.900	0.100	0.577	0.222	0.818	0.082
0.800	0.900	0.100	0.575	0.225	0.817	0.083
0.800	0.900	0.100	0.572	0.228	0.815	0.085
0.800	0.900	0.100	0.570	0.230	0.813	0.087
0.800	0.900	0.100	0.567	0.233	0.812	0.088
0.800	0.900	0.100	0.565	0.235	0.810	0.090
0.800	0.900	0.100	0.562	0.237	0.808	0.092
0.800	0.900	0.100	0.560	0.240	0.807	0.093
0.800	0.900	0.100	0.558	0.243	0.805	0.095
0.800	0.900	0.100	0.555	0.245	0.803	0.097
0.800	0.900	0.100	0.553	0.248	0.802	0.098

The following be the program on fuzzy labeled double star graph K1,n,n which has the apex vertex v0, internal vertices ui , internal edges and ei=(v0,vi) , pendent vertices wi and pendent edges ei*=(vi,wi) whose membership functions □(v0), □(vi), μ(v0,vi) , □(wi) and μ(vi,wi) represented as lv0, lu[i], le[i], lw[i], and le1[i] for i = 1 to n.

Program 4.3:

```
   #include<stdio.h>
   #include<conio.h>
   void main()
    {
        float i,j,lv[500],le[500],lw[500],le1[500];
         clrscr();
         printf(" Fuzzy  Double Star Graph atmost 30
edges \n");

printf("____________________________________________
___________\n");
        printf(" lv[0]  lv[i]  le[i]  lw[i]  le1[i]\n");
         lv[0]=0.8;
         for(i=1;i<=30; i++)
         {
            lv[i]=((200+i)/(400));
            le[i]=((120-i)/(400));
            lw[i]=((400+3*(i+1))/(500));
            le1[i]=((61.2+(0.7*i))/(200));
             printf(" %1.3f   %1.3f  %1.3f   %1.3f  %1.3f
",lv[0],lv[i], le[i], lw[i], le1[i]);
             printf("\n");
            }
       getch();
      }
```

Fuzzy Labeled Double Star Graph atmost 30 edges

lv[0]	lv[i]	le[i]	lw[i]	le1[i]
0.800	0.502	0.298	0.812	0.310
0.800	0.505	0.295	0.818	0.313
0.800	0.507	0.292	0.824	0.317
0.800	0.510	0.290	0.830	0.320
0.800	0.512	0.287	0.836	0.324
0.800	0.515	0.285	0.842	0.327
0.800	0.517	0.282	0.848	0.331
0.800	0.520	0.280	0.854	0.334
0.800	0.522	0.278	0.860	0.338
0.800	0.525	0.275	0.866	0.341
0.800	0.527	0.273	0.872	0.345
0.800	0.530	0.270	0.878	0.348
0.800	0.533	0.268	0.884	0.352
0.800	0.535	0.265	0.890	0.355
0.800	0.538	0.262	0.896	0.359
0.800	0.540	0.260	0.902	0.362
0.800	0.543	0.257	0.908	0.366
0.800	0.545	0.255	0.914	0.369
0.800	0.548	0.252	0.920	0.373
0.800	0.550	0.250	0.926	0.376

Fuzzy Labeled Double Star Graph atmost 30 edges

lv[0]	lv[i]	le[i]	lw[i]	le1[i]
0.800	0.553	0.248	0.932	0.380
0.800	0.555	0.245	0.938	0.383
0.800	0.558	0.243	0.944	0.387
0.800	0.560	0.240	0.950	0.390
0.800	0.562	0.237	0.956	0.394
0.800	0.565	0.235	0.962	0.397
0.800	0.567	0.233	0.968	0.400
0.800	0.570	0.230	0.974	0.404
0.800	0.572	0.228	0.980	0.407
0.800	0.575	0.225	0.986	0.411

V. CONCLUSION

We discussed fuzzy labeling of a star graph $S_{1,n}$,bi star graph $B_{n,n}$ and double star graph $K_{1,n,n}$. It has been proved that star graph at most 89 edges are fuzzy graceful iff fuzzy magic labeling. Also proved that bi star graph $B_{n,n}$ of 59 edges are fuzzy graceful iff fuzzy bi magic labeling. We have proved that fuzzy labeled double star graph $K_{1,n,n}$ at most 30 edges is fuzzy graceful. Further research work to be extended for some special graphs.

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